Supply function equilibria and carbon pricing

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Outline

- Why we should use supply function equilibria (SFE) to model electricity markets.
- What an SFE looks like and how can we calculate it.
- Using an SFE to predict changes in emissions with changes in carbon price.
- The key problems in using SFE.
Wholesale electricity markets

- We consider a real time ‘spot’ market using a pool arrangement.
- Generators make supply function offers; (stochastic) demand occurs; a price is determined; and a quantity of electricity is dispatched from each generator in order to meet demand.
- Each generator is paid the spot price for electricity it supplies (no matter what its offer price).
- Assume each player has full information, there are no network effects, and the game is played just once.
Generator offers

- Combined offer stack
  - Clearing price
  - Demand
  - Quantity
  - Price
Supply function equilibria

- Need to understand the equilibrium behaviour as an important guide to actual market behaviour.
- Using Cournot models is like insisting on very specific L-shaped supply functions and gives too much of a role to elastic demand.
- But finding equilibria is hard. Numerical solutions using ODE solvers are problematic.
- Equilibria may not exist or they may not be unique.
Basic model

- \( n \) suppliers: cost functions \( c_i(x) \) (convex, smooth).
- Supplier \( i \) has maximum capacity \( \bar{q}_i \)
- Demand is \( D(p, \varepsilon) = D(p) + \varepsilon \) where \( D(p) \) is strictly decreasing, smooth and concave.
- Demand shock \( \varepsilon \) has density function \( f \) with \( f(x) > 0 \) for \( x \) between \( \varepsilon_{\min} \) and \( \varepsilon_{\max} \)
- Supply functions are non-negative, non-decreasing and piecewise continuous.
- \( \pi_i(p, \varepsilon) = pq_i - c_i(q_i), \quad q_i = D(p) + \varepsilon - \sum_{j \neq i} s_j(p) \)
Strong supply function equilibrium

- On each residual demand curve find the best response subject to constraints on price and quantity. Join these points for an optimal response.
- Strong SFE when this is monotonic (for each firm).
Optimality conditions at a smooth point

(Klemperer and Meyer)

\[ s_i(p) = \left[p - c'_i(s_i(p))\right]\left[\sum_{j \neq i} s'_j(p) - D'(p)\right], \quad i = 1, 2, ..., n \]

From this we can deduce:

\[ p \leq c'_i(0) \text{ if and only if } s_i(p) = 0 \]

Suppliers always start supplying at their marginal cost
Conditions for a discontinuity

- **Theorem:**
  
  In a strong SFE each supply function is smooth except at prices \( p = c_j'(0) \) for \( j=1,2,\ldots, n \), where supply function \( s_i(p) \) may have a discontinuity, but only if \( i \) is the only supplier which is unconstrained (so neither at zero nor its capacity) at this point.
Multiple equilibria appear at a jump

- We can prove that the family of possible equilibria are ordered
Numerical solutions for an SFE

\[ s_i(p) = [p - c_i'(s_i(p))] \left[ \sum_{j \neq i} s_j'(p) - D'(p) \right], \quad i = 1, 2, \ldots, n \]

Solve a set of nonlinear equations linking values and slopes at \( p_k \)

Intersection points need to be between successive \( p_k \) values
Example: multiple solutions without capacity constraints
Capacity bounds pick out just one equilibrium
Example: Unique solution with capacity constraint

- Coal generator 300 MW, \( c_1(x) = 12x + 0.06x^2 \)
- Gas generator 700 MW, \( c_2(x) = 20x + 0.08x^2 \)
Actual bids for Eraring, NSW

- 2 September 2008. Four 660 MW generators
Supply functions compared with marginal cost bidding
Calculating emissions

- The demand is $350 + \varepsilon - 10p$ where the random shock $\varepsilon$ is uniformly distributed between 0 and 500
- Coal = 0.24 tons CO$_2$ per GJ; Gas = 0.12 tons
High carbon prices reverse merit order

- What happens with a carbon price of 23 euros?
Emissions as a function of carbon price

SFE gives less emissions as prices are higher, but reduction in emissions also happens more gradually.
A hidden problem

- For higher carbon prices (after the merit order reverses) the numerical method fails to find an exact optimal solution.
- Explore what happens with fixed marginal costs where we can solve the ODE’s exactly.
- Assume demand is $D(p) = A - bp + \epsilon$.
- The differential equations are:

$$s_1(p) = (p - c_1)(s'_2(p) + b)$$

$$s_2(p) = (p - c_2)(s'_1(p) + b)$$

This section of the curve ends up vertical and COINPOPT is unable to solve the appropriate non-linear equation set exactly.
Exact solution

- After putting in boundary condition $s_2(c_2) = 0$ we get

\[
\begin{align*}
    s_1(p) &= -(p - c_1) \log(p - c_1) + b(c_2 - c_1) + C_1(p - c_1) \\
    s_2(p) &= -(p - c_2) \log(p - c_1) + C_1(p - c_2)
\end{align*}
\]
An example with no SFE

- When $c_1 = 20$, $c_2 = 30$, $\bar{q}_2 = 300$, $b = 10$
When best responses decrease

- The supply function tracks best response to different demand outcomes
When best responses decrease

- We need to offer a horizontal segment, with every feasible deviation from it reducing the profit.

The right choice depends on the distribution of demand. It is no longer a strong SFE.
An example with no SFE

- When $c_1 = 20$, $c_2 = 30$, $\bar{q}_2 = 300$, $b = 10$

Also it seems that there is no mixed strategy equilibrium!

Flattening the curve here forces a different response and leads away from equilibrium!
Three problems with SFE: which is worst?

- Computational difficulties make SFE hard to work with.
  In practice computational difficulties are a signal that an SFE does not exist.
- There may be multiple SFE – which one should be chosen?
  Large demand uncertainties (or fixed offers) that are sufficient to remove multiple SFE may not occur in practice
- An SFE may not exist.
  This is much more common than we previously thought
Conclusions

- Supply function equilibria with capacity constraints are a natural way to model market power in electricity markets.
- This approach shows that merit order predictions will be misleading for emissions: deep cuts will need higher carbon prices.
- Recent work has helped in computing SFE but has thrown up significant problems with non-existence.
For more details on the form of SFE and how to calculate them: