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Modelling the costs of energy crops: A case study of U.S. corn and Brazilian sugar cane

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Aurélie Méjean, Chris Hope

Abstract

High crude oil prices, uncertainties about the consequences of climate change and the eventual decline of conventional oil production raise the prospects of alternative fuels, such as biofuels. This paper describes a simple probabilistic model of the costs of energy crops, drawing on the user's degree of belief about a series of parameters as an input. This forward-looking analysis quantifies the effects of production constraints and experience on the costs of corn and sugar cane, which can then be converted to bioethanol. Land is a limited and heterogeneous resource: the crop cost model builds on the marginal land suitability, which is assumed to decrease as more land is taken into production, driving down the marginal crop yield. Also, the maximum achievable yield is increased over time by technological change, while the yield gap between the actual yield and the maximum yield decreases through improved management practices. The results show large uncertainties in the future costs of producing corn and sugar cane, with a 90% confidence interval of 2.9 to 7.2 $/GJ in 2030 for marginal corn costs, and 1.5 to 2.5 $/GJ in 2030 for marginal sugar cane costs. The influence of each parameter on these costs is examined.

Keywords Biofuels; Uncertainty; Experience; Land suitability

JEL Classification Q42; Q47; Q55

Contact a.mejean@jbs.cam.ac.uk
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Modelling the costs of energy crops: A case study of U.S. corn and Brazilian sugar cane

Aurélie Méjean [1]

Chris Hope

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Introduction

There are growing concerns about whether a petroleum-based economy can be sustained in the coming decades, (Greene et al., 2005). Highly uncertain and potentially high crude oil prices, uncertainties about the consequences of climate change and the eventual depletion of conventional oil resources raise the prospects of alternative fuels, such as non-conventional oil and biofuels, (Farrell and Brandt, 2006). In particular, sugar crops like corn and sugar cane can be converted to bioethanol, a substitute for petrol. This paper describes a simple probabilistic model for projecting the costs of supplying U.S. corn and Brazilian sugar cane.

Climate change is a "serious and urgent issue" (Stern, 2006). The transport sector is the fastest growing source of CO_2 emissions in Annex I countries[2] and remains fundamentally dependent upon petroleum (Grubb, 2001 and UNFCCC, 2005). These anthropogenic CO_2 emissions accumulate in the atmosphere, leading to enhanced greenhouse effects and climate change. There are large uncertainties associated with this issue, from the scale of the impacts of climate change to the costs of mitigation (Stern, 2007 p33), but there is a growing consensus that this is an issue that the oil industry cannot ignore (Browne, 2006). With growing concerns about climate change, its social and economic consequences and the decline of conventional oil production (starting with non-OPEC oil supplies, see for instance IEA, 2007), the choice for solving the problem

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[1] Corresponding Author – Judge Business School, Trumpington Street, Cambridge, CB2 1AG, UK Tel: +44 (0) 7910245908. *E-mail address*: a.mejean@jbs.cam.ac.uk,
[2] Annex I Parties include the industrialised countries that were members of the OECD in 1992, plus countries with economies in transition (the EIT Parties), including the Russian Federation, the Baltic States, and several Central and Eastern European States, (UNFCCC, 2007).

of energy supply for transport could lie with lower-carbon alternatives like biofuels, (IEA, 2004 p171, IPCC, 2007 p10-13).

The role of technological change and learning has been well studied for low-carbon and other energy technologies (see for instance Grübler et al., 1999 and McDonald and Schrattenholzer, 2001). As is the case for most emerging technologies, the cost reduction resulting from experience or cumulative production is an argument in favour of investing in new, less carbon intensive energy technologies. Growing importance has been given to the role of learning curves in modelling as a way to "identify technologies that might become competitive with adequate investment" (Grübler et al., 1999). As stated in Grubb (2001), the study carried out by Grübler et al. (1999) shows that "innovation in renewable energy sources potentially makes them competitive compared to long-term fossil fuel resources as the conventional cheap petroleum resources deplete". Developing accurate experience curves for biofuels is essential for calculating their potential competitive position against other alternatives, like non-conventional oil.

- **Theoretical framework**

Decision theory, uncertainty and subjective probabilities

Decision theory is "designed to help a decision maker choose among a set of alternatives in light of their possible consequences"; each alternative is associated with one or more probability distributions (Web Dictionary of Cybernetics and Systems, 2007).

One approach to measure the uncertainty of events is to use subjective probabilities that are based on reasonable assessments by experts. Those probabilities are subjective as they depend on the subject making the judgements, (Lindley, 1985 p20). Bayesian theory uses these probabilities to represent the degree of belief of a subject. According to Lindley, probabilities are assumed to express a relationship between a person and the world. In practice, two observers may assign different probabilities to the same event and Lindley suggests that this difference arises due to different levels of information available to the observers.

The aim here is to express our uncertainty about the future costs of supplying alternative fuels. Numerical modelling is used as a tool to help decision-making: a model is introduced that draws on the user's degree of belief about a series of parameters as an input (for another example, see Hope, 2006). A probability distribution is assigned to these parameters and the basis of these probabilities is "up-to-date knowledge from science and economics", (Stern, 2006 p33). The uncertainty associated with the input data is examined, together with the influence of each parameter on the output.

Biofuel resources

Biofuels are "transportation fuels derived from biological sources", (IEA, 2004 p27). They can be liquid (such as bioethanol or biodiesel) or gaseous (such as biogas or hydrogen). Biofuels can be produced from crop sources (either food crops or non-food crops) and non-crop sources (e.g. forestry residues, industrial waste), (IEA, 2004 p123). Bioethanol is produced by fermentation of sugars found in a variety of feedstock. There are currently three main feedstock types for ethanol production: sugarcane or sugar beet, grains such as wheat or corn, and lignocellulosic materials such as wood and straw from agriculture and forest residues, (IEA, 2004 p34).

First generation biofuels are made from food crops (Shell, 2007). The production of first generation bioethanol mainly occurs in Brazil where it is made from sugarcane, and the U.S. where the feedstock used is corn. A major issue concerning the use of food crops for biofuel production is land availability. The area of land required to produce biofuels depends on crop yields and conversion yields from crop input. Large-scale biofuel production from food crops would dramatically reduce the area of land available for food production, (IEA, 2004 p124). In practice, land requirement puts an upper limit on the potential production capacity of first generation biofuels. This paper models the costs of the energy crops used to produce first generation bioethanol. The costs of producing bioethanol will be addressed in future work.

Learning and technological change

Experience curves are a powerful tool for energy policy making, they are used to "estimate technical change as a result of innovative activities", (Jamasb, 2007 p54). They give an indication of the investments that are needed to make a technology competitive, (IEA, 2000). Experience curves are usually described by the following mathematical expression:

$$C = C_0 \cdot \left(\frac{X}{X_0}\right)^{-b}$$

(1)

with C = unit costs
C_0 = initial unit costs
X = cumulative production
X_0 = initial cumulative production
b = experience curve parameter or learning coefficient, $b \geq 0$.

The figure below is an illustration of decreasing costs through accumulated experience:

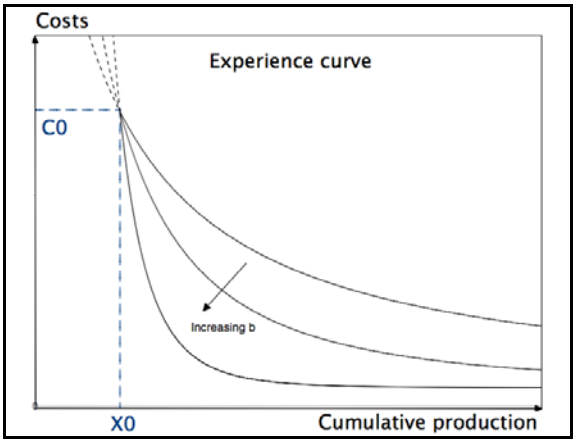


Figure 1 - Experience curve

The experience curve parameter b characterises the slope of the curve, (IEA, 2000). The learning rate (LR) is a parameter that expresses the rate at which costs decrease each time cumulative production doubles, and is given by: $LR = 1 - 2^{-b}$.

Land resources

Land is a limited and heterogeneous resource. In classical economics, rent is "the income derived from the ownership of land and other natural resources in fixed supply", (Britannica, 2008). As Ricardo explains:

> "If all land had the same properties, if it were unlimited in quantity, and uniform in quality, no charge could be made for its use, unless where it possessed peculiar advantages of situation. It is only, then, because land is not unlimited in quantity and uniform in quality, and because in the progress of population, land of an inferior quality, or less advantageously situated, is called into cultivation, that rent is ever paid for the use of it. When in the progress of society, land of the second degree of fertility is taken into cultivation, rent immediately commences on that of the first quality, and the amount of that rent will depend on the difference in the quality of these two portions of land", (Ricardo, 1817).

The need to use less suitable land should therefore be taken into account when assessing the prospects for the costs of supplying biofuels. Land is heterogeneous and of limited supply, and it is economically rational to use the low cost, high quality resources first. If we assume that crop production costs are negatively correlated to the suitability of land, it follows that the most suitable land will be first taken into production.

By definition, rent is the difference between total costs and total revenue, and "competition for land ensures that the landowner gets the excess of total revenue over total cost", (O'Sullivan, 2005 p18). If we assume that the price of crops is determined by the costs of production on marginal land (Friedman, 1998),

suitable land will show higher rent than land with lower productivity, as shown below. The scarcity of suitable land, which leads to the existence of rent, will affect the costs of producing on marginal land, (Friedman, 1998).

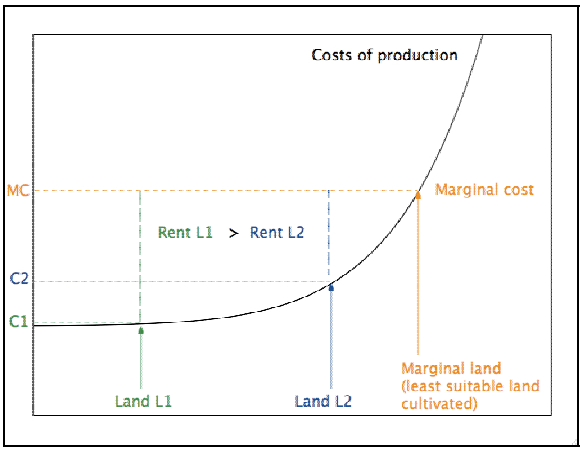


Figure 2 - Land rent

The least suitable land, also called marginal land (in orange on Figure 2 and Figure 3), earns no rent. Moreover, the rent of the most suitable land increases as more and more land is brought into production, as shown in Figure 3 below:

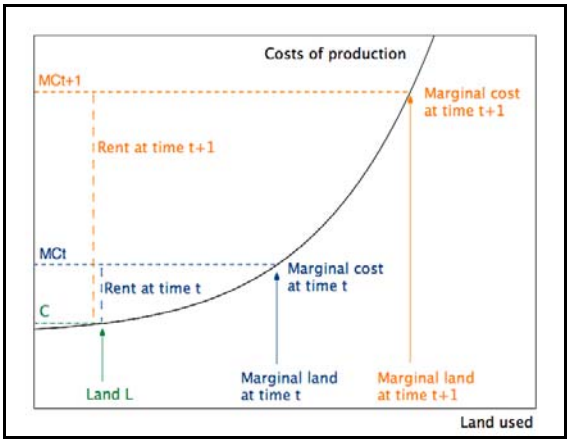


Figure 3 - Land rent over time

The approach taken by modellers is to try to reflect how the marginal productivity of land could evolve as more land is brought into production, and under a given state of knowledge. Van Meij et al. (2006), Eickhout et al. (2008) and Bakkes et al. (2008) introduce land productivity curves into IMAGE showing crop productivity as a function of cumulative land (cf. Appendix I.1).

Our model focuses on the marginal cost of producing crops, i.e. the cost of producing crops on the marginal hectare of land at a given time. The marginal production cost is more broadly relevant as it will reflect the costs faced by land-renting farmers on every type of land under cultivation at that time. These farmers will encounter the specific cost of production associated with the suitability of the land they cultivate, which by definition will be lower than the

marginal cost on the least suitable hectare of land, plus the rent owed to the landowner, i.e. the difference between the marginal cost at that time and their specific costs of production. In total, every crop-producing farmer will therefore face the cost of producing crops on the marginal hectare of land. With a given state of knowledge and experience, every farmer will thus see increasing costs of production (cf. Figure 3).

To conclude, both technological advances and the limited supply of land are driving the supply of crops and both need to be taken into account to forecast future crop costs.

- **Research design**

Methodology

This is a forward-looking analysis of the upstream liquid fuel industry, which describes the effects of both learning and production constraints on the costs of supplying energy crops.

Achievable yield

Equation (2) summarises the maximum yield model for first generation crops and builds on Equation (c) from Appendix I.1. A constant is added to the equation as the marginal yield is not necessarily zero when all agricultural land is used. The achievable marginal yield is a decreasing function of the area of land Q in cultivation, as the most suitable land is used first:

$$Y_{MAX} = Y_{\min} + (Y_{initial} - Y_{\min}) \cdot \left(1 - \frac{Q}{Q_T}\right)^{\gamma} \qquad (2)$$

With Q = area of land used for energy crop production (L in equation c)
Y_{MAX} = maximum achievable marginal yield
Q_T = total potentially available agricultural land (a in equation c)
$Y_{initial}$ = initial value of the maximum achievable marginal yield (most suitable hectare of land)
Y_{min} = minimum value of the achievable yield (least suitable hectare of land)
γ = exponent of the land productivity curve (1/p in equation c)

The exponent of the land productivity curve γ defines the pace at which land use is driving down the marginal achievable yield. We also have $Y_{MAX}(Q=0) = Y_{initial}$ and $Y_{MAX}(Q=Q_T) = Y_{\min}$. An illustration of the land productivity curve, i.e. the maximum yield as a function of the share of land used, is shown below.

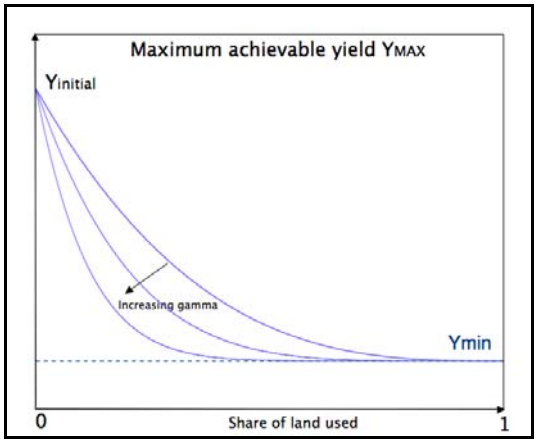


Figure 4 - Maximum achievable yield - illustration

The maximum yield will also benefit from developments in biotechnology. We assume that the maximum value of the achievable yield ($Y_{initial}$), on the most suitable hectare of land, will benefit from technological developments. $Y_{initial}$ is the value of the maximum achievable yield Y_{MAX} when the share of land used is zero, it is the starting point of the maximum yield curve. $Y_{initial}$ will increase with cumulative production according to the following equation:

$$Y_{initial} = Y_{initial\,0} \cdot \left(\frac{X}{X_0}\right)^{-b_{YMAX}}$$

(3)

with $Y_{initial0}$ = initial value of the initial maximum yield
X = cumulative production
X_0 = initial cumulative production
b_{YMAX} = learning coefficient, associated with the learning rate LR_{YMAX}

The minimum value of the achievable yield (Y_{min}), i.e. on the least suitable hectare of land, will also benefit from these developments, the difference $Y_{initial}$ - Y_{min} is therefore assumed to be constant, and equal to $Y_{initial0}$ - Y_{min0}, with Y_{min0} the initial value of the minimum achievable yield Y_{min}.

Actual yield

The gap between the actual marginal yield and the maximum achievable marginal yield is the yield gap, which decreases with cumulative production, through improved production technologies and management practice. We define the yield gap g as follows. The yield gap comes closer to zero with experience (or cumulative production):

$$g = g_0 \cdot \left(\frac{X}{X_0}\right)^{-b_{Yactual}} \quad , \quad g = 1 - \frac{Y_{actual}}{Y_{MAX}}$$

(4)

With Y_{MAX} = maximum achievable marginal yield
Y_{actual} = actual marginal yield
X = cumulative production

X_0 = initial cumulative production
g = yield gap
g_0 = initial yield gap
$b_{Yactual}$ = learning coefficient, associated with the learning rate $LR_{Yactual}$

Figure 5 illustrates how the yield gap between the maximum achievable marginal yield and the actual marginal yield decreases with cumulative production.

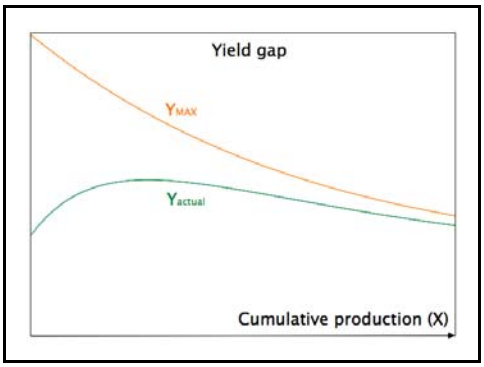


Figure 5 - Yield gap

Marginal costs

Marginal costs are primarily driven by the actual marginal yield. We define the marginal costs of producing the crops as the inputs, in US$ per hectare, divided by the actual marginal yield in GJ per hectare.

$$C = \frac{I}{Y_{actual}} \quad (5)$$

With C = marginal costs of producing crops
I = input, i.e. the amount of capital and labour used per hectare of land
Y_{actual} = actual marginal yield

While marginal yields are assumed to increased through induced technical change (cf. equations 6 and 7), the input I is assumed to be decreasing as a result of generalised (autonomous) technical progress in the economy: the level of input needed to obtain the same level of output is assumed to decrease over time, see (Grubb et al., 2002) for a description of induced vs. autonomous technical change. So, if we assume a constant marginal yield (e.g. 1 GJ/ha), the amount of capital and labour that is needed to produce that GJ will decrease with time as the economy performs better. Accordingly, we introduce the following form for the input I:

$$I = I_0 \cdot e^{-\alpha t} \quad (6)$$

with I_0 = initial input
α = rate of technical progress
t = time

For instance, as new fertilisers become cheaper as the economy performs better, the capital needed to produce one GJ/ha decreases. Because the mechanism

allowing for the lower fertiliser costs is not directly linked to the corn and sugar cane industries, but rather to the performance of the economy as a whole, the input variable is chosen to be autonomous rather than induced for simplification purposes. The approach to include both induced (as it is the case here for the yields) and autonomous technical change is standard practice. This approach was used for instance in the MERGE model (Manne and Richels, 2004 p4,9), where both learning-by-doing and autonomous improvements in the productivity of labour and energy were incorporated.

The initial input I_0 is derived from the costs (C_0) and actual marginal yield ($Y_{actual0}$) at time t_0:

$$I_0 = C_0 \cdot Y_{actual0}$$

(7)

Crop production and land use

The cumulative production (X) and the amount of land used (Q) are linked. The production rate (x) and the actual marginal yield (Y_{actual}) will determine the area of land that is needed to meet demand. More precisely, the production rate is the sum of the yield over the whole spectrum of land, as described in the following equation:

$$x(Q) = \int_{u=0}^{u=Q} Y_{actual}(q)dq$$

(8)

The production rate is illustrated below as the shaded area shown in blue:

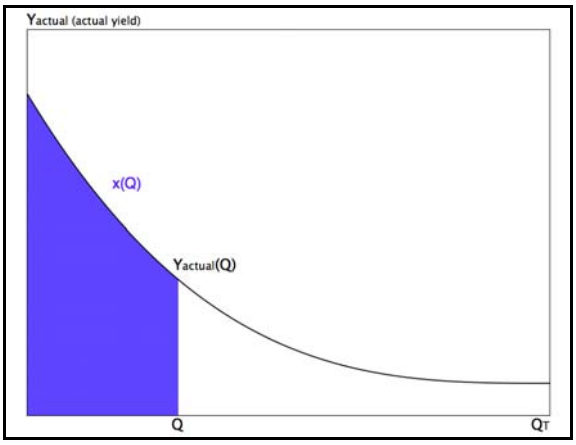


Figure 6 - Production rate and share of land used

This part of the model is not entirely satisfactory, as the production rate is determined exogenously:

$$x_{t+1} = x_t \cdot (1+d)$$

(9)

With d = rate of increase in demand (no unit)
x = production rate (GJ per year)

In practice, the production rate will depend on ethanol and petrol prices, which in turn can be influenced by the production costs, and this feedback loop should be a matter for further research.

Illustrative use of the model

The model aims at calculating the cost of energy crops, and focuses on U.S. corn and Brazilian sugar cane. In the first approximation, a triangular distribution is assigned to each parameter. Each distribution is defined by a minimum, a maximum and a most likely value. The direction of the skew of the triangular distribution is set by the size of the most likely value relative to the minimum and the maximum, (Palisade, 2007). A literature review is conducted in order to define the ranges of estimates associated with each parameter. The following parameters are used in the model:

Parameters - definitions	
Land resources and yields	
Q_T: Total agricultural land available	$Y_{initial0}$: Initial maximum crop yield
Q_0: Initial land used	Y_{min0}: Initial minimum crop yield
$Y_{actual0}$: Marginal crop yield at $Q = Q_0$	
Learning and technical progress	
LR_{YMAX}: Maximum achievable yield: learning rate	$LR_{Yactual}$: Marginal crop yield: learning rate
α: Generalised technical progress	
Costs	
C_0: Initial crop costs	
Production and demand	
d: Rate of increase in demand	X_0: Initial cumulative crop production

Table 1 - Parameters: definitions

Land resources and yields parameters

a) a. Land areas Q_0 and Q_T

In 2002, IIASA and the FAO produced a major report on global agro-ecological zones (GAEZ), which described the areas of very suitable (VS), suitable (S), moderately suitable (MS) and marginally suitable (mS) land for agricultural production.

The very suitable to marginally suitable area lies between 1.09E+08 and 2.69E+08 ha for corn production in the U.S. and 3.9E+07 and 3.9E+08 ha for sugarcane production in Brazil, depending on the levels of input and irrigation. GAEZ estimates of land suitability don't account for competing land uses. In total, 7% of the gross potential arable land (rainfed) in North America consists of settlements and protected land, (FAO, 2000 p39). We assume that this percentage is applicable to the gross potential arable land of the USA of 3.5E+08 ha (FAO, 2009d), i.e. 0.25E+08 ha are used for settlements and protected area in

the USA. The range of Q_T for U.S. corn is thus chosen as 1.09E+08 - 2.45E+08 ha, i.e. 11 to 24% of total country area, (FAO, 2009c). In 2005, the harvested area for U.S. corn totalled 3.04E+07 ha (FAO, 2009a), which gives a share of suitable land used between 0.12 and 0.27. In the case of South and Central America, 6.5% of the gross potential arable land (rainfed) consists of settlements and protected land, (FAO, 2000 p39). We assume that this percentage is applicable to the gross potential arable land of Brazil of 5.5E+08 ha (FAO, 2009d), i.e. 0.4E+08 ha are used for settlements and protected area in Brazil. This estimate doesn't take into account the fact that some suitable land for sugar cane production might be occupied by forest. Following Young (1999, p15), we assume that 10 to 20% of cultivable land would be occupied by forest that should be preserved, i.e. 0.4E+08 to 0.8E+08 ha, which gives 3.1E+08 as the upper bound of the range of Q_T for Brazilian sugar cane. The range for Q_T is thus chosen as 3.9E+07 - 3.1E+08 ha for sugar cane production in Brazil, i.e. 5 to 36% of total country area, (FAO, 2009c). In 2005, the harvested area for Brazilian sugar cane totalled 5.8E+06 ha (FAO, 2009b), which gives a share of suitable land used between 0.02 and 0.15.

b) b. Exponent of the land productivity curve, γ

In (Eickhout et al., 2006 p67), land productivities are expressed "on a relative scale between 0 and 1 on the basis of the potential crop productivity". The exponent of the land productivity curve is calculated using the simple model described in equation (11). The simplest approach is to consider the logarithmic form of equation (5):

$$Ln(Y_{MAX} - Y_{\min}) = Ln(Y_{initial} - Y_{\min}) + \gamma \cdot Ln\left(1 - \frac{Q}{Q_T}\right)$$

(12) The value of γ is thus calculated from the following parameters: Y_{min0}, $Y_{initial0}$, Y_{MAX0}, Q_0 and Q_T.

c) Yields

Initial maximum crop yield $Y_{initial0}$ at time t_0

We define the maximum achievable crop yield as the yield, in metric ton per ha, that could be reached on a given plot of land with optimal climatic conditions, using the best crop variety available, and assuming the full recovery of dry matter during harvest. The terms "theoretical yield potential" (Tollenaar, 1983), "yield potential" (Evans and Fisher, 1999), "biological maximum grain yield" (Fisher and Palmer, 1983) and "potential crop productivity" (Eickhout et al., 2006) are used in the literature. We assume that these all refer to the maximum achievable crop yield defined above and used in the model.

The potential crop productivity used in (Eickhout et al., 2006) for corn in temperate areas is 24.4 ton/ha, (Stehfest, 2008). This value is obtained from the IMAGE potential crop productivity module, which was based on an earlier version of the GAEZ model developed by IIASA and FAO. Tollenaar (1983) estimates at 25 ton/ha (dry matter) the theoretical yield potential of corn in North America. The following table summarises the estimates of maximum achievable corn yield found in the literature.

Grain yield (dry weight, t/ha)	**Yield** (low, GJ/ha)	**Yield** (high, GJ/ha)	**Region**	**Source**
28	462	496	World	Fisher and Palmer 1983 p162
31	512	549	World	Fisher and Palmer 1983 p162
24.4	403	432	World	Stehfest 2008
25	413	443	U.S.	Tollenaar 1983, 2002

Table 2 - Maximum achievable yields: corn

These values are above the maximum corn yields actually occurring in very suitable land in the U.S., which are between 12.7 and 17.5 ton/ha (high inputs), (IIASA, 2002). The gross and net heating values of corn (whole crop) used in the above table are 17.7 and 16.5 GJ/ton respectively, (BIOBIB, 2008). The range for $Y_{initial0}$ for U.S. corn is thus chosen as 24.4 – 31 ton/ha (dry matter), i.e. 400 – 550 GJ/ha.

The following table summarises the estimates of maximum achievable sugar cane yields found in the literature for some regions with good to very high suitability for rain-fed and irrigated sugar crops (IIASA, 2002).

Cane yield (fresh weight, t/ha)	**Stalk yield** (dry weight, t/ha)	**Yield** (low, GJ/ha)	**Yield** (high, GJ/ha)	**Region**	**Source**
250	75*	1298	1335	Australia	Irvine 1983 p372
219	66*	1137	1169	Colombia	Irvine 1983 p372
242	73*	1256	1292	Louisiana	Irvine 1983 p372
200	60*	1038	1068	Zimbabwe	Irvine 1983 p372
200	60*	1038	1068	Brazil	Janick 2002
218	65**	1126	1159	World	Stehfest 2008

*Experimental maximum; **Potential productivity

Table 3 - Maximum achievable yields: sugar cane

The stalk yield (dry weight) is obtained by multiplying the cane yield (fresh weight) by 0.3, as presented in (Irvine, 1983 p372). The dry stalks contains from 42 to 68% of saccharose, and 32 to 58% of fibre, (CGEE and BNDES, 2008 p68). The heating values of saccharose and fibre are 16.5 GJ/ton and 19.2 GJ/ton respectively, (Vieira da Rosa, 2005), which gives a heating value for dry stalk of 17.3 - 17.8 GJ/ton (which gives the low and high yield values in the above table). The range for $Y_{initial0}$ is thus chosen as 60 – 75 ton/ha (dry matter), or 1000 – 1350 GJ/ha. These values are above IIASA estimates for maximum sugar cane yields actually occurring in very suitable land in Brazil, which are between 17.2 and 23.3 ton/ha (dry matter), (IIASA, 2002), and above the maximum sugar cane yield that occurred in Brazil between 1975 and 2005: 81 – 178 ton/ha (fresh weight), i.e. 24 - 54 ton/ha (dry matter), (IPEADATA, 2008).

Minimum crop yield Y_{min0} at time t_0

We estimate the yield occurring in the least suitable part of the marginally suitable land by assuming that the yields in marginally suitable land have a

similar variation to those occurring on very suitable land. The average values for corn yield on marginally suitable land in the U.S and the estimates of the minimum corn yield on marginal land are presented in the appendix. The range for parameter Y_{min0} is chosen as: 3 – 26 GJ/ha. Following the same methodology, the range for parameter Y_{min0} for Brazilian sugar cane is chosen as: 28 – 77 GJ/ha (cf. appendix). These values are consistent with CGEE (2008 p218) estimates of biomass productivity on marginal lands between 2 and 5 ton/ha (dry matter), i.e. 35 - 78 GJ/ha (with a heating value of 17.5 GJ/ton).

Average crop yield ($Y_{average0}$), yield gap g_0, marginal crop yield ($Y_{actual0}$) and maximum achievable yield (Y_{MAX0}) at time t_0

The average U.S. corn yield was between 133 and 145 GJ/ha (9-10 ton/ha, 15.5% moisture) between 2004 and 2006 (FAO, 2009a).

The yield gap of corn is calculated on the basis of a "biological maximum grain yield" for corn of 31 ton/ha (dry) in 1983, as estimated by Fisher and Palmer (p162), i.e. 36.7 ton/ha (wet).

Year	State	Average Yield (ton/ha)	Maximum yield (ton/ha)	Yield gap (no unit)
1980	California	9	10	0.71
1985	Oregon	10	11	0.68
1990	Washington	11	13	0.65
1995	Washington	12	14	0.61
2000	Arizona	12	13	0.61
2005	Washington	13	15	0.58

Source: USDA, 2009a (wet basis)

Table 4 – Corn yield gap in U.S. 1980 – 2005

These values can be compared to the yield gap between the initial maximum achievable yield, $Y_{initial0}$, and the average yield $Y_{average0}$, obtained by using equation (7): 0.63 – 0.76. We use $Y_{average0}$ instead of $Y_{actual0}$ in the calculation of the yield gap because the range for $Y_{actual0}$ is not known at this stage. This leads to a slight underestimate of the yield gap, as $Y_{actual0}$ should be lower than $Y_{average0}$. The range for the U.S. corn yield gap is chosen as 0.6 – 0.76.

The marginal crop yield is not observable, so the corresponding range of $Y_{actual0}$ is obtained from $Y_{average0}$:

$$Y_{average0} = \frac{1}{Q_0} \int_{q=0}^{q=Q_0} Y_{actual}(q) \cdot dq \qquad (13)$$

Equation (13) is solved numerically: $Y_{actual0}$ is set so that $Y_{average0}$ calculated from the model matches the range obtained from the literature, which gives $Y_{actual0}$ = 115-135 GJ/ha. According to the amount of land used for corn production in 2005 and GAEZ data, 'suitable' land was the marginal land used in 2005 to grow corn in the U.S. The calculated range is compatible with corn yields in suitable land in the U.S. from (IIASA, 2002): 100 – 140 GJ/ha (with high inputs and heating value of corn 17 GJ/ton (dry)).

Y_{MAX0} is calculated from the most likely value of the yield gap g_0 (0.68) and the range assigned to the marginal crop yield $Y_{actual0}$ at time t_0: Y_{MAX0} = 359 – 422 GJ/ha.

The average Brazilian sugar cane yield was between 371 and 383 GJ/ha (72-75 ton/ha, fresh weight) between 2004 and 2006 (IPEADATA, 2009). The yield gap of Brazilian sugar cane is calculated from the initial maximum yield, $Y_{initial0}$ (1000-1350 GJ/ha) and the average yield, $Y_{average0}$ (371-383 GJ/ha), which gives g_0: 0.61 – 0.72.

The corresponding range of $Y_{actual0}$ is 345 - 368 GJ/ha. According to the amount of land used for sugar cane production in 2005 and GAEZ data, 'very suitable' land was the marginal land used in 2005 to grow sugar cane in Brazil. The actual yield $Y_{actual0}$ obtained is of the same order as maximum occurring sugar cane yields in very suitable land in Brazil from (IIASA, 2002), ranging from 306 to 415 GJ/ha.
Y_{MAX0} is calculated from the most likely value of the yield gap g_0 (0.67) and the range assigned to the marginal crop yield $Y_{actual0}$ for Brazilian sugar cane at time t_0: Y_{MAX0} = 1030 – 1099 GJ/ha.

▪ Production and demand parameters

According to the data gathered from the FAO (2009a), the cumulative production of corn in the U.S. between 1961 and 2005 totalled about 1.16E+11 GJ. The USDA (2008) gives slightly different numbers for U.S. cumulative corn production in 2005, as their data goes back to 1866. The cumulative production in year 2005 is estimated at about 1.94E+11 GJ, which is thus chosen as the upper bound of the range for X_0 for U.S. corn. According to the FAO (2009b), the cumulative production of sugar cane in Brazil between 1961 and 2000 totalled 1.0E+10 tonnes, i.e. about 1.75E+11 GJ. Van den Wall Bake et al. estimate the cumulative production between 1941 and 1974 at 1.11E+9 tonnes (2009, p7), and van den Wall Bake estimates the cumulative production between 1974 and 2004 at about 7.7E+9 tonnes (2006, p72), i.e. about 1.34E+11 GJ. The range for X_0 is thus chosen as 1.30E+11 – 1.75E+11 GJ for Brazilian sugar cane. The following table summarises the demand growth rates for all crops and livestock products for industrial countries and Latin America and Caribbean countries, in percent par annum, as estimated by the FAO.

Period	**Growth rate** (percent per annum)		**Source**
	Industrial countries	**LA &C**	
1999-01 to 2030	0.7	1.8	FAO 2006 p33
2015 to 2030	0.4	1.0	FAO 2006 p33

Table 5 – Demand growth rates, all crops and livestock products, industrial and Latin American & Caribbean countries

The USDA (2009b p33) projects that future corn annual production rate will reach 5.2E+09 GJ (1.45E+10 bushels) in 2018. This corresponds to a demand

growth rate of 0.074 between 2005 and 2018. The range of the demand parameter is thus set at 0.004 – 0.074 in the U.S. corn model. Van den Wall Bake et al. (2009) assume 8% annual growth of sugar cane production between 2005 and 2020 in their continued growth scenario. The range of the demand parameter is thus set at 0.01 – 0.08 in the Brazilian sugar cane model.

Costs and inputs

C_0 is the cost of supplying energy crops at time t_0. The table below summarises the values found in the literature for corn and sugar cane production costs. Unless stated otherwise, costs and prices are expressed in 2005 US$ throughout this paper.

Original data			In the model	
Feedstock costs	**Unit**	**Source**	**Feedstock costs**	**Unit**
Corn				
1.08 – 2.98	US$/bu	USDA, 2006 p6	2.9 – 8.1[a]	2001 US$/GJ
1.94 – 3.24	US$/bu	NREL, 2000 p18	5.3 – 8.8[a]	2001 US$/GJ
0.21	US$/litre EtOH	F.O. Lichts, 2003 in (IEA, 2004 p72)	4.9 – 5.5[b]	US$/GJ
0.23	US$/litre EtOH	F.O. Lichts, 2003 in (IEA, 2004 p72)	5.4 – 6.0[b]	US$/GJ
Sugar cane				
13-15.6*	2005 US$/ton	van den Wall Bake et al., 2009 p7, 13	2.4 - 3.0[c] (2000-2004)	2005 US$/GJ
0.127	1990 US$/litre	C&T Brazil (2002) in (IEA, 2004 p75)	2.2[d] (1990)	2005 US$/GJ
33.16*	2005 R$/ton	Unicamp in (Hourcade et al., 2008 p27)	2.56[e]	2005 US$/GJ

[a]With an assumed moisture content of 15.5% (market standard, Hellevang, 1995): 1 U.S. bushel of corn = 25.4 kg (wet basis). Heating value of dry corn = 16.5-17.7 MJ/kg (BIOBIB, 2008).
[b]With ethanol energy content = 21.1-23.4MJ/litre (BFIN, 2008) and conversion efficiency = 0.55-0.56 (DFT, 2006 p16)
[c]With heating value of dry stalk of 17.3 – 17.8 GJ/ton
[d]With the 2002 average conversion efficiency 90 litres ethanol per tonne of cane (IEA, 2004 p60)
[e]With exchange rate of 1US$ for 2.5 R$ in 2005, and heating value of dry stalk of 17.3 – 17.8 GJ/ton
*Fresh weight

Table 6 – C_0 – Corn and Sugar cane initial production costs

The range for the costs of producing corn at time t_0 is therefore chosen as: 2.9 – 8.8 US$/GJ. The range for the costs of producing sugar cane at time t_0 is therefore chosen as: 2.2 – 3.0 US$/GJ.

The initial input I_0 is derived from the average costs (C_0) and the average yield at time t_0: $I_0 = C_0 \cdot Y_{average0}$. The average corn yield in the U.S. was estimated as between 133 and 145 GJ/ha in 2005. This yield combined with the previous range for corn production costs gives I_0 for U.S. corn between 389 and 1276 (2005)US$/ha. The average sugar cane yield in Brazil was estimated between

371 and 383 GJ/ha in 2005, which gives I_0 between 816 and 1149 US$/ha for Brazilian sugar cane.

Learning parameters

The experience curve theory, when used for technology forecasting, assumes that the learning rate will remain constant over time, and the model implies that the rate of learning for emerging technologies will be greater than for mature technologies. According to Margolis (2002), "the process of innovation is inherently uncertain". The potential for breakthroughs is difficult to quantify and is not fully captured in the experience curve theory. Also, the ability of a technology to continue benefiting from learning is uncertain (IEA, 2000 p92), as the learning curve theory ignores theoretical and technical limitations that may hinder further cost reductions. For these reasons, and in order to capture the uncertainty associated with the future learning pace of these technologies, a range of estimates is assigned to the learning rate parameter.

According to (Duvick, 2005), both plant breeding and improved management practices are responsible for the rise in U.S. corn yield that occurred between the 1930s and today. The following numbers for learning rates for corn, sugar cane and ethanol production are found in the literature:

Progress ratio	Learning rate	Source
Corn		
0.55 +/- 0.02*	0.43 – 0.47	Hettinga et al. 2009 p6
-	0.238* (starch)	de Wit and Faaij 2008 p21
-	0.18* (ethanol)	IEA 2007 p6
Sugar cane		
0.68 +/- 0.03* (1975-2004)	0.29 – 0.35	van den Wall Bake et al. 2009 p7
0.71* (1985-2002)	0.29 (ethanol)	Goldemberg et al. 2004 p302
-	0.20* (ethanol)	IEA, 2007 p6

*Original data

Table 7 – Learning rates for corn and sugar cane production

The learning rate of 0.47 for corn production (Hettinga et al., 2009) and 0.35 for sugar cane production (van den Wall Bake et al., 2009) are at the upper end of learning rate estimates for energy technologies from (McDonald and Schrattenholzer, 2001). The IEA (2007 p6) estimates at 0.18 (corn) and 0.2 (sugar cane) the learning rates for ethanol production, and points out that feedstock production shows higher learning rates than industrial processing, The ranges for the overall learning rate of crop production are thus chosen as follow: 0.24 – 0.47 for corn and 0.2 – 0.35 for sugar cane production.

The introduction of genetically modified varieties and improved management practices will allow further cost reductions in cane production, but there is great uncertainty about the relative influence of improved practice, which will be reflected in $LR_{Yactual}$, and better varieties, which will be reflected in LR_{YMAX}. We attempt to separate these effects when setting parameter values.

a) Sugar cane

The yield gap between the actual yield and the maximum potential yield is assumed to decrease with experience, and Irvine predicts higher sugar cane yields through improved plant breeding (1983, p378). Two main sources are used to estimate $LR_{Yactual}$ and LR_{YMAX}. Firstly, according to Burnquist, cited in (van den Wall Bake et al., 2009, p10), the introduction of "optimal logistic systems", the wider adoption of state-of-the-art technologies and larger scale transportation systems should allow further cost reductions. Burnquist estimates that these factors alone, without the introduction of genetically modified crops, could decrease sugar cane production costs by 20 to 40% in the next fifteen years. Van den Wall Bake et al. (2009) estimate the learning rate for sugar cane production between 1974 and 2004 at 0.29 – 0.35, and expect further reduction in the costs of sugar cane production in Brazil of 32 to 49% between 2004 and 2020, assuming an 8% annual growth of the sugar cane industry. Secondly, Edmé et al. (2005) and Hogarth (1976) (quoted in Domaingue et al., no date, p6) report cane yield improvements of 1 to 2% per year, of which about half could be attributed to better plant varieties.

These figures are used to estimate the relative effect of genetic manipulation and improved management and derive LR_{YMAX} and $LR_{Yactual}$, assuming 1%, 5% and 8% annual growth in sugar cane production and no impact of the decreasing suitability of the marginal land, as that effect was not explicitly taken into account in the referenced sources. In order to do so, the model is extended to the period 1975 – 2005 and the average yield is calculated and fitted to historical yield data from IPEADATA. The ranges of $LR_{Yactual}$ and LR_{YMAX} are chosen so that the average yield increases between 1 and 2% per year over the period 2005 – 2055, which gives $LR_{Yactual} = 0.04 – 0.1$ and $LR_{YMAX} = 0.04 – 0.1$.

Technical change reflects "improvements in the way the inputs are used", (McKibbin et al., 2004 p12). In the farming industry, technical change has resulted in "less input per required unit output", (Gardner, 2003). Once the ranges of $LR_{Yactual}$ and LR_{YMAX} have been set, the range of α is chosen so that the projected average costs coincide with the estimates from van den Wall Bake and Burnquist mentioned previously. The resulting range is $\alpha = 0.005 – 0.02$. These ranges are compatible with the overall learning rate of 0.2 – 0.35 for sugar cane production costs.

b) Corn

Reilly and Fuglie estimate maize yield increase between 1.3% and 3.2% per year from 1994 to 2020, using a linear model (1998 p280). According to Cardwell (1982) cited in (Tollenaar et al., 1994 p189), changes in management practices were responsible for 43% of the total increase in corn yields from 1930 to 1980 in the U.S. The contribution of genetic improvements to historical corn yield improvement was also measured in various regions: these results are reported in (Tollenaar et al., 1994 p189) and reproduced below:

Region	Period	Portion due to genetic improvement	Source
U.S.	1930-1960	57-79%	Castleberry et al., 1984
France	1950-1985	42-69%	Derieux et al., 1987
Ontario	1959-1980	2.6% per year	Tollenaar, 1989

Table 8 – Genetic improvement in corn

Also, Long et al. (2006 p315) argue that genetic manipulations could improve the corn yield potential by 50%, potentially within 10 to 15 years. These figures are used to estimate the relative effect of genetic manipulation and improved management and calibrate LR_{YMAX} and $LR_{Yactual}$, assuming no impact of the decreasing suitability of the marginal land. In order to do so, the model is extended to the period 1995 – 2005 and the average yield is calculated and fitted to historical yield data from USDA (2008). This gives ranges of $LR_{Yactual}$ = 0.07 – 0.22 and LR_{YMAX} = 0.1 – 0.33. Again, it should be noted that the higher estimates of the learning rates for corn and sugar cane production might not be sustained in the very long term, as was discussed earlier.

Gardner reports an average rise in multifactor productivity (i.e. output divided by inputs) of 2% p.a. between 1930 and 2000 in U.S. agriculture, which exceeded the productivity rise in manufacturing. Saunders (1992) estimates technical progress at about 1.2% per year. Once the ranges of $LR_{Yactual}$ and LR_{YMAX} have been set, the range of α is chosen so that the projected average costs coincide with the estimates from Hettinga (2009). The resulting range is α = 0 – 0.01. That range is of the same order as the rate of technical progress across the U.S. economy mentioned above. These ranges give results that are compatible with the overall learning rate of 0.24 – 0.47 for corn production costs.

Summary

Table 9 summarises the ranges that are assigned to each parameter in the model. The wide ranges reflect the large uncertainty on these parameters. These ranges are illustrative: they are better than guesses but they are not the result of a formal elicitation exercise.

Parameters		U.S. corn			Brazilian sugar cane		
		Min	Most likely	Max	Min	Most likely	Max
Land resources and yields	**Unit**						
Total agricultural land available Q_T	ha	1.1E+08	1.8E+08	2.5E+08	0.4E+08	1.7E+08	3.1E+08
Share of land used Q_0/Q_T	no unit	0.12	0.20	0.27	0.02	0.09	0.15
Yield gap g_0	no unit	0.6	0.68	0.76	0.61	0.67	0.72
Initial maximum yield $Y_{initial0}$	GJ/ha	400	475	550	1000	1175	1350
Minimum yield Y_{min0}	GJ/ha	3	15	26	28	53	77
Actual marginal yield $Y_{actual0}$	GJ/ha	115	125	135	345	357	368
Average yield $Y_{average0}$	GJ/ha	133	139	145	371	377	383
Production and demand							
Rate of increase in demand d	no unit	0.004	0.039	0.074	0.01	0.045	0.08
Cumulative crop production X_0	GJ	1.9E+11	1.95E+11	2.0E+11	1.3E+11	1.53E+10	1.75E+11
Learning							
Marginal yield: Learning rate $LR_{Yactual}$	no unit	0.1	0.22	0.33	0.04	0.075	0.1
Maximum yield: Learning rate LR_{YMAX}	no unit	0.12	0.18	0.24	0.04	0.075	0.1
Generalised technical progress α	no unit	0	0.005	0.01	0.005	0.013	0.02
Costs							
Cost C_0	US$/GJ	2.9	5.9	8.8	2.2	2.6	3.0

Table 9 – Parameters ranges, U.S. corn and Brazilian sugar cane

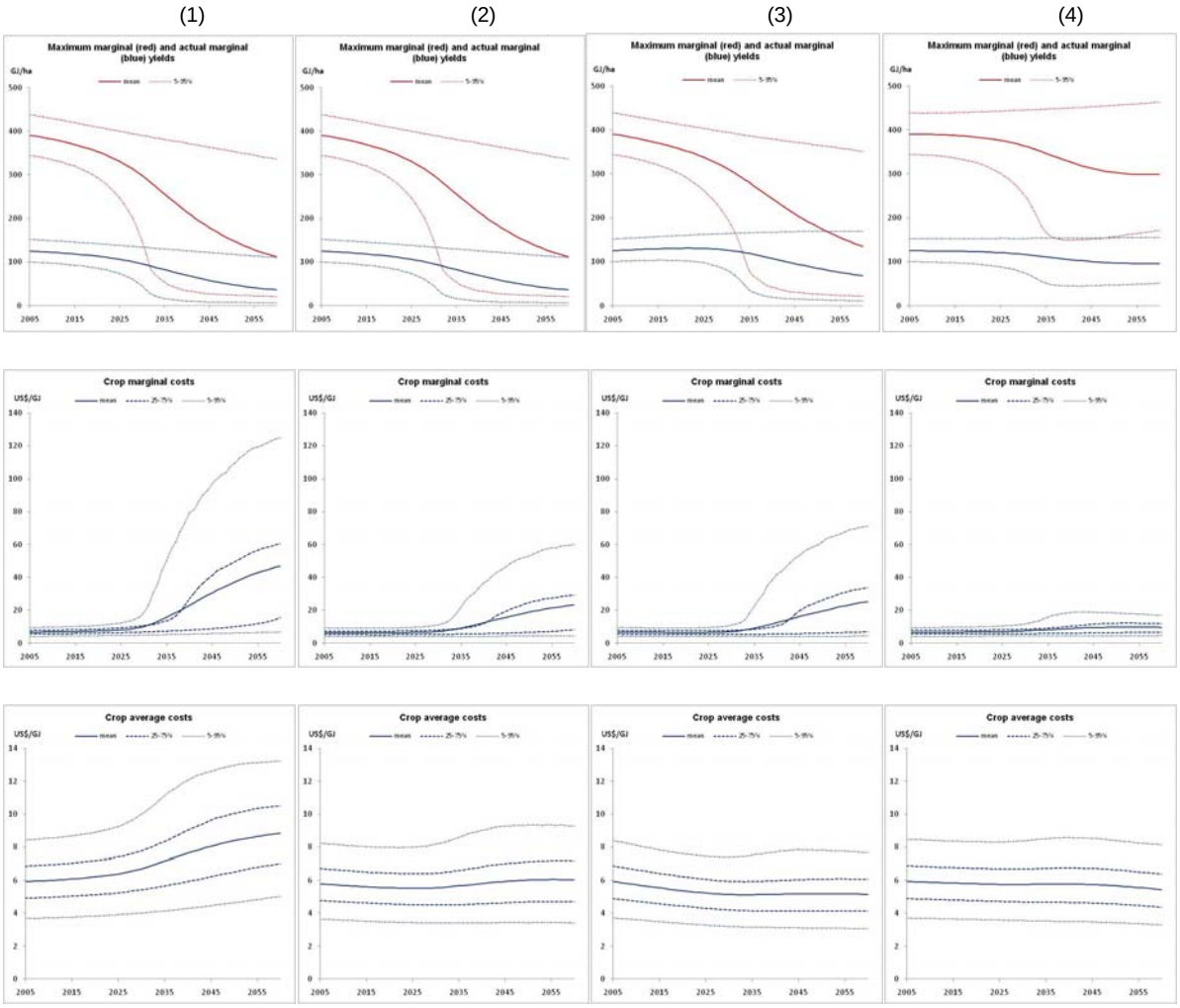
It is assumed that all these parameters are independent. These ranges are fed into the model to obtain some preliminary results.

- **Preliminary results: U.S. corn**

Yields and costs with some learning or technical progress

The following results are obtained for the maximum marginal and actual marginal yields, the marginal costs and the average costs of corn production in the U.S. with different assumptions about the learning and technical progress parameters ($LR_{Yactual}$, LR_{YMAX}, α). In our model, the marginal costs are defined as the costs of producing crops on the least suitable unit of land under cultivation. The first set of graphs (1) shows the results when all learning parameters are set at zero. The learning parameters are then introduced one at a time to illustrate

the structure of the model and to show the relative influence of each parameter on the results.



(1) α, $LR_{Yactual}$ and LR_{YMAX} set at zero

(2) α (0-0.01), $LR_{Yactual}$ and LR_{YMAX} set at zero

(3) $LR_{Yactual}$ (0.12-0.24), LR_{YMAX} and α set at zero

(4) LR_{YMAX} (0.1-0.33), α and $LR_{Yactual}$ set at zero

Figure 7 – Yields and crop costs, corn

The centre lines (full) show the mean values. On the cost charts, the two lines above the mean are the 75th and 95th percentiles. The two lines below the mean are the 25th and 5th percentiles: the narrower the band, the less the uncertainty about the results. The uncertainty about future yields increases with time. The ranges coincide with the literature estimates listed in Table 9 for all the parameters except the three learning parameters ($LR_{Yactual}$, LR_{YMAX}, α).

When all learning parameters are set at zero (1), the results show decreasing marginal yields, with for example mean values of the actual marginal yield dropping from about 125 GJ/ha in 2005 to 36 GJ/ha in 2060. Costs are increasing from 6.5 US$/GJ in 2005 to almost 50 US$/GJ in 2060 (mean values): with no learning and technical progress, the decreasing suitability of the marginal land drives the costs up. When the technical progress parameter, α, is set at 0-0.01 (2), the results show decreasing yields, but costs are increasing more slowly, from 6.5 US$/GJ in 2005 to 23 US$/GJ in 2060: the amount of inputs needed to

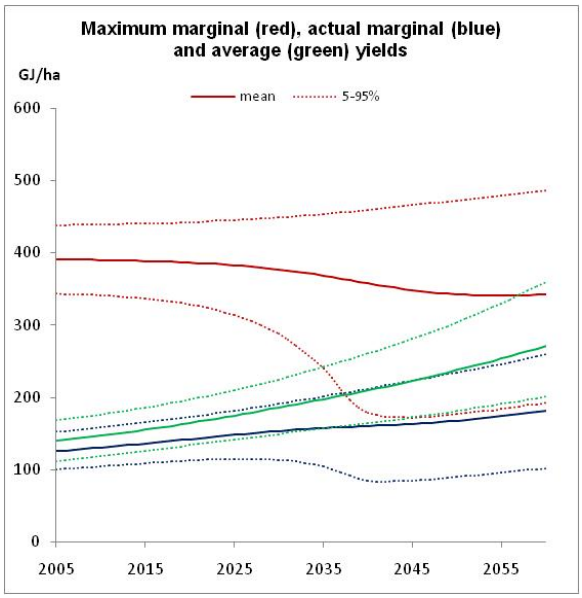
produce the same amount of crops is decreasing. When the learning parameter associated with the actual yield, $LR_{Yactual}$, is set at 0.12-0.24 (3), the results still show decreasing maximum yields ($LR_{Yactual}$ does not influence the marginal maximum yield) but the marginal actual yield is now slightly increasing in the 95th percentile: the yield gap narrows thanks to improved management practice. When the learning parameter associated with the maximum yield, LR_{YMAX}, is set at 0.1-0.33 (4), the results show increasing marginal actual and maximum yields at the end of the period: in 2060 the maximum marginal yield reaches 300 GJ/ha (mean value). These increasing yields are driving crop costs down.

The steep decrease of the maximum marginal yield shown as the 5th percentile of the yield curve (in case (1) from 345 to 55 GJ/ha in 30 years) corresponds to high demand growth (the maximum of this parameter is 7.4% p.a.). In that case, the amount of land used for corn production reaches the upper bound of the total suitable land available for corn production before the end of the period.

The absolute values mentioned above should be considered with caution, as they were obtained by excluding some of the learning and technical progress parameters that are believed to influence crop yields and costs. We consider the results with all parameters active in the next section.

Yields and costs with all learning and technical progress

Figure 8 shows the maximum and actual marginal yields, the average yield, the average costs and the marginal costs of corn production in the U.S. with all input ranges as shown in Table 10.



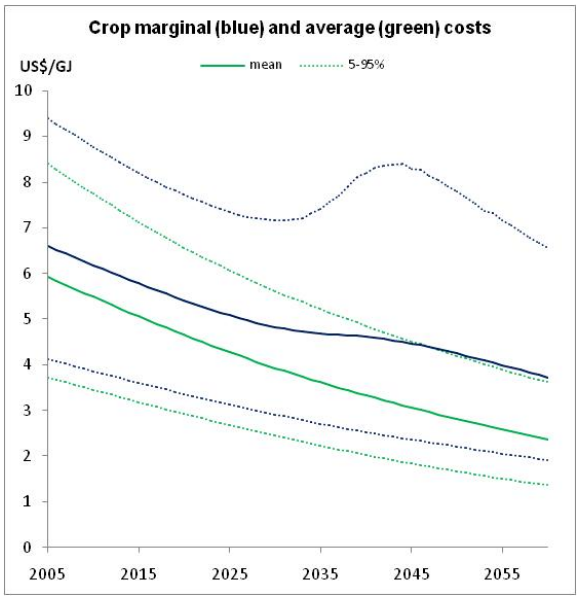


Figure 8 - Yields and crop costs, U.S. corn, all

This model reveals the kind of uncertainties that need to be dealt with when designing policies. All three learning parameters are now included to obtain the results shown in Figure 8. The mean value for the maximum marginal yield benefits from new biotechnologies and is increasing in the second half of the period (as it was shown on exhibit 4 of Figure 7). The marginal yield is now influenced by increasing maximum achievable yield and decreasing yield gap, as both learning effects are now included (learning parameters $LR_{Yactual}$, LR_{YMAX}, were taken separately in 0): the maximum achievable yield is driving up the marginal actual yield as the yield gap is decreasing with learning. Crop costs are calculated from the marginal yield, and are influenced by the generalised technical progress parameter α. The results show large uncertainties, with marginal costs falling in the range of 2.9 to 7.2 US$/GJ in 2030 (2005 US$). Costs decrease by over 40% between 2005 and 2060 (mean value). The 5th percentile line shows strictly decreasing marginal costs, while the 95th percentile line shows decreasing marginal costs in the first half of the period, increasing marginal costs between 2030 and 2045, and decreasing costs after 2045. In the first half of the period, the total amount of suitable land hasn't been reached, and experience and technological developments are driving down the marginal costs. As more marginally suitable land is used, the decreasing productivity overtakes experience, and marginal costs increase until the total amount of suitable land is used. In 2045, all suitable land is used (in the 95th percentile). The suitability of land is thus fixed, and marginal costs will then only be influenced by experience and technological developments.

Influences

The influence of each parameter on these results is examined further by using the sensitivity analysis in Palisade's @RISK. The influences shown in Figure 9 are obtained from a simulation of 10,000 iterations.

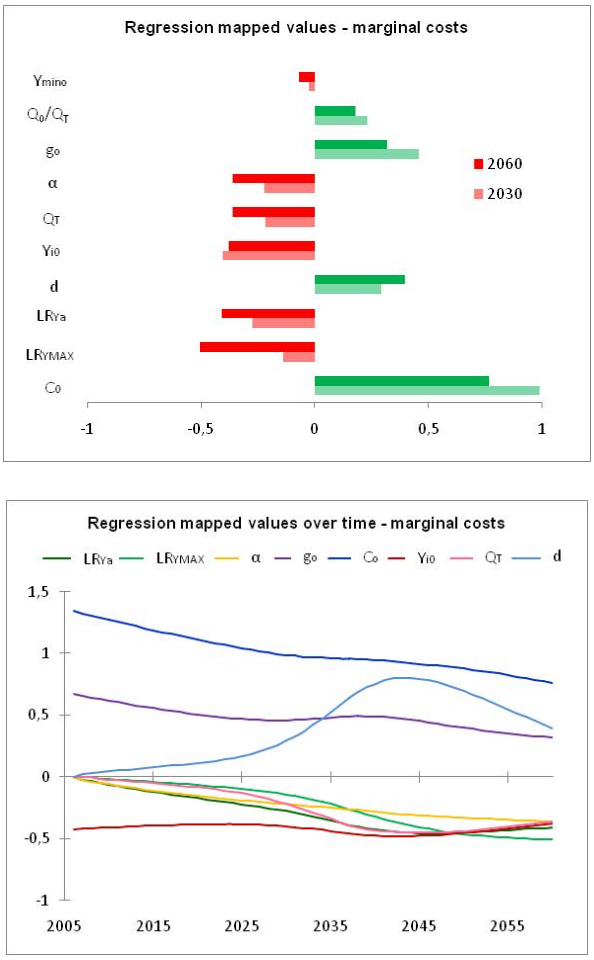


Figure 9 - Regression (mapped values): corn marginal costs

The longer bars represent the most significant variables. Mapped values show the change in marginal crop costs if one parameter changes by one standard deviation while all other parameters are constant, (Palisade, 2008).

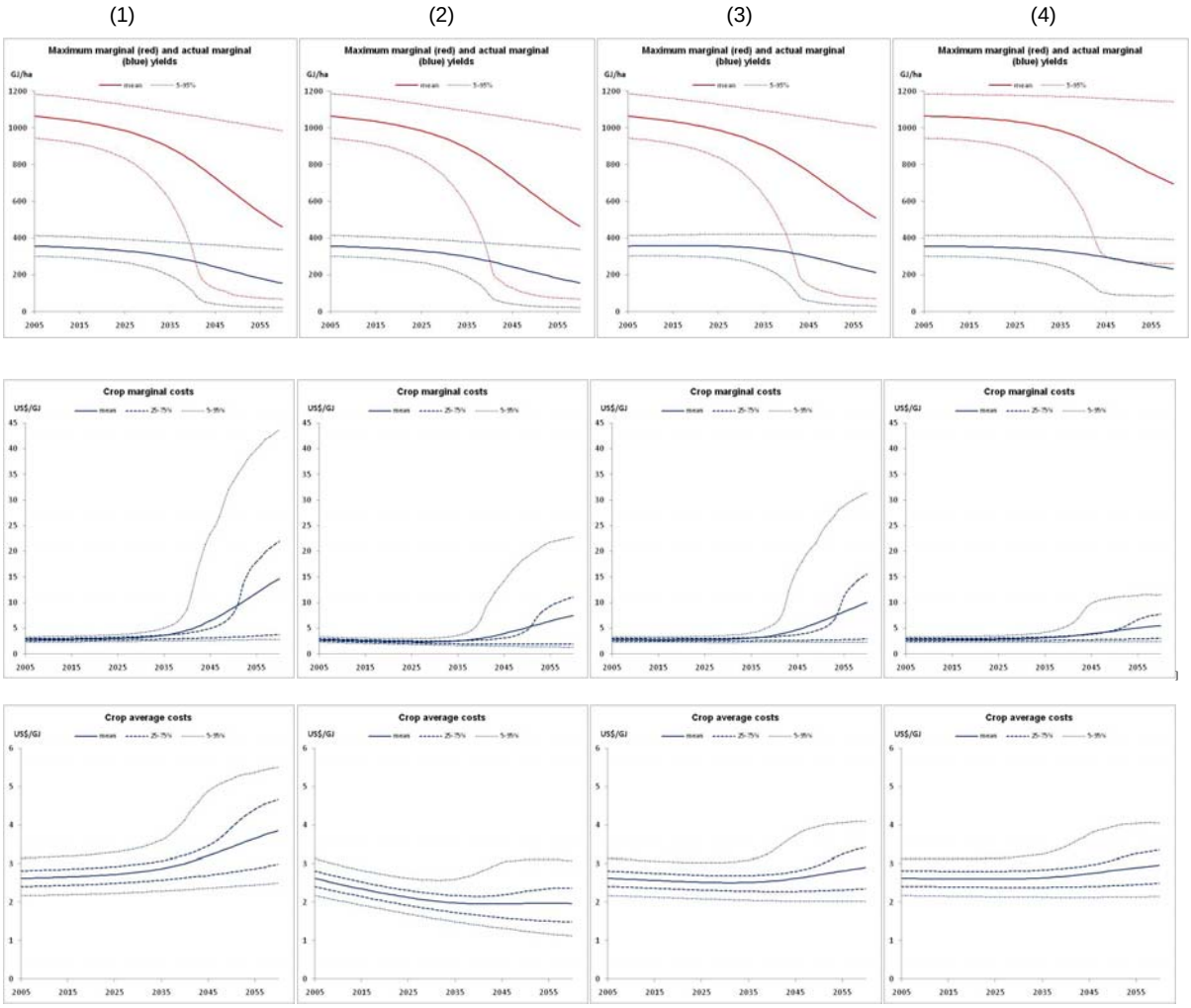
The results show that parameters C_0 and g_0 have the biggest influence on the marginal costs in 2030 (light) and C_0 and LR_{YMAX} have the biggest influence on the marginal costs in 2060 (dark): the results show that an increase of one standard deviation of C_0 in 2030 would increase marginal costs by 0.99 US$/GJ, or about 21% of its mean value, while an increase of C_0 in 2060 would increase marginal costs by 0.76 US$/GJ, or about 20% of its mean value. In 2030, an increase of one standard deviation of g_0 would increase marginal costs by 0.46 US$/GJ (10%). In 2060, it would increase marginal costs by 0.32 US$/GJ (9%), while an increase of one standard deviation in LR_{YMAX} would decrease marginal costs by 0.5 US$/GJ (13%). Higher initial costs (C_0) and a higher initial yield gap (g_0) induce higher marginal costs. Although $Y_{initial0,}$ the initial maximum achievable yield in 2005, is the third most influential parameter in 2030, its influence decreases towards the end of the period. LR_{YMAX} is the learning rate associated with the maximum achievable yield. A high LR_{YMAX} will enhance the maximum yield (as it will benefit more quickly from improved technology) and will therefore lower the costs, hence the negative sign of the sensitivity. $LR_{Yactual}$ is the learning rate associated with the yield gap, and a higher $LR_{Yactual}$ means

higher actual yields, hence lower costs. The amount of land suitable for corn production, Q_T, is positively correlated with the marginal yields (Y_{MAX} and Y_{actual}), so the costs of producing the crops will be lower if more land is available. α is the rate of technical progress across the economy, and a higher α means that more crops can be produced with less input, which translates into lower production costs. Higher demand induces higher marginal costs. Learning and the decreasing suitability of the marginal land are driven by production: costs decrease with experience and the marginal productivity of land decreases as more land is brought into production, so the regression coefficient associated with demand suggests that the dominant effect is the decreasing suitability of the marginal land over the whole period.

Two effects are driving the costs in opposite ways: technological change and the decreasing suitability of the marginal land. In order to illustrate the evolution of both effects, we examine the influences on costs over time. The learning parameters LR_{YMAX} (technological change), α (general technical progress) and $LR_{Yactual}$ (experience) are gaining influence on costs over time. Q_T affects the onset of the decreasing suitability of the marginal land: the influence of Q_T is positive and reaches a peak in 2040. The influence of demand growth (d) reaches a peak after 2040, following the evolution of the influence of Q_T. In the beginning of the period, higher demand drives the decreasing suitability of the marginal land, driving up the costs. As the maximum marginal yield comes closer to the minimum value of the achievable yield Y_{min}, driven by higher demand (in the 5th percentile of the yield chart on Figure 8), the potential for further cost increase linked to the decreasing suitability of the marginal land is reduced, and the influence of d decreases. To conclude, the effect of learning slowly overtakes the effect of the decreasing suitability of the marginal land on marginal costs. Looking at these influences helps us to concentrate on the most influential parameters to refine the study in the future.

▪ Preliminary results: Brazilian sugar cane

The following results are obtained for the maximum marginal and actual marginal yields, the marginal costs and the average costs of sugar cane production in Brazil with various ranges assigned to learning parameters ($LR_{Yactual}$, LR_{YMAX}, α). Again, the marginal costs are defined as the costs of producing crops on the least suitable unit of land under cultivation. The first set of graphs (1) shows the results when all learning parameters are set at zero. The learning parameters are then introduced one at a time to illustrate the structure of the model and to show the relative influence of each parameter on the results.



(1) $LR_{Yactual}$, LR_{YMAX} and α set at zero

(2) α (0.05-0.02), LR_{YMAX}, and $LR_{Yactual}$ set at zero

(3) $LR_{Yactual}$ (0.04-0.1), LR_{YMAX} and α set at zero

(4) LR_{YMAX} (0.04-0.1), $LR_{Yactual}$ and α set at zero

Figure 10 - Yields and crop costs, sugar cane

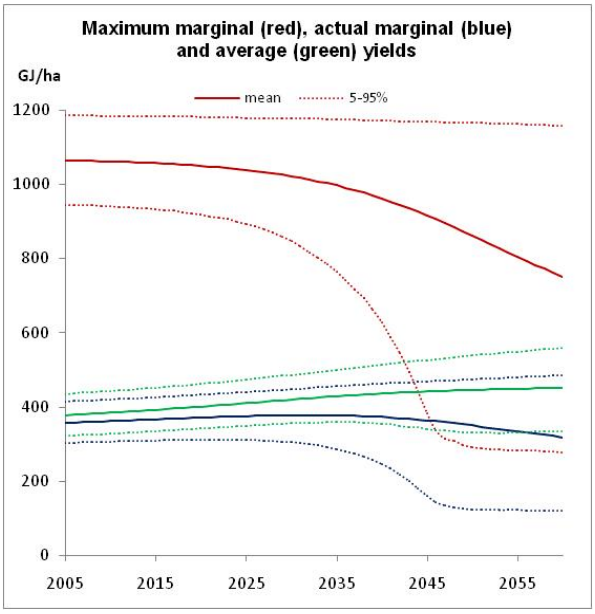
When all learning parameters are set at zero (1), the results show decreasing marginal yields, with mean values of the actual marginal yield halving between 2005 and 2060 (from 357 GJ/ha to 155 GJ/ha). Costs are increasing sharply in the 95th percentile, and mean values of the marginal costs increase from below 3 US$/GJ in 2005 to almost 15 US$/GJ in 2060: with limited learning and technical progress, the decreasing suitability of the marginal land drives up the costs. The generalised technical progress parameter α has no effect on yields, but mean marginal costs are increasing more slowly in (2): from about 3 US$/GJ in 2005 to 7 US$/GJ in 2060, and average costs are decreasing in the first half of the period. The introduction of the learning parameter $LR_{Yactual}$ slows down the decrease of the actual yield Y_a (3): the yield gap narrows thanks to improved management practice, and the marginal actual yield is almost constant in the 95th percentile, around 415 GJ/ha. The mean value of the actual marginal yield decreases from 356 GJ/ha in 2005 to 213 GJ/ha in 2060 (-40%).

The learning parameter LR_{YMAX} has a similar effect on the maximum yield: the maximum achievable yield is decreasing more slowly in (4) than in the previous cases, from 1064 GJ/ha in 2005 to 694 GJ/ha in 2060 (-35%, mean values). This

in turns affects mean actual marginal yields, which are decreasing more slowly, from 356 GJ/ha to 233 GJ/ha (-35%), even with a constant range for the yield gap. Costs are increasing much more slowly in this case, from 3 GJ/ha in 2005 to only 5.5 GJ/ha in 2060.

The steep decrease of the maximum marginal yield shown as the 5th percentile of the yield curve (in case (1) from 945 to 336 GJ/ha in 35 years) corresponds to high demand growth (the maximum of this parameter is 8.0% p.a.). In that case, the amount of land used for sugar cane production reaches the upper bound of the total suitable land available for sugar cane production in Brazil before the end of the period.

Figure 11 shows the results obtained for the maximum marginal and actual marginal yields, the average yield, the marginal costs and the average costs of sugar cane production in Brazil with the parameter ranges shown in Table 10.



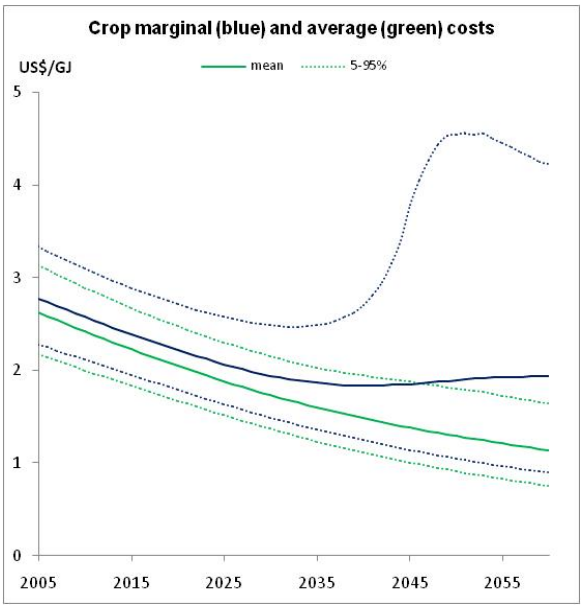


Figure 11 – Crop yields and costs, sugar cane, all

All three learning parameters are now included to obtain the results shown in Figure 11. The combination of the learning parameters $LR_{Yactual}$ and LR_{YMAX} induces slowly increasing mean marginal yields in the first half of the period, from 356 GJ/ha in 2005 to 377 GJ in 2030 (while the mean marginal yield Y_{actual} was strictly decreasing in every configuration shown on Figure 10). Increasing marginal yields, in association with the learning parameter α, drive sugar cane costs down in the first half of the period. The results show marginal costs falling in the range of 1.5 to 2.5 US$/GJ in 2030. After 2030, marginal yields are slowly decreasing, driving the marginal costs up from 1.8 US$/GJ in 2040 to 1.9 US$/GJ in 2060 (mean values). The 5th percentile line shows strictly decreasing marginal costs, while the 95th percentile line shows decreasing marginal costs in the first half of the period, increasing marginal costs between 2035 and 2050, and decreasing costs after 2050. As in the case of corn, experience and technological developments are driving down the marginal costs in the first half of the period. As more marginally suitable land is used, the decreasing suitability of the marginal land overtakes experience, and marginal costs increase until the total amount of suitable land is used. In 2050, all suitable land is used (in the 95th percentile), and marginal costs will then only be influenced by experience and technological developments.

Influences

The influence of each parameter on these results is examined further by using the regression sensitivity analysis in Palisade's @RISK. The following influences are obtained from a simulation of 10,000 iterations.

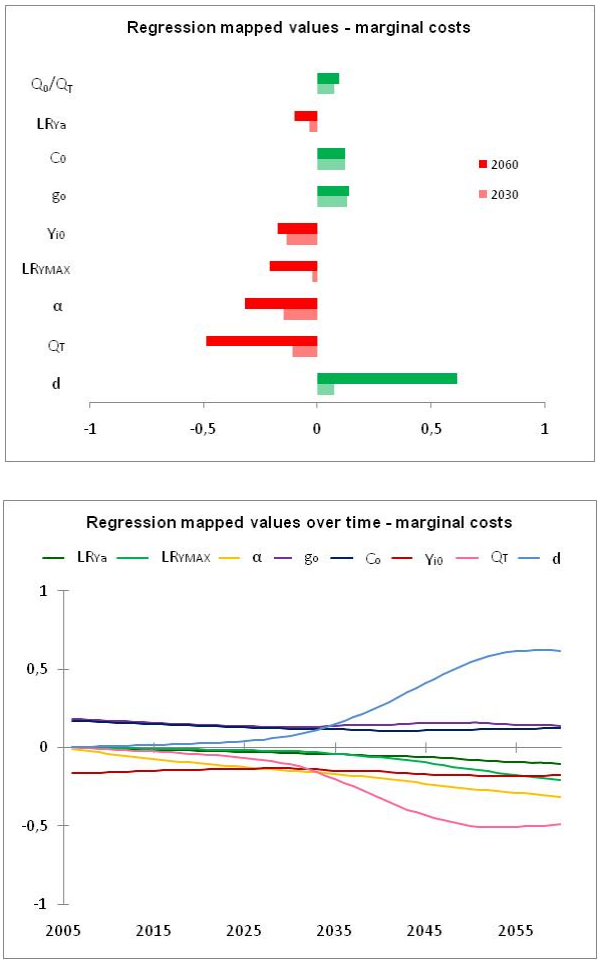


Figure 12 – Regression (mapped values): sugar cane marginal costs

The results show that the parameters Q_T and d have the biggest influence on the supply costs in 2030 (light) and 2060 (dark): the results show that in 2060, an increase in d would increase marginal costs by 0.62 US$/GJ, or about 32% of its mean value, while an increase of one standard deviation of Q_T would decrease marginal costs by 0.50 US$/GJ, or about 25% of its mean value. The amount of land suitable for sugar cane production (Q_T) is driving down the marginal yields (Y_{MAX} and Y_{actual}), and crop production costs will be lower if more land is available, hence the negative influence of Q_T. Higher demand (d) induces higher marginal costs, weakly so in 2030, but strongly in 2060. α is the rate of technical progress across the economy, and a higher α means that more crops can be produced with less input, which translates into lower production costs.

Two effects are driving the costs in opposite ways: technological change and the decreasing suitability of the marginal land. In order to illustrate the evolution of both effects, we examine the correlation sensitivities of costs over time. The learning parameters LR_{YMAX} (technological change), α (general technical progress) and $LR_{Yactual}$ (experience) are gaining influence on costs over time. The influence of Q_T is also increasing and peaks around 2055. The influence of the growth rate of demand (d) is increasing over time and follows the same evolution as the influence of Q_T. In the case of sugar cane, the peak of the influence of the demand parameter is reached later than in the case of corn. The

shift in the effect of demand growth between corn and sugar cane is explained by the fact that the initial share of suitable land used for sugar cane production in Brazil is lower than the initial share of suitable land used for corn production in the U.S., as shown below.

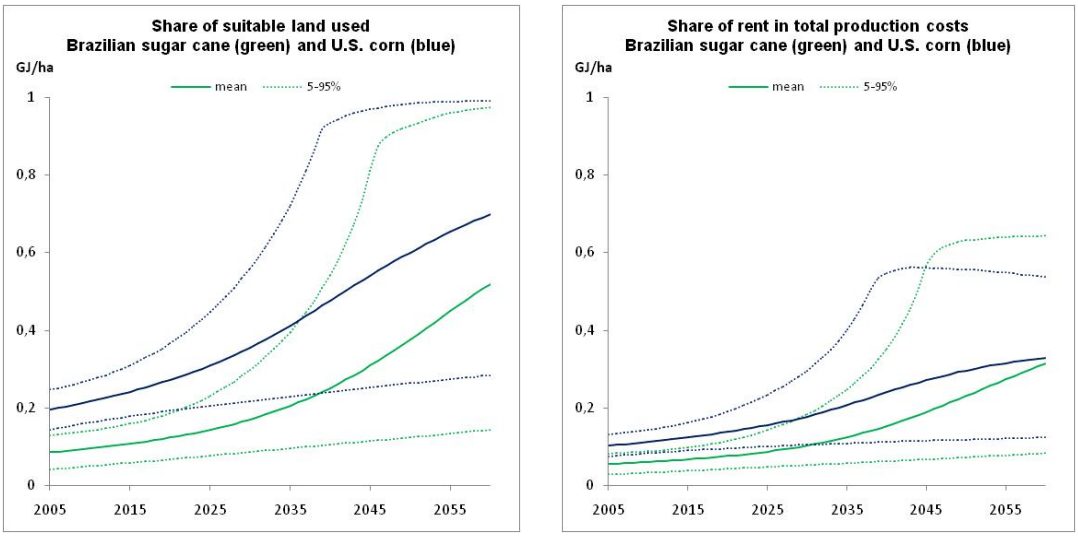


Figure 13 - Share of suitable land used and rent over costs for Brazilian sugar cane and U.S. corn

The share of rent in total production costs is increasing as more land is brought into production. In the case of U.S. corn, the share of rent in total production costs peaks in 2040 and slowly decreases afterwards (95th percentile). This corresponds to the time when the share of suitable land used for corn in the U.S. becomes close to one (95th percentile). When all suitable land is used, experience is the only driver of marginal costs: marginal costs peak before 2045 (95th percentile on Figure 8) and lower marginal costs induce lower rents.

The results of the model are compared to cost estimates found in the literature in table 10.

crop	year	Model output						Literature estimates	
		average costs (no rent)			marginal costs			National average	Source
		5th	mean	95th	5th	mean	95th		
corn	2020	2.9	4.7	6.6	3.4	5.4	7.7	5.2 -5.3	Hettinga 2009
sugar cane	2020	1.7	2.0	2.5	1.8	2.2	2.7	1.7 – 2.0	van den Wall Bake 2009
								1.5 – 1.9	

Table 10 - Expected costs in 2020 – literature estimates and model output (2005 US$/GJ)

The estimates of average corn and sugar cane production costs from the literature fall in the range obtained for average costs in the model (5th and 95th percentiles). The results from the model are compatible with these cost estimates.

▪ Conclusion and further work

This research ultimately aims to reveal the effects of experience, technological developments and production constraints on the costs of supplying alternative fuels. In this paper, a model describing the effects of learning and decreasing suitability of the marginal land on the costs of supplying corn and sugar cane has been introduced. The learning, resources and production parameters of the model are not known precisely, and uncertainty was introduced by assigning a distribution to each parameter.

The results show large uncertainties in the future costs of supplying corn and sugar cane, with a 90% confidence interval of 2.9 to 7.2 $/GJ in 2030 for marginal corn costs in the U.S., and 1.5 to 2.5 $/GJ in 2030 for marginal sugar cane costs in Brazil. The sensitivity analysis shows that production is first driving costs up, as the productivity of the marginal land decreases. As the maximum marginal yield comes closer to its theoretical minimum, the potential for further cost increase linked to the decreasing suitability of the marginal land is reduced, and learning dominates in the longer term. This phenomenon occurs first in the U.S., as the total area of suitable land for corn production in the U.S. is used earlier than the total area of suitable land for sugar cane production in Brazil. The share of rent in total production costs increases as more land is brought into production. In the case of U.S. corn, the share of rent in total production costs reaches a plateau when all suitable land is used. Marginal costs then decrease thanks to experience and induce lower rents.

Bioethanol is obtained by the fermentation of sugars found in the crop feedstock. A model for conversion costs will be introduced: the costs of producing ethanol will be calculated from the costs of crop and the conversion yield, driven by accumulated experience. The environmental costs associated with the production of biofuels are not presently included in the cost estimates. In particular, the cost of carbon will be considered when assessing the cost-competitiveness of these fuels. Carbon emissions from crop production and conversion stages will be assessed, and the social cost of carbon will be used to calculate the total carbon costs associated with ethanol production from U.S. corn and Brazilian sugar cane. High carbon prices will impact on investment into alternative fuels supplies, and will therefore influence the scale of production and trend in supply costs. The costs of supplying ethanol will be later compared to the costs of petrol from non-conventional fossil resources, including learning, depletion and carbon costs. It is expected that the study will inform decision makers on the type of policy and the scale and timing of investments that will be needed to meet the growing demand for liquid fuels while satisfying CO_2 constraints, and the model described here is a step in this direction.

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- **Appendix**

Equations

a) Productivity

The form used in (Bakkes et al., 2008 p123) is shown below:

$$L = a - \frac{b}{f(1/y)} \qquad \text{(a)}$$

With L the amount of land on the land supply curve (ha)
y the land productivity (GJ/ha)
a the maximum potentially available agricultural land (ha)
b a positive parameter
f(1/y) an increasing function of the inverse of land productivity

Under the assumption that the land price is inversely proportional to the marginal yield, Eickhout et al. (2008 p19) use the following form for equation (a):

$$L = a - \frac{b}{r^p}$$

(b)

With r the land rental rate
p a positive parameter

and the price elasticity ε given by: $\varepsilon = \frac{p \cdot b}{a \cdot r^p - b}$

In that case, we have $f(x) = x^p$. Equation (b) can thus be written as: $L = a - c \cdot y^p$, with c a positive parameter. The marginal yield y is then expressed as a function of the share of land used L/a:

$$y = \left(\frac{a-L}{c}\right)^{1/p} = \left(\frac{a}{c}\right)^{1/p} \cdot \left(1 - \frac{L}{a}\right)^{1/p} = A \cdot \left(1 - \frac{L}{a}\right)^{1/p}$$

(c)

b) Production rate

From equation (7), it follows that $\frac{dx}{dQ} = Y_{actual}(Q)$. If we assume that changes in the production rate x and the land used Q between time t-1 and time t are very small: $\frac{dx}{dQ} \approx \frac{\Delta x}{\Delta Q} = \frac{x_t - x_{t-1}}{Q_t - Q_{t-1}}$, hence $\Delta Q = \frac{\Delta x}{Y_{actual}(t)} = \frac{\Delta x}{Y_{actual}(Q_t)}$. If we assume again small changes in the area of land used between time (t-1) and time t ($Q_t \approx Q_{t-1}$), we obtain:

$\Delta Q \approx \frac{\Delta x}{Y_{actual}(Q_{t-1})}$, and finally $Q_t = Q_{t-1} + \frac{\Delta x}{Y_{actual}(t-1)}$

- **Parameters**

Initial minimum yield

a) Corn yield occurring in U.S. marginally suitable land

	Average yield (marginal land)		Minimum yield divided by average yield (very suitable land)	Estimated minimum yield (marginal land)
	Y(mS)*	Y(mS)**	Ymin(VS) / Y(VS)	Y(mS) *Ymin(VS)/ Y (VS)
	ton/1000ha	GJ/ha	No unit	GJ/ha
l	1350	23	0.54	12.4
i	2051	35	0.66	23.1
h	2824	48	0.53	25.4
m	1218	21	0.23	4.8
xi	2064	35	0.55	19.3
xh	2841	48	0.39	18.7
xm	1286	22	0.17	3.7

*(IIASA, 2002)
** Results obtained with heating value of corn of 17 GJ/ton

b) Sugar cane yield occurring in Brazilian marginally suitable land

	Average yield (marginal land)		Minimum yield divided by average yield (very suitable land)	Estimated minimum yield (marginal land)
	Y(mS)*	Y(mS)**	Ymin(VS) / Y(VS)	Y(mS) *Ymin(VS)/ Y (VS)
	ton/1000ha	GJ/ha	No unit	GJ/ha
l	2726	47.43	0.88	41.9
i	3872	67.37	0.77	52.2
h	5101	88.76	0.86	76.5
m	2734	47.57	0.86	41.0
xi	3872	67.37	0.61	41.2
xh	5093	88.62	0.59	52.7
xm	2734	47.57	0.59	28.3

*(IIASA, 2002)
** Results obtained with heating value of sugar cane of 17.4 GJ/ton