Questions Addressed by Strategic Market Models

What might be the effect of policies concerning...

- **Generation structure** (mergers, ownership, distributed resources, entry…)
- **Transmission investment** (new lines …)
- **Market rules**
  - Transmission pricing (taxes, congestion, counterflows, zonal …)
  - Access (retail load, generators, arbitragers …)
  - Environmental markets (green certs., CO2 trading …)

...upon...

- **Economic efficiency** (allocative & productive efficiency)
- **Income distribution** (TSO revenues, profits, consumer surplus)
- **Emissions**

...considering generator strategic behavior?

- **Bidding**
- **Capacity withdrawal**
- **Manipulation of transmission** (deliberate congestion, decongestion)
Course Outline

I. Bottom-up Models of Markets: Philosophy
II. Review of Operations & Planning Models
   A. Dispatch
   B. Generation mix
   C. Linearized DC load flow
III. Perfect Competition Market Models
   A. Equivalency Result: Samuelson’s Principle
   B. General Equilibrium Model
IV. Strategic Market Models
   A. Basic concepts
   B. Simple Nash-Cournot example
   C. Transmission-Constrained Cournot model
   D. Advanced Models
V. Conclusions

Overview of Lecture

Multifirm Market Models with Strategic Interaction

Single Firm Models
Design/Investment Models
Operations/Control Models

Single Firm Models
Design/Investment Models
Operations/Control Models

Demand Models

Market Clearing Conditions/Constraints
I. “Process” or “Bottom-Up” Analysis: Company & Market Models

What are bottom-up/engineering-economic models? And how can they be used for policy analysis?

- Explicit representation & optimization of individual elements and processes based on physical relationships

Process Optimization Models

Elements:
- Decision variables. E.g.,
  - Design: MW of new combustion turbine capacity
  - Operation: MWh generation from existing coal units
- Objective(s). E.g.,
  - Maximize profit or minimize total cost
- Constraints. E.g.,
  - $\Sigma$ Generation = Demand
  - Respect generation & transmission capacity limits
  - Comply with environmental regulations
  - Invest in sufficient capacity to maintain reliability

Traditional uses:
- Evaluate investments under alternative scenarios (e.g., demand, fuel prices) (3-40 yrs)
- Operations Planning (8 hrs - 5 yrs)
- Real time operations (<1 second - 1 hr)
Bottom-Up/Process Models
vs. Top-Down Models

- Bottom-up models simulate investment & operating decisions by an individual firm:
  - E.g., capacity expansion, production costing models
  - Individual firm models can be assembled into market models
- Top-down models start with an aggregate market representation (e.g., supply curve for power, rather than outputs of individual plants).
  - Often consider interactions of multiple markets
  - E.g., National energy models

Functions of Process Model: Firm Level Decisions

Real time operations:
- **Automatic protection** (<1 second): auto. generator control (AGC) methods to protect equipment, prevent service interruptions. (Responsibility of: Independent System Operator ISO)
- **Dispatch** (1-10 minutes): optimization programs (convex) min. fuel cost, s.t. voltage, frequency constraints (ISO or generating companies GENCOs)

Operations Planning:
- **Unit commitment** (8-168 hours). Integer NLPs choose which generators to be on line to min. cost, s.t. “operating reserve” constraints (ISO or GENCOs)
- **Maintenance & production scheduling** (1-5 yrs): schedule fuel deliveries & storage and maintenance outages (GENCOs)
Firm Decisions Made Using Process Models, Continued

Investment Planning
- **Demand-side planning** (3-15 yrs): implement programs to modify loads to lower energy costs (consumer, energy services cos. ESCOs, distribution cos. DISCOs)
- **Transmission & distribution planning** (5-15 yrs): add circuits to maintain reliability and minimize costs/ environmental effects (Regional Transmission Organization RTO)
- **Resource planning** (10 - 40 yrs): define most profitable mix of supply sources and D-S programs using LP, DP, and risk analysis methods for projected prices, demands, fuel prices (GENCOs)

Pricing Decisions
- **Bidding** (1 day - 5 yrs): optimize offers to provide power, subject to fuel and power price risks (suppliers)
- **Market clearing price determination** (0.5- 168 hours): maximize social surplus/match offers (Power Exchange PX, marketers)

Emerging Uses of Process Models
- Profit maximization rather than cost minimization guides firm’s decisions
- Market simulation:
  - Use model of firm’s decisions to simulate market.
  - Paul Samuelson:
    \[
    \text{MAX \{consumer + producer surplus\}} \Leftrightarrow \text{Marginal Cost Supply = Marg. Benefit Consumption} \Leftrightarrow \text{Competitive market outcome}
    \]
  - Other formulations for imperfect markets
- Price forecasts
- Effects of environmental policies on market outcomes (costs, prices, emissions & impacts, income distribution)
- Effects of market design & structure upon market outcomes
Advantages of Process Models for Policy Analysis & Market Design

Explicitness:
- Model changes in technology, policies by altering:
  - decision variables
  - objective function coefficients
  - constraints
- assumptions can be laid bare

Descriptive uses:
- Texture! Detailed impacts of policy
- Costs, emission, technology choices, market prices, consumer welfare

Normative:
- identify better solutions through use of optimization
- show tradeoffs among policy objectives


Basic model
- Cost minimization, pure thermal system, deterministic

In words:
- Choose level of operation of each generator (decision variable),
- …to minimize total system cost (objective)
- …subject to load, capacity limit (constraints)

Decision variable:
\[ y_{it} = \text{megawatt [MW] output of generating unit } i \ (i=1,\ldots,I) \ \text{during period } t \ (t=1,\ldots,T) \]

Coefficients:
- \( CY_{it} \) = variable operating cost [$/MWh] for \( y_{it} \)
- \( X_i \) = MW capacity of generating unit \( i \).
- \( LOAD_t \) = MW demand to be met in period \( t \)
- \( H_t \) = length of period \( t \) [hours/yr]. (Note: in pure thermal system, periods do not need to be sequential)
MIN Variable Cost = $\sum_{i,t} H_t CY_{it} y_{it}$

subject to constraints:

Meet load:

$\sum_i y_{it} = \text{LOAD}_t \quad \forall t$

Generation no more than capacity:

$y_{it} \leq X_i \quad \forall i,t$

Nonnegativity:

$y_{it} \geq 0 \quad \forall i,t$

This is a “Linear Program” (i.e., objective, constraints are linear in decision variables)

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**Operations LP Exercise**

- Two generation types
  - A: Peak: 800 MW, MC = $70/MWh
  - B: Baseload: 1500 MW, MC = $25/MWh

- Load
  - Pk: Peak: 2200 MW, 760 hours/yr
    - OP: Offpeak: 1300 MW, 8000 hours/yr

- Assignment:
  - Write down LP
  - What is best solution (by inspection?)
EXCEL Solver Model for Cost Minimization

Operating Model Formulation, Continued: Complications

- Other objectives
  - Max Profit? Min Emissions?
- Energy storage
  - Pumped storage, batteries, hydropower
- Explicitly stochastic
  - Usual assumption: forced outages are random and independent
- Transmission constraints
- Commitment variables
  - E.g., start-up costs
- Cogeneration
II.A.2. Unit Commitment: A Mixed Integer Program

- Disregard forced outages & fuels; assume:
  - \( u_{it} = 1 \) if unit \( i \) is committed in \( t \) (0 o.w.)
  - \( CU_i \) = fixed running cost of \( i \) if committed
  - \( MR_i \) = “must run” (minimum MW) if committed
  - Periods \( t = 1,\ldots,T \) are consecutive, and \( H_t = 1 \)
  - \( RR_i \) = Max allowed hourly change in output

\[
\begin{align*}
\text{MIN} & \quad \sum_{i,t} CY_{it} y_{it} + \sum_{i,t} CU_i u_{it} \\
\text{s.t.} & \quad \sum_i y_{it} = \text{LOAD}_t & \forall t \\
& \quad \text{MR}_i u_{it} \leq y_{it} \leq X_i u_{it} & \forall i,t \\
& \quad -RR_i \leq (y_{it} - y_{i,t-1}) \leq RR_i & \forall i,t \\
& \quad y_{it} \geq 0 & \forall i,t; \; u_{it} \in \{0,1\} & \forall i,t
\end{align*}
\]

II.A.3. Using Operating Models to Assess NO\(_x\) Regulation

(NO\(_x\): an ozone precursor)

\[
\begin{align*}
N_2 + O_2 + \text{heat} & \rightarrow NO_x \\
NO_x + VOC + & \rightarrow O_3
\end{align*}
\]

- Power plants emit ~1/3 of anthropogenic NO\(_x\) in USA

- Policy question: How effectively can NO\(_x\) limits be met by changed operations (“emissions dispatch”)?
Framework

We want less cost and less NO$_x$

Cost

Efficient

Inefficient

=Alternative dispatch order

How To Generate Alternatives

Solve the following model for alternative levels of the regulatory constraint:

MIN $\sum_i CY_i y_i$

s.t. 1. $MR_i \leq y_i \leq X_i$ (note nonzero LB)

2. $\sum_i y_i \geq LOAD \text{ (MW)}$

3. $\sum_i E_i y_i \leq MASS \text{ CAP (tons)}$

Note: MR, X, LOAD vary (used a stochastic programming method: probabilistic production costing with side constraints)

- Data: 11,400 MW peak and 12,050 MW of capacity, mostly gas and some coal. Most of capacity has same fuel cost/MBTU. Plant emission rates vary by order of magnitude (0.06 - 0.50 lb/MBTU)
**Cost-Emissions Tradeoffs**

- The cost of reducing emissions by 20% is $70M (a 5% increase in fuel cost).

![Graph showing cost vs. Tons NOx/year](chart.png)

### II.B.1. Deterministic Investment Analysis: LP Snap Shot Analysis

Let generation capacity $x_i$ [MW] now be a variable, with (annualized) cost = $CX_i$ [$/MW/yr]

MIN \[ \sum_{i,t} H_t CY_{it} y_{it} + \sum_i CX_i x_i \]

s.t. \[ \sum_i y_{it} = LOAD_t \quad \forall t \]

\[ \sum_f y_{it} - x_i \leq 0 \quad \forall i,t \]

\[ \sum_i x_i \geq LOAD_1 (1+M) \quad \text{("reserve margin" constraint)} \]

\[ y_{it} \geq 0 \quad \forall i,t \]

\[ x_i \geq 0 \quad \forall i \]
Some Complications

- Dynamics (timing of investment)
- Plants available only in certain sizes
- Retrofit of pollution control equipment
- Construction of transmission lines
- “Demand-side management” investments
- Uncertain future (demands, fuel prices)
- Other objectives (profit)

Planning LP Exercise

- Two generation types
  A: Peak: 800 MW, MC = $70/MWh
    - Operating Cost = $70/MWh
    - Capital Cost = $70,000 / MW/yr
  B: Baseload:
    - Operating Cost = $25/MWh
    - Capital Cost = $120,000 / MW/yr

- Load
  - Peak: 2200 MW, 760 hours/yr
  - Offpeak: 1300 MW, 8000 hours/yr
  - Reserve Margin: 15%

- Assignment:
  - Write down LP
  - What is best solution (by inspection?)
Example Capacity Expansion Analysis: Costs of Maryland Joining the Regional Greenhouse Gas Initiative

- **RGGI**: CO₂ trading program for generators in Northeastern US states
- **Maryland Healthy Air Act (2006)**: Requires a study of the reliability and cost impacts of Maryland joining RGGI
- Also: what are the emissions effects? What is the effect of CO₂ “leakage”? How is this affected by market power?
- **Tools**:
  - Haiku (Resources for the Future competitive market model—includes capacity expansion)
  - JHU market power model

Maryland Joining the Regional Greenhouse Gas Initiative: Findings

*Report, University of Maryland, January 2006 (www.cier.umd.edu)*

- Emissions modestly lower
  - -10% for Maryland; -4% for RGGI
  - Some offset by leakage
- Electricity demand decreases due to DSM programs, consumers save money
- Generator profits drop, but few plant closures
- Maryland Generation Mix in 2015
II.C Including Transmission: or Why Power Transport is Not Like Hauling Apples in a Cart

- Review of “Laws”
- Weird implications
- Calculating “Load Flow”

Review of DC Circuit Laws

- **Ohm’s Law:**
  - $V_A - V_B = I_{AB} \cdot R_{AB}$
  - Voltage difference proportional to current * resistance

- **Kirchhoff’s Current Law:**
  - No net current inflow to a node
  - $\sum_j I_{Aj} = 0$

- **Kirchhoff’s Voltage Law:**
  - Sum of voltage differences around any loop = 0
  - $(V_A - V_B) + (V_B - V_C) + (V_C - V_A) = 0$
  - Sub in Ohm’s Law: $I_{AB} \cdot R_{AB} + I_{BC} \cdot R_{BC} + I_{CA} \cdot R_{CA} = 0$

- **Losses:**
  - $L_{AB} = I_{AB}^2 \cdot R_{AB}$
  - Doubling the current implies four times the losses
Implications of Laws

Use laws to calculate flows

- If you know power generation and consumption at “bus” except the “swing bus”, then ...
- The “load flow” (currents in each line, voltages at each are uniquely determined by Kirchhoff’s two laws!
- This is the “load flow” problem

Some odd byproducts of laws:

- Can’t “route” flow
- Parallel flows
- Transmission paths (e.g., choosing which German-Dutch interface to buy) are a fiction
- What you do affects everyone else
- Adding a line can worsen transmission capacity of system

AC Load Flow is More Complex

- Voltage at each bus is sinusoidal (with RMS amplitude and phase angle), as are line currents
- “Reactive” (vs. “real” power) a result of “reactance” (capacitance and inductance)
- This is the power stored and released in magnetic fields of capacitors and inductors as the current changes direction
- Although reactive power doesn’t do useful work, it causes resistance losses & uses up capacity
"DC" Linearization of AC load flow

Assumptions
- Assume reactance >> resistance
- Voltage amplitude same at all buses
- Changes in voltage angles $\theta_A - \theta_B$ from one end of a line to another is small

Results:
- Power flow $t_{AB}$ proportional to:
  - current $I_{AB}$
  - difference in voltage angle $\theta_A - \theta_B$
- Analogies to Kirchhoff’s Laws:
  - Current law at A: $\Sigma_i y_{iA} = \Sigma_{\text{neighboring } m} t_{Am} + \text{LOAD}_A$
  - Voltage law: $t_{AB}R_{AB} + t_{BC}R_{BC} + t_{CA}R_{CA} = 0$
- Given power injections at each bus, flows are unique

Example of “DC” Load Flow

All lines have reactance = 1

Kirchhoff’s Current Law at C:
$+33 + 67 - 100 = 0$

Kirchhoff’s Voltage Law:
$1*33 + 1*33 + 1*(-67) = 0$

Proportionality!
Proportionality means “Power Transmission Distribution Factors” can be used to calculate flows

All lines have reactance = 1

$PTDF_{mn,jk}$ = the MW flowing from $j$ to $k$, if 1 MW is injected at $m$ and 1 MW is removed at $n$

E.g., $PTDF_{AC,AB} = 0.33$

Principle of Superposition
Using PTDFs to Calculate Total Flow

Total flow from B to C = \( PTDF_{AC,BC} \times 100 + PTDF_{BC,BC} \times 50 \)

\[ = 0.33 \times 100 + 0.67 \times 50 = 67 \text{ MW} \]

Exercise in Transmission Modeling

**Assumptions**

- **Triangle network, equal reactances**
  - Line from A to C: 100 MW limit
- **Two plants:**
  - A: MC = 25 $/MWh
  - B: MC = 70 $/MWh
- **Load:**
  - A: 400 MW
  - B: 500 MW

**What’s the optimal dispatch?**

**What’s the marginal cost of meeting an increase of 1 MW of load at A; at B; at C?**
Linearized Transmission Constraints in Operations LP

\( y_{imt} = \text{MW from plant } i, \text{ at node } m, \text{ during } t \)
\( z_{mt} = \text{Net MW injection at node } m, \text{ during } t \)

\[
\text{MIN Variable Cost} = \sum_m \sum_{i,t} H_t CY_{im} y_{imt}
\]

subject to:

- Net Injection: \( \sum_i y_{imt} - \text{LOAD}_{tm} = z_{mt} \) \( \forall t, m \)
- Injection Balance: \( \sum_m z_{mt} = 0 \) \( \forall t \)
- GenCap: \( y_{imt} \leq X_{im} \) \( \forall i, m, t \)
- Transmission: \( -100 \leq [ \sum_m \text{PTDF}_{mk} z_{mt} ] \leq 100 \) \( \forall k, t \)

\( y_{imt} \geq 0 \) \( \forall i, m, t \)

---

Linearized Transmission Constraints in Operations LP: Exercise Example

\[
\text{MIN Variable Cost} = 25y_A + 70y_B
\]

subject to:

- Net Injection: 
  \( y_A - 400 = z_A \)
  \( y_B - 500 = z_B \)
- Injection Balance: \( z_A + z_B = 0 \)
- Transmission: \( -100 \leq [ 0.33z_A + 0.0z_B ] \leq +100 \)
- Nonnegativity:

Note: In calculating PTDFs, I assume that all injections “sink” at node B

- E.g., injection \( z_A \) at A is assumed to be accompanied by an equal withdrawal -\( z_A \) at B
III. Mathematical Programming Models of Perfectly Competitive Energy Markets

A. An Equivalency Result

- Definition of pure competition market equilibrium:
  - Each player maximizes their profit, subject to fixed prices (no market power)
  - Market clears (supply = demand)

- Assemble:
  - “First order” optimization conditions for players
  - Market clearing
  This yields set of simultaneous equations that can be solved for a market equilibrium

- Same set of equations are first order conditions for a single optimization model (MAX net social welfare)
  - MAX (Area under demand curves)-(Cost)
  - Results in intersection of demand + supply curves

- Widely used in energy policy analysis

Applications of the Pure Competition Equivalency Principle

- MARKAL: Used by Intl. Energy Agency countries for analyzing national energy policy, especially CO₂ policies
- US Project Independence Evaluation System (PIES) & successors
- ICF Coal and Electric Utility Model (http://www.epa.gov/capi/capi/frcst.html)
  - Acid rain, Clear Skies, Clean Air Interstate Rule
- POEMS (http://www.retailenergy.com/articles/cecasum.htm)
  - Economic & environmental benefits of US restructuring
- Some of these modified to model imperfect competition (price regulation, market power)
B. Equilibrium Model Formulations

Meet the FOC’ers:
“First Order Conditions” for Optimization

Let an optimization problem be:

\[
\begin{align*}
\text{MAX } & F(X) \\
\{X\} & \\
\text{s.t.: } & G(X) \leq 0 \\
& X \geq 0
\end{align*}
\]

Assume \( F(X) \) smooth/concave, \( G(X) \) smooth/convex.

A solution \( \{X,\lambda\} \) to the KKT (“Karush-Kuhn-Tucker”) conditions below is optimal for the above problem, and vice versa. “KKTs necessary & sufficient for optimality.”

\[
\begin{align*}
X & \geq 0; \quad \partial F/\partial X - \lambda \partial G/\partial X \leq 0; \\
0 & = X(\partial F/\partial X - \lambda \partial G/\partial X)
\end{align*}
\]

\[
\begin{align*}
\lambda & \geq 0; \quad G(X) \leq 0; \\
\lambda & \geq 0; \quad G(X) = 0
\end{align*}
\]

“Perp” Notation for the FOC’ers:

Let an optimization problem be:

\[
\begin{align*}
\text{MAX } & F(X) \\
\{X\} & \\
\text{s.t.: } & G(X) \leq 0 \\
& X \geq 0
\end{align*}
\]

The KKT’s, written in “perp” notation, are:

\[
\begin{align*}
X & \downarrow \leq X \downarrow \partial F/\partial X - \lambda \partial G/\partial X \leq 0 \downarrow \\
\lambda & \downarrow \leq \lambda \downarrow G(X) \leq 0
\end{align*}
\]
Notation: Each node \( i \) is a separate commodity (type, location, timing)

Consumer: Buys \( d_i \)

Other suppliers

Supplier \( f \): Uses inputs \( x_{fi} \) to produce & sell \( s_f \)

Transporter/Transformer: Uses exports \( t_{Eij} \) from \( i \) to provide imports \( t_{Iij} \) to \( j \)

Players’ Profit Maximization Problems

Consumer at \( i \):
\[
\text{MAX} \quad B_i(d_i) - p_i^* d_i
\]
\[
s.t. \quad d_i \geq 0
\]

Supplier \( f \) at \( i \):
\[
\text{MAX} \quad p_i^* s_f - C_{Sfi}(x_{fi})
\]
\[
s.t. \quad s_{fi} \cdot x_{fi} \geq 0
\]

Transporter for nodes \( i,j \):
\[
\text{MAX} \quad p_j^* t_{Iij} - p_i^* t_{Eij} - C_{Tij}(t_{Eij}, t_{Iij})
\]
\[
s.t. \quad G_{Tij}(t_{Eij}, t_{Iij}) \leq 0 \quad (\theta_{ij})
\]
\[
\quad t_{Eij}, t_{Iij} \geq 0
\]
Players’ Profit Maximization Problems

**Supplier FOCs/KKTs at** \(i\):
--one for each decision variable
--one for each constraint

**Consumer FOCs/KKTs at** \(i\):
--one for each decision variable
--one for each constraint

**Transporter FOCs/KKTs for** \(i,j\):
--one for each decision variable
--one for each constraint

KKTs for All Players in Market Game + Market Clearing Condition

**Consumer FOCs/KKTs, \(\forall i\):**
--one for each decision variable
--one for each constraint

**Supplier FOCs/KKTs, \(\forall i\)**
--one for each decision variable
--one for each constraint

**Transporter/Transformer KKTs, \(\forall ij\)**
--one for each decision variable
--one for each constraint

**Market Clearing, \(\forall i\):**

\[
p_i^* = \frac{\sum_j s_{ji}}{\sum_{j \in I(i)} t_{ji} - \sum_{j \in E(i)} t_{Eij}} - d_i = 0
\]

\(N\) conditions
& \(N\) unknowns!
An Optimization Model for Simulating a Commodity Market

\[
\text{MAX} \quad (\text{Value of Consumption}) - (\text{Production, Transport Cost})
\]

\[
\text{MAX} \quad \sum_i B_i(d_i) - \sum_i C_{Gi}(x_{fi}) - \sum_{ij} C_{Ti}(t_{Eij}, t_{lij})
\]

s.t.  Production Functions for each firm:

\[
G_{Si}(s_i, x_i) \leq 0, \forall i
\]

\[
G_{Ti}(t_{Eij}, t_{lij}) \leq 0, \forall \text{ij}
\]

Market Clearing for each commodity:

\[
\sum_t s_{fi} + \sum_{j \in I(i)} t_{jii} - \sum_{j \in E(i)} t_{Eij} - d_i = 0, \forall i
\]

...and the usual nonnegativity conditions

Its FOC conditions = market equilibrium conditions for the purely competitive commodities market! So:

- a single NLP can simulate a market
- a purely competitive market maximizes social surplus

---

An Optimization Model for Simulating a Competitive Energy Market

\[
\text{MAX Social Surplus} = \sum_t B(d_t) - \sum_{i,t} H_t CY_{it} y_{it}
\]

subject to constraints:

Meet load:

\[
-\sum_i y_{it} + d_t = 0 \quad \forall t
\]

Generation no more than capacity:

\[
y_{it} \leq X_i \quad \forall i, t
\]

Nonnegativity:

\[
y_{it} \geq 0 \quad \forall i, t
\]

This is a “Quadratic Program” (i.e., objective, constraints are either linear or quadratic in decision variables)
**Perfect competition**

- Company 1: \( CY_1 = 2, \quad X_1 = 5 \)
- Company 2: \( CY_2 = 3, \quad X_2 = 5 \)
- Demand: \( p = 10 - d = 10 - (y_1 + y_2) \)

**Demand function**

\[ p = 10 - (y_1 + y_2) \]

**Equilibrium in perfect competition**

\[ d = y_1 + y_2 \]

**Competitive supply function**

**Integral under demand curve minus production costs**

CONSUMER SURPLUS

PROFIT

\[ d = y_1 + y_2 \]
Excel Solver Perfect Competition Model

General Procedure for Building Equilibrium Models

Not all equilibrium problems can be formulated as optimization problems

- Complementarity models are more general
  - Some but not all complementarity equilibrium problems have an equivalent optimization problem
  - But all convex optimization problems have an equivalent equilibrium (KKT) problem

Five steps:
1. Formulate optimization submodel for each market party
2. Derive KKTs for each party’s submodel
3. Create a complementarity problem consisting of those conditions for all parties plus market clearing
   - Should be as many conditions (either perp or equality) as variables. As check, associate one variable with each condition
   - Types of complementarity problems include linear/nonlinear, nonmixed/mixed (without or with equality conditions, each with a matching unrestricted variable)
4. Analyze resulting problem for existence, uniqueness, other properties
5. Parameterize & solve
III.C. Commodity Modeling

Exercise

1. Draw a diagram representing the following market structure:
   - Two electricity companies in California
     - Use two commodities as inputs:
       1. NOx emissions allowances
       2. Natural gas
     - Sell power in offpeak and peak electricity markets
   - Supply of NOx emission allowances auctioned by EPA
   - Natural gas produced by companies in Texas, and piped to California

2. Write an optimization problem that gives an equivalent solution

3. Homework: Write optimization problem for each party & derive a complementarity problem (in very general terms) that would represent a competitive equilibrium
   - Assume all parties are ‘price takers’

IV. Strategic Market Modeling: Oligopoly

A. Concepts

- Oligopoly or imperfect competition is the most representative market structure in real electric power markets
  - Small number of large generating firms.

- Imperfect market analysis and modelling is more complex
  - Each generator must bear in mind the interdependence between its decisions and the decisions of all other agents
  - This strategic interdependence varies with the time horizon of the decisions to be made
Market Power = Ability to manipulate prices persistently to one’s advantage, independently of the actions of others

**Generators**: The ability to raise prices above marginal cost by restricting output

Generators may be able to exercise market power because of:
- economies of scale
- large existing firms
- transmission costs, constraints
- siting constraints, long lead time for generation construction
- dumb market designs

---

**Types of Games**

- **Noncooperative Games (Symmetric):** Each player has same “strategic variable”
  - Each player implicitly assumes that other players won’t react.
  - “Nash Equilibrium”: no player believes it can do better by a unilateral move

- Let $\pi_i(X_i, X_{-i}) = i$’s profit, a function of $i$’s strategy $X_i$ and everyone else’s strategy $X_{-i}$

- Nash equilibrium $\{X_i^*, X_{-i}^*\}$ occurs if:
  $$\pi_i(X_i^*, X_{-i}^*) \geq \pi_i(X_i, X_{-i}^*)$$
  for all feasible $X_i$, and for all $i$
Types of Games (Continued)

- **Examples of Nash Games:**
  - **Bertrand (Game in Prices).** Implicit: You believe that market prices won’t be affected by your actions, so by cutting prices, you gain sales at expense of competitors.
  - **COMPETITIVE COMMODITY MODEL!**
  
  - **Cournot (Game in Quantities):** Implicit: You believe that if you change your output, your competitors will maintain sales by cutting or raising their prices
  
  - **Supply function (Game in Bid Schedule):** Implicit: You believe that competitors won’t alter supply functions they bid

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Digression: History Quiz

**What was the profession of John Nash’s father?**

Electric power engineering
Types of Games (Continued)

- **Noncooperative Game (Asymmetric/Leader-Follower):** Leader knows how followers will react.
  - E.g.: strategic generators anticipate:
    - how a passive ISO prices transmission
    - competitive fringe of small generators, consumers
  - “Stackelberg Equilibrium”
  - Multiple leaders possible:
    - Several large generators competing a la Nash with each other, but each anticipating reaction of ISO (transmission pricing) and fringe generators (outputs)

- **Cooperative Game (Exchangable Utility/Collusion):** Max joint profit.
  - E.g., competitors match your changes in prices or output

Three General Generator-Transmission Games

**Simultaneous Game:** Each takes other’s decisions as fixed

- Gen 1 Sales 1 ... Gen n Sales n Trans price ISO

**Sequential Game:**
- Single Leader anticipates Follower’s reactions
- Follower takes leader’s decisions as fixed

**Multiple Leader-Follower:**
- Each leader anticipates follower’s reactions
- Each leader takes other’s decisions as fixed
B. Computation Methods for Nash (Simultaneous) Games

**Simple Example**

1. **Payoff Matrix:** Enumerate all combinations of player strategies; look for stable equilibrium

2. **Iteration/Diagonalization/Alternate Play/Gauss-Seidel:** Simulate player reactions to each other until no player wants to change

3. **Direct Solution of Equilibrium Conditions:** Collect FOCs/KKTs for all players; add market clearing conditions; solve resulting system of conditions directly
   - Usually involves complementarity conditions

4. **Equivalent Optimization:** May exist a single optimization model that gives same solution (“Hashimoto”)

---

**Strategic Modeling Exercise**

- Two Cournot generators (competing on quantity)
  - Sell output in ISO day ahead market
    - Strategic variables is quantity bid
  - “Locational marginal pricing” – “first price auction” -- market clearing price
  - Equivalent to bilateral contracting with efficient arbitrage

- Solve example with 4 methods

- Variant: “Pay as Bid”
  - Strategic variable is price bid
  - No single price; if cut price, you might sell more, but at a lower price
  - Also try to solve with payoff matrix
Strategic Modeling Exercise: Cournot/Quantity Model

- Each firm \( i \)'s marginal cost = \( y_i \), \( i = A, B \) (Total cost = \( 0.5 y_i^2 \))
- Demand function: \( p = 100 - d/2 \) [\$/MWh]

**Firm A:**
\[
\max \quad \pi_A(y_A, y_B) = P(y_A + y_B)y_A - C_A(y_A) = (100 - 0.5(y_A + y_B))y_A - 0.5y_A^2
\]
\[\text{s.t. } y_A > 0\]
- KKTs: \( 0 \leq y_A \perp P + P y_A - MC_A \leq 0 \)
  \( \text{or} \quad 0 \leq y_A \perp (100 - y_A - 0.5y_B) - y_A \leq 0 \)

**Firm B:**
\[
\max \quad \pi_B(y_A, y_B) = \max (100 - 0.5(y_A + y_B))y_B - 0.5y_B^2
\]
\[\text{s.t. } y_B > 0\]
- KKTs: \( 0 \leq y_B \perp (100 - y_A - 0.5y_B) - y_B \leq 0 \)

**Market clearing:** \( d = y_A + y_B \)
- The market participant’s KKTs + market clearing form a complementarity problem
**Method 1:**
Find cell such that $\pi_A$ is highest in column (Firm A maximizes its profit given $y_B$) and $\pi_B$ is highest in row (Firm B maximizes its profit given $y_A$). In the below table, **Bold italics** represents Firm A’s best response to $y_B$, while **Bold** represents Firm B’s best response to $y_A$. The format of the table is:

<table>
<thead>
<tr>
<th>$y_A$</th>
<th>$y_B$</th>
<th>$\pi_A$</th>
<th>$\pi_B$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>$y_A$</th>
<th>$y_B$</th>
<th>$\pi_A$</th>
<th>$\pi_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1650</td>
<td>1520</td>
<td>1734</td>
</tr>
<tr>
<td>32</td>
<td>1604</td>
<td>1664</td>
<td>1700</td>
</tr>
<tr>
<td>34</td>
<td>1590</td>
<td>1632</td>
<td>1666</td>
</tr>
<tr>
<td>36</td>
<td>1500</td>
<td>1600</td>
<td>1632</td>
</tr>
<tr>
<td>38</td>
<td>1530</td>
<td>1568</td>
<td>1592</td>
</tr>
<tr>
<td>40</td>
<td>1500</td>
<td>1536</td>
<td>1564</td>
</tr>
<tr>
<td>42</td>
<td>1500</td>
<td>1536</td>
<td>1564</td>
</tr>
<tr>
<td>44</td>
<td>1500</td>
<td>1536</td>
<td>1564</td>
</tr>
<tr>
<td>46</td>
<td>1500</td>
<td>1536</td>
<td>1564</td>
</tr>
<tr>
<td>48</td>
<td>1500</td>
<td>1536</td>
<td>1564</td>
</tr>
<tr>
<td>50</td>
<td>1500</td>
<td>1536</td>
<td>1564</td>
</tr>
</tbody>
</table>

**Method 2: Diagonalization/Iteration Method**

- **Optimal reaction of Firm A to $y_B$** is found by maximizing $\pi_A(y_A,y_B)$ w.r.t. $y_A$. The resulting KKT condition that defines the optimal response $y_A$ is:
  
  
  $0 \leq y_A \perp d\pi_A(y_A,y_B)/dy_A \leq 0$, or:

  $0 \leq y_A \perp (100 - y_A - 0.5y_B) - y_A \leq 0$

- If the optimal $y_A > 0$, then $y_A = 50 - y_B/4$ is the optimal reaction. A similar development given B’s optimal reaction to $y_A$, $y_B = 50 - y_A/4$.

- Tennis anyone?

<table>
<thead>
<tr>
<th>Iteration #</th>
<th>$y_A$</th>
<th>$y_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>70</td>
<td>initial point</td>
</tr>
<tr>
<td>1</td>
<td>32.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>41.875</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>39.531</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>40.117</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>39.971</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>40.007</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>39.998</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>40.0005</td>
<td></td>
</tr>
</tbody>
</table>
Method 3: Mixed Linear Complementarity

Problem Statement

- Mixed LCP statement: *Find* \( \{y_A, y_B, d\} \) *such that the following conditions are satisfied:*
  - *Firm A:* \( 0 \leq y_A \perp (100 - 0.5y_B) - y_A \leq 0 \)
  - *Firm B:* \( 0 \leq y_B \perp (100 - 0.5y_A - y_B) - y_B \leq 0 \)
  - *Market clearing:* \( d = y_A + y_B \)

- Mixed LCP: Has equalities as well as complementarity conditions
- Well-formulated problem will have equal number of variables and conditions
- Could use PATH to solve this problem
  - Lemke’s algorithm
  - Iteratively linearizes for NCP
  - Solution: \( y_A = y_B = 40 \text{ MW}; \ d = 80 \text{ MW}; \ P(y_A + y_B) = 60 \$/MWh

Method 4: Single Equivalent Optimization

Problem (Hashimoto 1985)

- Consider the following MP:
  \[
  \text{MAX} \quad \int_0^d \left(100 - q/2\right) dq - \left(y_A^2/4 + y_B^2/4\right) - C_A(y_A) - C_B(y_B)
  \]
  \[
  \text{s.t.} \quad d - y_A - y_B = 0; \quad y_A, y_B \geq 0
  \]
- First term: integral of demand curve. If the underlined term was omitted, this would be the standard welfare max (perfect competition) model.
  - Underlined term modifies customer value term (integral) so that the derivative of \{integral + underscored term\} w.r.t. \( y_f \) is the marginal revenue (MR) for a Cournot firm \( f \) rather than price.
- KKT conditions =equilibrium conditions (Method 3)
- But it is not always possible to define a single optimization problem whose KKTs match the equilibrium conditions of a hypothetical market
Method 4: Excel Solver Cournot Model

Example of Nonexistence of Pure Strategy Equilibria

**Definitions:**

- **Pure strategy equilibrium:** A firm $i$ chooses $X_i^*$ with probability 1

- **Mixed strategy:** Let the strategy space be discretized $\{X_{ih}, \ h = 1, \ldots, H\}$. In a mixed strategy, a firm $i$ chooses $X_{ih}$ with probability $P_{ih} < 1$. The strategy can be designated as the vector $P_i$
  
  - Can also define mixed strategies using continuous strategy space and probability densities
  
  - Let $P_i^c = \{P_{j}, \ \forall j \neq i\}$

- **Mixed strategy equilibrium:** $\{P_i^*, \ \forall i\}$ is mixed strategy Nash Equilibrium iff:
  
  $$\pi_i(P_i^*, P_i^c) > \pi_i(P_{ij}, P_i^c), \ \forall i; \ \forall P_{i}; \ \Sigma_h P_{ih} = 1, \ P_{ih} > 0$$

- By Nash’s theorem, a mixed strategy equilibrium always exists (perhaps in degenerate pure strategy form) if strategy space finite.
Another Example But with a Difference: Pay as Bid Case:

Payoff Matrix method: Find cell such that \( n_A \) is highest in column (Firm A bids to maximize its profit given \( BID_B \)) and \( n_B \) is highest in row (Firm B maximizes its profit given \( BID_A \))

**Bold italics** represents Firm A’s best bid response to \( BID_B \)

**Bold** represents Firm B’s best bid response to \( BID_A \)

<table>
<thead>
<tr>
<th>BID_A \ BID_B</th>
<th>2’s Bid</th>
<th>1’s Bid</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( n_A )</td>
<td>( n_B )</td>
</tr>
</tbody>
</table>

Note that there is no single cell that is the best response by both firms!

---

C. A Cournot Transmission-Constrained Model

**Features:**

- Bilateral market (generators sell to customers, buy transmission services from ISO)
- Cournot in power sales
- Generators assume transmission fees fixed; linearized DC load flow formulation
- If there are arbitragers, then same as POOLCO Cournot model
  - In which generators sell to “single buyer”
- Mixed LCP formulation: allows for solution of very large problems
Adding Transmission (and other commodities!)

NOx Market

Power Market

Transmission Constraint

Simple Example of Model:
Generation Duopoly with Arbitrage and Transmission Constraint

\[ P_1(d_1) \]

\[ P_2(d_2) \]
Perfect Competition Model

Everyone a price taker w.r.t. nodal energy prices $P_1$, $P_2$, and transmission price $W_{1,2}$

**Equilibrium problem:** Find $\{p_1, p_2, w_{1,2}, s_{A1}, s_{A2}, s_{B1}, s_{B2}, a_{1,2}, y_{1,2}\}$ that simultaneously solve the following problems:

Gen A: Given $\{p_1, p_2, w_{1,2}\}$:

$$\text{MAX } p_1s_{A1} + p_2s_{A2} - w_{1,2}s_{A2} - C_A(s_{A1} + s_{A2})$$

$s_{A1}, s_{A2} \geq 0$

Gen B: Given $\{p_1, p_2, w_{1,2}\}$:

$$\text{MAX } p_1s_{B1} + p_2s_{B2} + w_{1,2}s_{B1} - C_B(s_{B1} + s_{B2})$$

$s_{B1}, s_{B2} \geq 0$

Arbitrager: Given $\{p_1, p_2, w_{1,2}\}$:

$$\text{MAX } (p_2 - p_1 - w_{1,2})a_{1,2}$$

$s_{A1}, s_{A2}, s_{B1}, s_{B2} \geq 0$

TSO: Given $\{w_{1,2}\}$:

$$\text{MAX } w_{1,2}y_{1,2}$$

$s_{1,2} \geq 0$

s.t. $-50 \leq y_{1,2} \leq 50$

Market clearing:

$$d_1 = -a_{1,2} + s_{A1} + s_{B1}$$

$$d_2 = +a_{1,2} + s_{A2} + s_{B2}$$

$$y_{1,2} = a_{1,2} + s_{A2} - s_{B1}$$

Consumer 1: Given $\{p_1\}$:

$$\text{MAX } \int_0^{d_1} P_1(x)dx - p_1d_1$$

s.t. $-50 \leq d_1 \leq 50$

Consumer 2: Given $\{p_2\}$:

$$\text{MAX } \int_0^{d_2} P_2(x)dx - p_2d_2$$

s.t. $-50 \leq d_2 \leq 50$

Arbitrager: Given $\{p_1, p_2, w_{1,2}\}$:

$$\text{MAX } (p_2 - p_1 - w_{1,2})a_{1,2}$$

s.t. $-50 \leq y_{1,2} \leq 50$

Under mild conditions, solution (1) exists, (2) is unique
**Oligopolistic Generation**

Naïve assumption that Generators are Bertrand (price takers) with respect to transmission costs $W$ (e.g., Wei & Smeers, 2000)

**Equilibrium Problem:** Find $\{w_{1,2}, s_{A1}, s_{A2}, s_{B1}, s_{B2}, a_{1,2}, y_{1,2}\}$ that simultaneously solve the following problems:

**Gen A:** Given $\{w_{1,2}, s_{B1}, s_{B2}, a_{1,2}\}$:

$\text{MAX } P_1(d_1) s_{A1} + P_2(d_2) s_{A2}$

$s.t.$

$d_1 = -a_{1,2} + s_{A1} + s_{B1}$
$d_2 = a_{1,2} + s_{A2} + s_{B2}$

**Gen B:** Given $\{w_{1,2}, s_{A1}, s_{A2}, a_{1,2}\}$:

$\text{MAX } P_1(d_1) s_{B1} + P_2(d_2) s_{B2}$

$s.t.$

$d_1 = -a_{1,2} + s_{A1} + s_{B1}$
$d_2 = a_{1,2} + s_{A2} + s_{B2}$

**Arbitrager:** Given $\{p_1, p_2, w_{1,2}\}$:

$\text{MAX } (p_2 - p_1 - w_{1,2}) a_{1,2}$

$s.t.$

$y_{1,2} = a_{1,2} + s_{A2} - s_{B1}$

**TSO:** Given $\{w_{1,2}\}$:

$\text{MAX } w_{1,2} y_{1,2}$

$s.t.$

$-50 \leq y_{1,2} \leq 50$

**Market clearing:** $y_{1,2} = a_{1,2} + s_{A2} - s_{B1}$

**Oligopolistic Generation Model**

Derive KKTs for each player’s problem; combine with market clearing conditions. After rearrangement, we get:

**Mixed LCP:** Find $\{p_1, p_2, w_{1,2}, s_{A1}, s_{A2}, s_{B1}, s_{B2}, a_{1,2}, y_{1,2}, \lambda_{1,2}, \lambda_{1,2}^+\}$ that simultaneously solves the following \textit{mixed complementarity problem}:

**Gen A:**

$0 \leq s_{A1} \perp (P_1 - P_1' s_{A1} - C_A') \leq 0$
$0 \leq s_{A2} \perp (P_2 - P_2' s_{A2} - w_{1,2} - C_A') \leq 0$

**Gen B:**

$0 \leq s_{B1} \perp (P_1 - P_1' s_{B1} + w_{1,2} - C_B') \leq 0$
$0 \leq s_{B2} \perp (P_2 - P_2' s_{B2} - C_B') \leq 0$

**Arbitrager:**

$P_1 - P_2 - w_{1,2} = 0$

**TSO:**

$w_{1,2} - \lambda_{1,2}^+ + \lambda_{1,2}^- = 0$
$0 \leq \lambda_{1,2}^+ \perp y_{1,2} - 50 \leq 0$
$0 \leq \lambda_{1,2}^- \perp -y_{1,2} - 50 \leq 0$

**Market clearing:**

$d_1 = -a_{1,2} + s_{A1} + s_{B1}$
$d_2 = a_{1,2} + s_{A2} + s_{B2}$
$y_{1,2} = a_{1,2} + s_{A2} - s_{B1}$

Under mild conditions, solution to resulting MCP (1) exists, (2) is unique, and (3) is equivalent to POOLCO Cournot equilibrium.
Simple Example of Model: Generation Duopoly with Arbitrage and Transmission Constraint

D. A Large Scale Cournot Bilateral & POOLCO Model

Features:
• Bilateral market (generators sell to customers, buy transmission services from ISO)
• Cournot in power sales
• Generators assume transmission fees fixed; linearized DC load flow formulation
• Mixed “linear complementarity” formulation: allows for solution of very large problems
A Large Scale Implementation: Eastern Interconnection Model

- 100 nodes representing control areas and 15 interconnections with ERCOT, WSSC, and Canada
- 829 firms (of which 528 are NUGs)
- 2725 generating plants (in some cases aggregated by prime mover/fuel type/costs); approximately 600,000 MW capacity
- Implemented by FERC staff
  - Spatial market power issues (congestion, addition of transmission constraints)
  - Effects of mergers

Eastern Interconnection Model

- 814 flowgates, each with PTDFs for each node (most flowgates and PTDFs defined by NERC; a mix of physical and contingency flowgate limits)
- 68 firms represented as Cournot players (with capacity above 1000 MW). Remainder is competitive fringe
Selected Market Power Results

Merger Example
(Firm A at Node A, Firm B at Node B)

pre-merger | merger
---|---
Competition | $22.17 | $22.17
Cournot w/ arbitrage | $22.85 | $22.86
Difference | 3.07% | 3.11%
Cournot Price Node A | $18.89 | $18.92
Cournot Price Node B | $27.65 | $27.66
Profits | A+B=$162,723 | AB=$162,400
Section E: Advanced Models

Desirable Improvements

(thanks to R. Baldick, 2006)

- Improved models of Physical system
  - Better representation of technology constraints
  - The economist’s “production function”

- Improved models of Commercial system
  - Definition of products / markets
  - “Settlement rules”: who gets paid what in each market

- Improved models of Economic system
  - Agent objectives
  - Agent strategic variables
  - Agent state of knowledge / expectations
  - Agent cooperation

Examples of Improved Models: Generation

- Better physical models
  - Multiple periods and hydro (Bushnell 2002)
  - Capacity additions
    - Make capacity & energy decisions at same time (“open loop”) (Wei, Smeers, 1999)
    - Make capacity decisions anticipating effect on energy market (“closed loop”) (Murphy, Smeers, 2004)
  - Emissions permits markets (*)

- Better commercial models
  - Locational operating reserves markets (Helman 2002; Bautista et al. 2005)
  - Two-settlement systems: day ahead (perhaps zonal pricing) and real-time (locational) (EPEC!) (Kamat & Oren, 2004)

- Better economic models
  - Forward contracts:
    - Exogenous contracts (Green 2002)
    - Endogenous contracts: Two stage models (EPECs!) (Yao, Oren, Adler 2005)
  - Anticipate supply response of rivals
    - Include fringe’s KKTs in leader’s constraint set (MPEC!) (Neuhoff et al.)
    - “Conjectured supply response” (Day/Hobbs 2002)
    - Inverse problem (estimate conjectural variations) (Garcia-Alcalde et al. 2002)
  - Tacit collusion (multiperiod “supergames”) (Liu, Harrington, Hobbs, Pang, 2005)
Examples of Improved Models: Transmission

- Better physical models:
  - Linearized DC Load flow model (*)
    - TSO constraints involve PTDFs
    - Quadratic losses
  - AC Load flow model (Anjos, Bautista et al. 2006)
  - Controllable DC lines, phase shifters in linearized DC load flow (Hobbs et al. 2006)

- Better commercial models:
  - Commercial rules result in (economically) imperfect transmission pricing
    - Path-based models (Hobbs, Rijkers et al. 2003)
    - No-netting of flows (use nonnegative flow variables for each direction) (Hobbs, Rijkers et al. 2003)
    - Average cost-based tariffs (Wei and Smeers, 2000)

- Better economic models:
  - Generators anticipate transmission price changes
    - Include TSO KKTs as constraints: MPEC (e.g., Cardell/Hitt/Hogan 1997; Hobbs/Metzler/Pang 2000; Borenstein/Busmen/Stoft 2000)
    - Or “conjectured transmission price response” (MCP) (Hobbs/Rijkers 2003)

Example of a Stackelberg (Leader-Follower) Model

- Large supplier as leader, ISO & other suppliers as followers in POOLCO market

- Problem: choose bids $B_{Li}$ to max $\pi_L$
  $$\text{MAX } \pi_L = \sum_i [P_i y_{Li} - C_i(y_{Li})]$$
  $$\text{s.t. } 0 < y_{Li} < X_i, \forall i$$
  KKTs for ISO (depend on $B_{Li}$’s)
  KKTs for other suppliers (price takers)

- The Challenge: the complementarity conditions in the leader’s constraint set render the leader’s problem non-convex (i.e., feasible region non-convex)

- Algorithms for math programs with equilibrium constraints (MPECs) and equilibrium programs with equilibrium constraints (EPECs) are improving
EPEC

Sophisticated assumption that Generators are Stackelberg leader with respect to transmission costs \( w \) (e.g., Hobbs, Metzler, Pang, 2000)

**Equilibrium Problem:** Find \( \{w_{1,2}, s_{A1}, s_{A2}, s_{B1}, s_{B2}, a_{1,2}, y_{1,2}\} \) that simultaneously solve the following problems:

**Gen A:** Given \( \{w_{1,2}, s_{B1}, s_{B2}, a_{1,2}\} \):
\[
\text{MAX} \quad P_1(d_1)s_{A1} + P_2(d_2)s_{A2} - w_{1,2}s_{A2} - C_A(s_{A1} + s_{A2})
\]
\[
s.t. \quad d_1 = -a_{1,2} + s_{A1} + s_{B1}
\]
\[
d_2 = +a_{1,2} + s_{A2} + s_{B2}
\]

**TSO:**
\[
50 \geq y_{1,2} - \lambda \leq 0 \quad -50 \leq y_{1,2} - \lambda \leq 0
\]

**Arbitrager:**
\[
p_1 - p_2 - w_{1,2} = 0
\]

**Market clearing:**
\[
y_{1,2} = a_{1,2} + s_{A2} - s_{B1}
\]

**Gen B:** Given \( \{s_{A1}, s_{A2}\} \):
\[
\text{MAX} \quad p_1(d_1)s_{B1} + p_2(d_2)s_{B2} + W_{1,2}s_{B1} - C_B(s_{B1} + s_{B2})
\]
\[
s.t. \quad d_1 = -a_{1,2} + s_{A1} + s_{B1}
\]
\[
d_2 = +a_{1,2} + s_{A2} + s_{B2}
\]

**TSO:**
\[
50 \geq y_{1,2} - \lambda \leq 0 \quad -50 \leq y_{1,2} - \lambda \leq 0
\]

**Arbitrager:**
\[
p_1 - p_2 - w_{1,2} = 0
\]

**Market clearing:**
\[
y_{1,2} = a_{1,2} + s_{A2} - s_{B1}
\]

Iteration/Diagonalization (Gauss-Seidel) among MPECs often used
Generally: pure strategy solutions may neither exist nor be unique

---

**Stackelberg Analysis**

**L’s decisions** \( x_L \):
- Allowances bought \( q_{NOx,L} \)
- Energy decisions \( y_{i,A}, s_{i,L} \)

**Stackelberg Leader**

**P^{NOx}(x_L)**
- \( P_i(x_L) \)
- \( W_i(x_L) \)

---

Stackelberg Leader’s Problem

The firm with a longest position in NOx market and greatest power sales is designated as the leader.

\[ q^w = \text{Stackelberg's NO}_x \text{ withholding variable [tons]} \]
\[ q_{f}^{\text{NO}_x} = \text{Firm's available NO}_x \text{ allowances [tons]} \]

\[
\text{MAX}_{s_{if}, g_{if}, q^w} \sum_i \left\{ p_i \left( s_{if} + \sum_{g 
eq f} s_{ig} \right) - W_i \right\} s_{if} - \left[ C_{if}(y_{if}) - W_i y_{if} \right] - p_{NO_x}^i \left[ E_{if}^{NO_x} - (q_{f}^{NO_x} - q^w) \right] \]
\]

\[ s.t.: \quad y_{if} \leq \text{CAP}_{if}, \forall i \]
\[ \sum_i s_{if} = \sum_i y_{if} \]
\[ s_{if}, y_{if} \geq 0, \forall i \]
\[ 0 \leq q^w \leq q_{f}^{NO_x} \]
\[ 0 \leq p_{NO_x}^i \downarrow \sum_i (E_{if}^{NO_x} - q_{f}^{NO_x}) + q^w \leq 0 \]

- Other Producer & TSO KKT Conditions
- Market Clearing Conditions

ISO Optimization Problem

Quadratic Loss Functions

- ISO’s decision variables:
  \[ z_i = \text{transmission service from hub to } i \]
  \[ q_{i}^{\text{Losses}} = \text{make-up loss from node } i \]
  \[ t_{ij} = \text{flow in arc } (i,j) \]
- ISO’s maximizes the “value of services”:

\[
\text{MAX}_{\pi_{ISO}(t_{ij}, z_i, q_{i}^{\text{Losses}})} = \sum_i \left( W_i z_i - p_i q_{i}^{\text{Losses}} \right) \\
\text{s.t.: } z_i - q_{i}^{\text{Losses}} + \sum_{j \in \mathcal{V}(i)} (t_{ij} - (1 - L_{ij}^{'} t_{ij}^{'}) t_{ij}^{''}) \leq 0, \quad \forall i \\
\sum_{i,j \in \mathcal{V}(k)} R_{ij} (t_{ij} - t_{ji}^{'}) = 0, \quad \forall k, (i,j) \in \mathcal{V}(k) \\
\sum_i z_i = 0 \\
0 \leq t_{ij} \leq T_{ij}, \quad \forall i, j \\
q_{i}^{\text{Losses}} \geq 0, \quad \forall i \\
\]

- Kirchhoff’s Current Law
- Kirchhoff’s Voltage Law
- Services Balance

- Solution allocates transmission to most valuable transactions
- Define the model’s KKTs (complementarity conditions), one per variable \( x_{ISO} \)
Model Statistics

- 18,618 variables; 9739 constraints
  - Order of magnitude larger than test problems in R. Fletcher and S. Leyffer, “Numerical Experience with Solving MPECs as NLPs,” Univ. of Dundee, 2002
- Solved by PATH and SQP (SNOPT, FILTER) (Thanks to Todd Munson & Sven Leyffer!)
- 9,536 seconds (1.8 MHz Pentium 4)
  - Other MPECs took much less time

Stackelberg Results

Compared to the Cournot Case:

- Stackelberg leader:
  - withholds 5,536 tons of allowances (7.2% of total available)
  - … increasing NO\textsubscript{x} price from 0 to 1,173 [$/ton]
- Output:
  - other producers shrink their power sales (87.4→83.5 \times 10^6 \text{ MWh}) due to increased NO\textsubscript{x} price
  - … while the leader expands its output (24.6→28.7 \times 10^6 \text{ MWh})
- Profit:
  - Stackelberg leader earns more profit (893 → 970 \text{ M$})
  - … at the expense of other producers (2394 → 2273 \text{ M$})
- Consumers:
  - are only marginally better off with a gain of 14 [M$] in consumer surplus, as power prices are essentially unchanged
V. Conclusion

- There are practical market models that capture key features of the market:
  - Kirchhoff’s current & voltage laws,
  - transmission pricing,
  - generator strategic behavior
- Market features present computational & analytical challenges for the power engineering/O.R. researcher
- Prices can’t be predicted precisely because games are repeated, and conjectural variations are fluid and more complex than can be modeled. Models most useful for exploring issues/gaining insight--thus, simpler models preferred
  - MODELS ARE FOR INSIGHT, NOT NUMBERS!!!
- Apply to market structure evaluation, market design, and strategic pricing
- Need comparisons of model results with each other, and with actual experience

Operations LP Answer: Model Formulation

\[
\begin{align*}
\text{MIN} & \quad 760(70y_{A,Pk} + 25y_{B,Pk}) \\
& \quad + 8000(70y_{A,OP} + 25y_{B,OP})
\end{align*}
\]

subject to:

Meet load:
\[
\begin{align*}
y_{A,Pk} + y_{B,Pk} &= 2200 \\
y_{A,OP} + y_{B,OP} &= 1300
\end{align*}
\]

Generation ≤ capacity:
\[
\begin{align*}
y_{A,Pk} &\leq 800; \quad y_{A,OP} \leq 800 \\
y_{B,Pk} &\leq 1500; \quad y_{B,OP} \leq 1500
\end{align*}
\]

Nonnegativity: \( y_{A,Pk}, y_{A,OP}, y_{B,Pk}, y_{B,OP} \geq 0 \)
Operations LP Answer: Load Duration Curve

Load Duration Curve

Planning LP Answer: Model Formulation

MIN \(760(70 y_{A,Pk} + 25 y_{B,Pk}) + 8000(70 y_{A,OP} + 25 y_{B,OP})\)
+ 70,000 \(x_A\) + 120,000 \(x_B\)

subject to:
Meet load:
\(y_{A,Pk} + y_{B,Pk} = 2200\)
\(y_{A,OP} + y_{B,OP} = 1300\)

Generation ≤ capacity:
\(y_{A,Pk} - x_A \leq 0\);
\(y_{A,OP} - x_A \leq 0\)
\(y_{B,Pk} - x_B \leq 0\);
\(y_{B,OP} - x_B \leq 0\)

Reserve:
\(x_A + x_B \geq 1.15 \times 2200\)

Nonnegativity:
\(y_{A,Pk}, y_{A,OP}, y_{B,Pk}, y_{B,OP} \geq 0\)
Exercise in Transmission Modeling: Answer

**Optimal Dispatch**

- **Two plants:**
  - A: Meet load at A (400 MW) plus inject maximum amount that transmission limit allows (100 MW/PTDF = 100/.33 = 300 MW)
    - = 700 MW
  - B: Serve the load at B not served by A (= 500 MW-300 MW)
    - = 200 MW

**Marginal Costs (“LMP”) to Load:**

- A: The cost of Plant A ($25)
- B: The cost of Plant B ($70)
- C: More complex! To bring 1 MW to C, you can back off 1 MW at B and expand 2 MW at A:
  - = -$70 + 2*$25 = -$20
Commodity Markets Exercise
(Rectangles are Optimizing Market Parties; Ovals are Markets with Clearing Conditions)

Market Simulation Model: Max Value to Power & Other Gas Consumers
minus Costs of power, gas production & transport
s.t. market clearing, production functions for power & gas, capacity limits