



Generation Capacity Expansion under uncertainty

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Private investors operate in a merchant world with different sources of uncertainty. These uncertainties are increasing tremendously and very hard to price.

Commodity Prices Risk

- Costs of fuels determine the marginal prices of the electrical system and the market prices; their relative behavior has an impact on the profitability of the different technologies

Residual Demand Risk

- Uncertainty in the total demand growth (or decline)
- Development of non competitive but CO₂ friendly technologies through various subsidies
- Decommissioning of nuclear and old conventional plants
- Demand behavior

Regulation risk

- Market architecture
- Carbon policy: uncertainty around the targets
- Sustainability of Subsidy Mechanisms

1	Capacity expansion models: from optimization to stochastic equilibrium
2	Endogenous cost of capital <ul style="list-style-type: none">• a fixed point model• Linear stochastic discount factor
3	Endogenous cost of capital <ul style="list-style-type: none">• risk function and nonlinear stochastic discount factor
4	Illustration: a small example
5	Conclusions

Typology of capacity expansion models

From the old days (50ties and 60ties in EdF)

- capacity expansion models of the optimization form
 - initially simple, later very complex
 - adapted to the regulated monopoly
 - where risk was largely passed to the consumer through average cost pricing

After restructuring

- adaptation of financial methods of the real option type
 - initially simple and still simple
 - adapted to the single project type
 - where price risk is exogenously given
 - cost of capital taken from CAPM type methodology

Typology of capacity expansion models

This presentation:

Very stylized two stage Investment model:

A two stage problem:

1. Decide investment today (2010–2011)
2. that will come on stream after 2016 (on which we know nothing)

Approach:

1. start from capacity expansion models because they allow for considerable details in the representation of the system
2. cast them in an economic equilibrium context because this better represents a competitive economy
3. and expand on the representation of risk because it can no longer be simply passed to the consumer

In this presentation: A basic two stage optimization model and the corresponding equilibrium model with fixed (price insensitive) load duration curve

Can be written as stochastic optimization model

- Benefit: some features of power systems are amenable to optimization but not to equilibrium
 - e.g. unit commitment characteristics

A stochastic version of the equilibrium model

Benefit: the equilibrium model can embed features that cannot be accommodated in optimization mode

- price sensitive storage possibilities arising from smart grids
- market imperfection such as average cost price

What we need is a margin by plant, indexed by scenario from an adequate short term model to make an investment decision

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Cost of capital are usually motivated by the CAPM

The CAPM links the cost of capital as the correlation of expected market and expected asset returns

$$r(k) = r^f + \frac{\text{cov}(R(k, \omega), R(M, \omega))}{\sigma^2[R(M, \omega)]} E_p[R(M, \omega) - R^f]$$

- For the use of the CAPM a forecast for the market for all scenarios ω is needed
- One might be tempted to use historic correlations (but most risk factors are new)

Iterative approach:

1. Calculate stochastic capacity expansion model
2. Derive correlation of cash flows and market outcomes
3. Update cost of capital; if changed back to 1.

Problem: very time consuming

An alternative: linear stochastic discount factor

Linear stochastic discount factors

Introduce a (CAPM compatible) linear stochastic discount factor (Cochrane 2005)

$$m(\omega) = a - b \times R(M, \omega)$$

Where a and b are chosen so that the pricing kernel prices the risk free asset and the market

$$\begin{aligned} E_p[m(\omega)] &= \frac{1}{R_f} \\ E_p[m(\omega) \times R(M, \omega)] &= \frac{1}{R_f} \end{aligned}$$

Note, the pricing kernel $m(\omega)$ does not span the entire space of the payoffs but only the one spanned by the market return $R(M, \omega)$:

$$X = X_m + X_{\perp m}$$

With X_m is the systematic risk (spanned by $R(M, \omega)$)
 $X_{\perp m}$ which has a zero covariance with X_m is the idiosyncratic risk

An alternative: linear stochastic discount factor

New stochastic optimization problem:

- All discounting at the risk free rate
- In optimization terms: We minimize the investment cost plus the expected operating costs weighted with the stochastic discount factor $m(\omega)$
- In equilibrium terms: we invest as long as investment costs – expected margins weighted by the stochastic discount factor $m(\omega)$ is negative

The formulation is CAPM compatible and bypasses the fixed point problem.

If one wants a CAPM consistency, one needs scenarios of market returns:

Data

Fuel	Load	CO ₂	Market return	$m(\omega)$
Low	Low	Low	.55	1.421
Low	Low	Medium	.6	1.380
Low	Low	High	.65	1.338
Low	Medium	Low	0.95	1.087
Low	Medium	Medium	1.0	1.045
Low	Medium	High	1.05	1.003
Low	High	Low	1.35	0.753
Low	High	Medium	1.4	0.711
Low	High	High	1.45	0.669
Medium	Low	Low	0.65	1.338
Medium	Low	Medium	0.7	1.296
Medium	Low	High	0.75	1.254
Medium	Medium	Low	1.05	1.003
Medium	Medium	Medium	1.1	0.962
Medium	Medium	High	1.15	0.920
Medium	High	Low	1.45	0.669
Medium	High	Medium	1.5	0.627
Medium	High	High	1.55	0.585
High	Low	Low	0.75	1.254
High	Low	Medium	0.8	1.212
High	Low	High	0.85	1.171
High	Medium	Low	1.15	0.920
High	Medium	Medium	1.2	0.878
High	Medium	High	1.25	0.836
High	High	Low	1.55	0.585
High	High	Medium	1.6	0.543
High	High	High	1.65	0.502

Preprocessing

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Coherent risk functions (nonlinear stochastic discount rates can be used to price idiosyncratic risk)

Reminder: The random payoff is split in two parts $X = X_{m^*} + X_{\perp m^*}$

- The first part is the systematic risk and can be priced with a linear stochastic discount factor (CAPM)
- The second part is the idiosyncratic risk
 - Should it be priced at all?
 - Is a regulatory intervention like nuclear lifetime extension systematic?
- Suppose we want to account for idiosyncratic risk:
 - Use coherent risk functions to price the second part. See Artzner et al. (1999) for the introduction of these functions and Shapiro et al. (2009) for their use in optimization

Using coherent risk functions in an optimization model leads to a convex program but is not an equilibrium model (but still useful)

Reminder: The optimization model minimizes expected system costs

- One can apply a risk function on the system costs
- The use of a stochastic discount rate on total system costs or welfare has a natural interpretation for a central planner or monopolist
- but it does not apply to the agents of a market where total cost of the system is the sum of the cost of the agents.
 - generator apply a stochastic discount rate on their margins
 - consumer apply a stochastic discount rate on costs
 - Stochastic discount rates of consumers and producers could be the same under perfect risk trading (Ralph and Smeers)
 - but that contradicts that fact that we talk about idiosyncratic risk

Applying stochastic discount factors on individual plants gives an equilibrium model of an incomplete market. Unfortunately it is not necessarily convex.

We start from the investment criterion of each project or for heterogeneous investors that already own different plant portfolios

- derive a project specific stochastic discount factor
- concatenate all investment criteria

The vector of discounted margins is not monotonous

- Since idiosyncratic risks are not traded
- which has an interpretation of incomplete markets

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A small system

- A three technology (coal, CCGT, OCGT) example
- Two stages (invest/operate)
- Investment and operation costs, CO₂ emission cost
- Annual investment costs are calculated at the risk free rate
- An energy only market with a price cap
- 6 time segments
- Randomness: uncertain load, fuel and CO₂ (3 possible states each)

Reminder: the representation of the short run market model is flexible; for instance

- Replace CO₂ price uncertainty by CO₂ cap uncertainty
- Introduce free allowances (no longer an optimization problem)
- Price responsive demand
- without cross time price elasticity remains an optimization problem
- with cross time price elasticity (smart grid) is no longer an optimization problem

Select a risk function

A CVaR that ignores the 5% best realizations

Two alternatives cases

1. pricing for systematic risk only (no CVaR)
2. the whole risk is assumed idiosyncratic (all risk goes into the CVaR)

	Coal	CCGT	OCGT	Total	Max Shortfall	Average Baseload Price
Deterministic	20000	40000	20000	80000	6000	60.98
Stochastic	20000	46000	6000	72000	22.6	60.70
Elastic demand	20340	40780	0	61120	0	60.89
Elastic demand DSM	24120	37690	0	61810	0	60.91
CO2 constraint	27200	38800	6000	72000	22600	59.85
Free Allocation	28890	43110	0	72000	22600	60.25
Linear discount factor	12000	54000	6000	72000	22600	61.71
CVAR	18000	48000	6000	72000	22600	61.16

Everything works and behaves as expected

- deterministic → stochastic → reinforces partial load
- smart grid → reduces investment in peak
- free allocation → induces investments

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- The **range** of uncertainty around **residual demand** and **environmental regulation risks** is **increasing** and may have a **substantial impact** on the revenue accruing from the operations of new investments. Most of those risks did not exist in the past.
- A model should be interpretable in terms of market equilibrium but it should also account for risk.
- Since the risk exposure of plants depends on the investment a (CAPM based) discounted cash flow analysis effectively poses a fixed point problem. Stochastic discount factors can bypass this problem
- If one insists on pricing idiosyncratic risk, the problem becomes far more complicated:
 - The optimization model loses its interpretation as equilibrium
 - The equilibrium model loses convexity

Coherent Risk/Value functions

- ▶ **A1. Convexity.** $\rho(tZ + (1 - t)Z') \leq t\rho(Z) + (1 - t)\rho(Z')$ for all $t \in [0, 1]$ and all $Z, Z' \in \mathcal{Z}$
- ▶ **A2. Monotonicity.** If $Z, Z' \in \mathcal{Z}$ and $Z \geq Z'$ then $\rho(Z) \geq \rho(Z')$.
- ▶ **A3. Translation equivariance** if $a \in R$ and $Z \in \mathcal{Z}$ then $\rho(Z + a) = \rho(Z) + a$
- ▶ **A4. Positive homogeneity.** If $t > 0$ and $Z \in \mathcal{Z}$ then $\rho(tZ) = t\rho(Z)$.
- ▶ **A'1. Concavity.** $\rho(tZ + (1 - t)Z') \geq t\rho(Z) + (1 - t)\rho(Z')$ for all $t \in [0, 1]$ and all $Z, Z' \in \mathcal{Z}$.

A representation theorem (in the sense of valuation):

$$\rho(X(\omega)) = \inf_{m \in \mathcal{M}} E_p[m(\omega) \times X(\omega)]$$

where \mathcal{M} is a convex set of probability measures

An alternative: linear stochastic discount factor

New stochastic optimization problem:

- All discounting at the risk free rate
- In optimization terms

$$\min_{x \geq 0} \sum_{k \in K} I(k) x(k) + R^f E_p[m(\omega) Q(x, \omega)]$$

- In equilibrium terms

$$0 \leq I(K) - R^f E_p[m(\omega) \mu(x, K, \omega)] \perp x(K) \geq 0$$

The formulation is CAPM compatible and bypasses the fixed point problem.