The robustness of agent-based models of electricity wholesale markets

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Abstract

Agent-based modelling is an attractive way of finding equilibria in complex problems involving strategic behaviour, particularly in electricity markets with transmission constraints. However, while it may be possible to demonstrate convergence of learning behaviour to a Nash equilibrium, that is not sufficient to establish that the equilibrium is robust against more sophisticated strategy choices. This note examines two particular forms of agent-based modelling used in electricity market models, both variants of mark-up pricing, and demonstrates that they are robust against other Nash strategies.

Keywords: agent-based modelling, electricity markets, mark-up equilibria, stability, oligopoly, learning

JEL Classification: C63, C73, D43, L10, L13, L94

1 Introduction

Liberalized electricity markets are frequently bedevilled by the persistence of a dominant incumbent generator, even if the industry has been unbundled and entry made potentially contestable. The EU has embarked on a project to create an Integrated Electricity Market by facilitating cross-border trade to increase competition within each Member State, although progress towards efficient use of interconnectors through market coupling has been until recently very slow.

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†The 2013 revision corrects the description of deviations from Stackelberg to Cournot and changes the description of the firms from leader/follower to deviant/other.

With the Third Package, the creation of the Agency for the Cooperation of Energy Regulators, ACER, and commitment to deliver the Target Electricity Model by 2014,[1] progress has recently accelerated, so that market coupling now extends over a wide area in Central West Europe, larger even that the PJM[2] Interconnection. It is therefore of considerable importance to examine the impact of these various reforms on the extent of market power, to test whether the reforms have been successful in mitigating such power, or whether further structural reforms are necessary. This is difficult, as it is hard to model generator behaviour in such markets, and even harder to model strategic behaviour with transmission constraints (and most interconnectors are heavily congested). Once one abandons the quest for analytic solutions, the way is open to computer simulation that can include more realistic features of markets, agent behaviour and constraints such as those imposed by transmission capacities. It is therefore attractive to adopt an agent-based modelling strategy that can handle such complexity.

1.1 Agent-based modelling[3]

Agent-based models have been used to describe the dynamic interaction of a possibly large number of agents in a spatially well-defined system that will typically not start in equilibrium, although the agents are pursuing goals that may lead to an equilibrium in which each agent is satisfied that he or she can make no further improvements. Until then agents acquire and process information, often by observing their neighbours' behaviour, receiving some information (e.g. from advertisements) and update their actions, experimenting to see if their choice delivers improvements (for consumers in their utility net of cost, for producers in their profits, etc.). In some cases the aim is to find an equilibrium when analytical models may be intractable, in others it is to test the robustness of the system, e.g. of a financial system facing periodic crises. The adoption of technology provides a good example of a dynamic learning process, where the slow rate of adoption or diffusion (often logistic or S-shaped) indicates that the innovation is not immediately perceived as being superior to current options. Thus Zhang and Nuttall (2007, 2011) use agent-based models to study the adoption of smart meters by retail electricity customers, and hence to evaluate the effectiveness of various public policies to promote their use. Such models concentrate on the interaction and learning of a large number of individually small agents, where the focus is on the psychology of choice, rather than assuming that consumers are fully informed

[1] Directive 2009/72/EC *Concerning common rules for the internal market in electricity*, Regulation 713/2009 *Establishing an Agency for the Cooperation of Energy Regulators*, and Regulation 714/2009 *On conditions for access to the network for cross-border exchanges in electricity*.

[2] Originally *P*ennsylvania New *J*ersey and *M*aryland but now covering a wider area.

[3] Wikipedia at http://en.wikipedia.org/wiki/Agent-based_model provides a concise and useful guide to the variety of fields, approaches and emergent behaviour that agent-based modelling encompasses.

about their preference and the choices available.

1.2 Agent-based modelling of wholesale electricity markets

In contrast, this paper follows the tradition of looking at the interaction of a small number of agents, each of which potentially has market power, but operates in a system where their interactions are complex and they are not initially well-informed about the decisions their competitors or customers are making. They therefore start out of equilibrium and must learn from their observations. Even if there is no uncertainty about demand and technology (and hence costs), each agent is uncertain about the choices its competitors will make, and therefore what is its own best strategy, hence the need for learning and experimentation.

There are a wealth of examples of using such models to explore learning in complex electricity markets, where generators (and in some cases other agents such as retailers or suppliers) might choose a variety of actions in their pursuit of profit or advantage. In particular, they can sign contracts ahead of time as well as deciding at what price to offer electricity into the spot market. Except for very stylized (e.g. Cournot) models, it has been analytically very difficult to solve for the optimal combination of forward contracts and spot sales in the presence of market power. Newbery (1998) was able to do this for a supply function equilibrium model by assuming that existing generators would choose contracts to deter entry that might otherwise lead to excess capacity. Unless average prices can be externally specified (in this case by free entry), there are too many possible combinations of contracts and spot offers to be tractable in a supply function setting (arguably the most natural choice for modelling electricity wholesale markets).

Bunn and Oliveira (2001) adopt an agent-based model to explore the possible implications of moving from the original Electricity Pool for England and Wales to the New Electricity Trading Arrangements (NETA) to see how this might impact on both generators and suppliers, given that generators may have market power and can choose to contract ahead but also be exposed to the Balancing Mechanism. Given that their model was developed before NETA went live some of their predictions were remarkably prescient (such as the predicted low volume of trading in the Balancing Mechanism), and illustrate the strength of this approach for studying proposed market design changes in a realistic setting that includes contracting.

Bunn and Oliveira (2003) extend this model to explicitly consider market power in a model of the English wholesale market in which the regulator, Ofgem, had imposed a Market Abuse Licence Condition that was appealed to the Competition Commission. Again, the development of their model was motivated by a practical policy question. Veit et al (2006) also study the case in which an oligopoly of generators sign contracts ahead of time and then compete in a transmission constrained spot market. In this case agent-based modelling is chosen to handle the complexity of the decision process (complicated by the transmission constraints), and confirms the prediction of simpler analytical models (e.g. Newbery, 1998) that forward contracting to sell leads to more competitive spot market behaviour and hence lower prices.[4] These and other agent-based models of wholesale electricity market are discussed in the excellent survey by Weidlich and Veit (2008), who compare different learning strategies and their results.

A key question facing such modelers is whether the resulting equilibrium is indeed a Nash equilibrium (where that is unique) in the space of actions allowed in the formulation of the game, and indeed what happens where there are multiple Nash equilibria. This is the question that Krause et al. (2005) address in the context of a simplified power market, and answer affirmatively for unique Nash equilibria.

The obvious problem with agent-based modelling where agents are assumed to learn about the profit consequences and adapt their strategies to increase profits is that the action space over which they make choices may be too restrictive and may allow other more sophisticated agents to exploit this limited strategy choice by choosing from a wider range of actions. In that sense the models may be dismissed as too simplistic to model the behaviour of more sophisticated firms (who certainly hire the brightest and best to examine their strategic choices). A good defence of adaptive learning would be to show that the equilibrium of the form of learning were robust against more sophisticated players choosing from a wider set of actions.

One proposed agent-based model known as Q-learning[5] has agents choosing a mark-up on their marginal cost schedule, thus departing from competitive behaviour (Krause et al, 2005). Models in which agents choose mark-ups on marginal costs are popular among agent-based models of electricity markets, and most of the examples surveyed in Weidlich and Veit (2008) have this form. This paper examines the robustness of such models as a contribution to the validity of adopting agent-based modelling more widely for studying wholesale electricity markets. It does so by applying the standard approach to Industrial Organization (I-O) to the new field of agent-based modelling. As such, the approach has much wider application than just to electricity markets, for oligopolies are pervasive in many sectors of the economy and modelling their behaviour faces similar problems to those of the electricity market. The next section sets out the benchmark I-O workhorse of the Nash-Cournot oligopoly model, and the following section describes the space of actions considered in learning or agent-based oligopoly models to test their robustness to deviations by more sophisticated agents, and hence their plausibility for finding a robust equilibrium. This leads to a discussion of robustness more generally, possible extensions

[4] In contrast, if producers buy in the forward market they can amplify their incentive to manipulate the spot market, as Mahenc and Salanie (2004) demonstrate in a Bertrand model.

[5] set out in Watkins (1989) and further developed by Littman (1994) and Hu and Wellman (2003), and critically compared to other reinforcement learning models in Weidlich and Veit (2008).

and a conclusion.

2 Standard oligopoly models

The profit maximizing strategy for an oligopolistic firm, which might be a generator supplying to a power market, will depend on the actions available and the responses of its competitors. The simplest Cournot oligopoly model has n firms simultaneously choosing their output levels taking the outputs of rivals as given - Nash behaviour. It has proven popular for modelling electricity wholesale markets as it is tractable, and can be extended to incorporate contracting (e.g. Allaz and Vila, 1993; Bushnell, 2007; Murphy and Smeers, 2010). Although Green and Newbery (1992) argue that supply function models are more appropriate for modelling wholesale electricity markets, Bushnell et al (2008) argue that Cournot models should not be dismissed for the study of contracting in electricity markets.

One immediate question is whether agents do better or worse following a Cournot strategy rather than choosing an optimal mark-up over their marginal cost. Suppose that all n firms are identical, each with cost function $C(q) = aq + \frac{1}{2}cnq^2$. This has a marginal cost, MC$= a + cnq$, with an zero output intercept of a. By measuring prices as the excess over a, and thus setting $a = 0$, the MC becomes

$$\mathrm{MC} = cnq_i, \quad (i = 1, .., i, .., n), \tag{1}$$

(which gives an aggregate competitive supply schedule that is independent of n). Aggregate demand is $Q(p)$, where the slope is set equal to unity by a suitable choice of the units in which to measure output:

$$Q(p) = A - p, \quad p = A - Q. \tag{2}$$

2.1 The perfectly competitive benchmark

The perfectly competitive solution is $p =$MC$= cQ$, for which the solution is

$$p_c = \frac{Ac}{(1 + c)}, \quad q_c = \frac{A}{n(1 + c)}. \tag{3}$$

In the constant marginal cost case, $c = 0$, $p_c = 0$ but otherwise if $c > 0$ the average cost is half the price p_c (both relative to the price level normalization, a).

2.2 Oligopoly: Nash-Cournot

Now consider the Nash-Cournot solution in which each identical firm takes the output decision of the other as given, and chooses q_i to maximize profit, $\pi_i(\boldsymbol{q}) = p(\boldsymbol{q})q_i - C(q_i)$ (where $\boldsymbol{q}$ is the vector $(q_1, .., q_i, .., q_n)$):

$$\pi_i(\boldsymbol{q}) = (A - q_i - \sum_{j \neq i} q_j)q_i - \frac{1}{2}cnq_i^2.$$

The first order condition (f.o.c.) satisfies

$$q_i = \frac{(A - \sum_{j \neq i} q_j)}{(2 + cn)}.$$

The symmetric oligopoly solution is

$$p_o = \frac{A(\frac{1}{n} + c)}{(\frac{1}{n} + 1 + c)} > p_c, \quad q_o = \frac{A}{n(\frac{1}{n} + 1 + c)} < q_c. \tag{4}$$

To illustrate, if $c = 1$, $n = 2$, $A = 2$, then $p_c = 1$, $q_c = \frac{1}{2}$. In this case the Cournot duopoly solution is $q_d = 2/5$, $p_d = 6/5$, so that quantities are reduced and as a result the price and profits are increased. Note that as $n \to \infty$, so $p_o \to p_c$, and that in the constant marginal cost case in which $c = 0$, $p_o = A/(n + 1)$, which is strictly positive.

3 Reinforcement or Q-learning

Under Q-learning, each firm selects a specified type of deviation from competitive bidding, and will continue to adjust the size of the deviation until the resulting market outcome converges, which, if there is a unique Nash equilibrium, should be that equilibrium. In the simple quadratic cost, linear demand model there are two simple types of deviation from competitive bidding - marking-up the offer schedule by a constant amount, or changing the slope of the offer schedule, as in Hobbs et al (2000). These are each considered and their stability against sophisticated players examined.

3.1 Q-learning with a constant mark-up

Suppose each firm offers its supply at a mark-up above marginal cost, MC, so its offer price $p =$ MC $+ m_i$, or for the present case in which MC $= ncq_i$, the supply schedule in price space is

$$q_i = \frac{(p - m_i)}{nc}, \quad (i = 1, .., i, .., n). \tag{5}$$

Under the mark-up form of Q-learning the firm keeps adjusting its mark-up m_i to improve its profit. The natural equilibrium in this "multi-agent system" is to find the value of m_i that maximizes firm i's profit, assuming the other firm chooses its mark-up independently (m_j is treated as fixed, i.e. Nash behaviour).[6] If firm i sets mark-up m_i, then the solutions for the

[6] Grant and Quiggin (1993) examine the same problem but with constant elastic supply and demand, and find similar results to those in Proposition 1 below.

market clearing price, MCP, $p(\boldsymbol{m})$ (where $\boldsymbol{m}$ is the vector $(m_1, .., m_i, .., m_n)$ solves

$$p(\boldsymbol{m}) = A - \sum q_i(\boldsymbol{m}) = \frac{(Anc + \sum m_i)}{n(1+c)}.$$

The appendix demonstrates that the symmetric solution is

$$m = \frac{Ac}{n(1+c)^2 - 1}, \tag{6}$$

$$p_m = n(1+c)m = \frac{Ac(1+c)}{(1+c)^2 - \frac{1}{n}}, \tag{7}$$

$$q_m = \frac{(p-m)}{nc} = \frac{A(1+c-\frac{1}{n})}{n(1+c)^2 - 1}. \tag{8}$$

Note that in the constant returns case in which $c = 0$, the Nash equilibrium is the competitive price, $p_m = 0$, in contrast to the Nash-Cournot case, suggesting the competition in mark-ups is more competitive than competition in quantities. In order to prevent mark-up competition collapsing into perfect competition marginal costs must be increasing. Note also that as $n \to \infty$, $p_m \to p_c$ as with the Nash Cournot case. As a numerical example, with the numbers before $(c = 1,\ n = 2,\ A = 2)$ $m = 2/7,\ p_m = 8/7$, which is below the Nash Cournot price (but above the competitive price), while $q_m = 3/7$. More generally we can check to see whether the mark-up equilibrium is always more competitive than the Cournot equilibrium. The answer is yes, and given in

Proposition 1 *In a symmetric oligopoly with linear demand and equal quadratic costs, the Nash equilibrium in which firms choose their mark-up on marginal cost always has a lower equilibrium price than the symmetric Nash-Cournot oligopoly solution.*

(All proofs are given in the appendix.) This confirms for the linear case Grant and Quiggen's (1994) finding for the constant elastic supply and demand case where competition in average mark-ups (in their model equivalent to competition in mark-ups over marginal cost) leads to more competitive outcomes than the Nash-Cournot equilibrium. It also suggests that a sophisticated firm observing other firms following a mark-up strategy might prefer to play a Cournot strategy in order to increase profits. We can explore that by supposing that the sophisticated firm optimizes against the mark-up firms, choosing output rather than a mark-up.

3.2 Cournot deviations from a mark-up equilibrium

If the deviant can choose not just a mark-up on marginal cost but an output (as in the Cournot model) one might expect higher profits as the Nash-Cournot equilibrium has higher profits than the mark-up equilibrium. In this case the deviant firm will choose an output assuming that the other players stick to their Nash mark-ups. Thus if the other firms, j, offer the supply schedule of (5), $q_j = (p - m)/(nc)$, then the residual demand schedule, $Q_r(p)$, facing the deviant, d, at price p is

$$Q_r(p) = Q(p) - (n-1)q_j = (A - p) - \frac{(n-1)(p-m)}{nc} \equiv \alpha - \beta p = q_d, \tag{9}$$

where $\alpha = A + (1 - \frac{1}{n})\frac{m}{c}$, $\beta = 1 + (1 - \frac{1}{n})/c$. The deviant's optimal response is to choose q_d to maximize profit, $pq_d - C(q_d)$, where $p = (\alpha - q_d)/\beta$. The question to address is whether choosing the Nash mark-up given in (6) is robust against a player choosing output rather than a mark-up. Surprisingly, the answer is positive, and in that sense we can demonstrate

Proposition 2 *In a Nash game with identical players each with a quadratic cost function facing linear demand, if each player assumes that other players will choose the optimal mark-up on its marginal cost, then each player will find it no more profitable to choose an optimal quantity to supply (the choice in the Nash-Cournot game) than the optimal mark-up on marginal cost. In that sense the Nash mark-up equilibrium is robust against Cournot deviations.*

Interestingly, in this case the best Nash quantity response to the Nash choice of mark-up by followers is the symmetric Nash equilibrium in mark-ups. In that sense, Nash behaviour in mark-ups is stable and can thus be justified as the outcome of a learning process in which players follow this strategy but a potential deviant can choose from a set of strategies that also includes quantities.

3.3 Q-learning with a choice of slope

Suppose instead of choosing a mark-up on the linear MC schedule, whose slope is nc, firms choose a linear offer schedule which is increasingly above the true MC schedule (in quantity space). Their supply schedule in price space can be written as

$$q_i = \frac{s_i p}{n}, \quad i = 1, ., i, .n, \tag{10}$$

instead of (5), where lower values of s_i indicate higher mark-ups (and again in the symmetric case aggregate supply will be independent of n). Aggregate demand now can be solved for the MCP, $p(\boldsymbol{s})$, where $\boldsymbol{s}$ is the vector $(s_1, ., s_i, ., s_n)$,

$$Q(p) = p\frac{1}{n}\sum s_j = A - p, \quad p = \frac{A}{1 + \frac{1}{n}\sum s_j}.$$

The profit function becomes $\pi_i(\boldsymbol{s}) = q_i(\boldsymbol{s})p(\boldsymbol{s}) - C(q_i)$ and the appendix demonstrates that the symmetric equilibrium solves

$$
\begin{aligned}
0 &= (n-1)cs^2 - (n - nc - 2)s - n, && (11)\\
s &= \frac{n - nc - 2 + \theta}{2(n-1)c}, \quad \theta = \sqrt{(n-2)^2 + n^2c(c+2)}, && (12)\\
p_s &= \frac{A}{1+s} = \frac{2A(n-1)c}{(1+c)(n-2)+\theta}, \; q_s = \frac{As}{n(1+s)}. && (13)
\end{aligned}
$$

In the constant returns case in which $c = 0$, $p_s = 0$, as with the mark-up case, but with a positive slope prices can be above the competitive solution. In the limit as $n \to \infty$, $p_s \to \frac{Ac}{1+c}$ or the competitive (and Nash-Cournot) limit. In the numerical example $n = 2 = A$, $c = 1$, $\theta = 2\sqrt{3} = 3.46$, $s = 0.732$, $p_s = 1.154$, compared to $p_m = 1.143$, and $p_o = 1.2$, and the competitive solution $p_c = 1$, so the slope solution is between the mark-up and Nash-Cournot solution in this case.

That the slope mark-up price is higher than the mark-up over marginal cost case is a general result:

Proposition 3 *In a Nash game, if players assume that the strategy space is the choice of the slope of its offer schedule, then with a quadratic cost function, linear demand and identical players, the equilibrium yields a price that lies between the Nash-Cournot and the Nash mark-up price, and hence yields higher profits that choosing the best mark-up over marginal costs.*

3.4 Cournot deviations from a slope mark-up equilibrium

Now consider the case in which other firms offer a supply schedule given in (10), $q_j = sp/n$, and the deviant chooses an optimal level of output. The residual demand, Q_r, facing the deviant at price p is

$$Q_r(p) = Q(p) - (n-1)q_j = A - (1 + \frac{rs}{n})p = q_d, \quad r \equiv n - 1.$$

The deviant chooses q_d to maximize profit $\pi_d = q_d p(q_d) - C(q_d)$, for which the (appendix) solution is

$$p_{sd} = \frac{A(1 + nc(1 + \frac{rs}{n}))}{(1 + \frac{rs}{n})(2 + nc(1 + \frac{rs}{n}))}, \quad q_d = \frac{A}{(2 + nc(1 + \frac{rs}{n}))}. \tag{14}$$

Taking the numerical values from above ($A = 2 = n$, $c = 1$, $r = 1$, $s = 0.732$), the slope price with this deviation, $p_{sd} = 1.15 = p_s$, $q_j = 0.423 = q_d$. The deviant has failed to improve upon the profit of simply choosing the slope mark-up. Again this a general result:

Proposition 4 *In a Nash game, if players assume that the strategy space is the choice of the slope of its offer schedule, then with a quadratic cost function, linear demand and identical players, then each player will find it no more profitable to choose an optimal quantity to supply (the choice in the Nash-Cournot game) than the optimal slope mark-up. In that sense the Nash slope mark-up equilibrium is robust against Cournot deviations.*

4 Wider robustness

Each of the mark-up Nash equilibria is robust against a firm choosing output instead of a mark-up, but one might argue that this is because both mark-up strategies are more competitive than Cournot behaviour, and that the real test of robustness is whether the less competitive slope mark-up equilibrium is robust against a more competitive fixed mark-up deviation. Again, somewhat surprisingly, the answer is that it is, and therefore mark-up equilibria seem quite generally robust against choices chosen from a wider strategy set.

Proposition 5 *In a Nash game, if players assume that the strategy space is the choice of the slope of its offer schedule, then with a quadratic cost function, linear demand and identical players, each player will maximize his profits regardless of whether another player (a potential deviant) chooses an optimal slope or an optimal mark-up on marginal cost in determining its offer to supply, and hence the Nash slope mark-up equilibrium is robust against fixed mark-up deviations.*

5 Extensions

The case for supply function equilibria is that generators need to submit their offers before the realization of (residual) demand and if these offers are required to hold for the next 24 hours, this demand will vary substantially, as Green and Newbery (1992) argued. A natural extension would be to test robustness with variable A, and compare expected profits of deviants. A more complex extension would examine the equilibria with heterogeneity of firms' cost functions. This is moderately tractable provided they differ in the slopes of their MC schedules but not in their zero intercept (the value of a that has been set to zero by the choice of the price intercept). It would be useful to know whether the argument that their is no incentive to deviate holds for firms regardless of their cost functions, or whether, for example, firms with lower MC schedules might be tempted to deviate. It will also be desirable to test whether Stackelberg behaviour, in which a leader commits an output in advance, is a profitable strategy, and hence whether in mark-up equilibria remain robust or are vulnerable to such deviations, but that will be left for a later paper.

6 Conclusion

Agent-based models are attractive in attempting to model outcomes in markets where some agents can act strategically, and there has been considerable interest in whether adaptive or Q-learning will lead to Nash equilibria, as these would seem natural equilibrium concepts. However, as with all attempts to model strategic behaviour, the resulting equilibrium is sensitive to the action space from which agents choose. Standard oligopoly models consider actions to be either quantities (supplies to the market), as in the Cournot formulation, or prices offered to the market (the Bertrand assumption). In the presence of uncertain or varying demand, supply function models, developed by Klemperer and Meyer (1989) and applied to electricity markets by Green and Newbery (1992), are attractive intermediate formulations, and their linear solutions[7] have been influential in motivating the kind of agent-based models considered here.

While the choice of action space in optimizing models is normally guided by the market structure and the actions that agents have, the choice of action space in agent-based models is normally guided by tractability, where a choice of a single parameter (such as the mark-up over marginal cost or the slope of the supply schedule) considerably simplifies the problem. This paper has shown that such equilibria are robust against deviations by firms choosing quantities or even other mark-up strategies instead of the chosen form of mark-up, and to that extent mark-up equilibria are robust Nash equilibrium concepts.

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[7] Supply function models typically have a continuum of solutions, one of which may be linear, providing there are no relevant capacity constraints. Where capacity constraints are important, there may be unique but non-linear solutions.

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Appendix

The Nash choice of the optimal mark-up can be found from the market clearing condition

$$\begin{aligned} p &= A - \sum q_i = \frac{(Anc + \sum m_i)}{n(1+c)}, \quad \frac{\partial p}{\partial m_i} = \frac{1}{n(1+c)}, \\ ncq_i &= p - m_i = \frac{(Anc + m_i + \sum_{j\neq i} m_j)}{n(1+c)} - m_i. \end{aligned}$$

The f.o.c. from maximizing w.r.t. m_i gives

$$\begin{aligned} \frac{\partial \pi_i}{\partial m_i} &= (p - \mathrm{MC})\frac{\partial q_i}{\partial m_i} + q_i \frac{\partial p}{\partial m_i}, \\ &= \frac{-m_i(1 - \frac{1}{n} + c)/c + q_i}{n(1+c)}. \end{aligned}$$

$$\begin{aligned} m_i(n - 1 + nc) &= ncq_i = p - m_i, \\ nm_i(1+c) &= p = \frac{(Anc + m_i + \sum_{j\neq i} m_j)}{n(1+c)}. \\ m_i(n^2(1+c)^2 - 1) &= Anc + \sum_{j\neq i} m_j, \end{aligned}$$

which gives the solutions (6) - (8) in section 3.1.

Proof of Proposition 1 *Evaluate*

$$\begin{aligned} p_o - p_m &= \frac{A(1+nc)}{n(1+c)+1} - \frac{nAc(1+c)}{n(1+c)^2 - 1}, \\ \mathrm{Sign}(p_o - p_m) &\propto (1+nc)(n(1+c)^2 - 1) - nc(1+c)(n(1+c)+1), \\ &= n - 1 > 0. \end{aligned}$$

QED.

Proof of Proposition 2 *Suppose the other firms adopt the Nash mark-up of (6), and the deviant chooses his supply optimizing against this, then as before the first order conditions for the deviant are given by*

$$\begin{aligned} 0 &= (p - MC) + q_d \frac{\partial p}{\partial q_d}, \quad \frac{\partial p}{\partial q_d} = -\frac{1}{\beta}, \qquad (15) \\ q_d &= \frac{\alpha}{(2 + \beta nc)} = \frac{\alpha}{n(1+c)+1}, \\ \beta p &= \alpha - q_d = \frac{\alpha(1+c)}{1 + c + \frac{1}{n}}, \quad p = \frac{\alpha c(1+c)}{(1+c)^2 - (\frac{1}{n})^2}. \end{aligned}$$

From (6)

$$\alpha = A + (1 - \frac{1}{n})\frac{m}{c} = \frac{(n(1+c)+1)(n(1+c)-1)A}{n^2(1+c)^2 - n},$$

so

$$q_d = \frac{\alpha}{n(1+c)+1} = \frac{(n(1+c)-1)A}{n^2(1+c)^2 - n}, \quad (16)$$

$$p = \frac{Ac(1+c)}{(1+c)^2 - \frac{1}{n}}, \quad (17)$$

exactly as in the mark-up equilibrium. QED.

The choice of optimal slope

Aggregate demand now can be solved for the MCP, $p(\boldsymbol{s})$,

$$\begin{aligned} Q(p) &= p\frac{1}{n}\sum s_j = A - p, \quad p = \frac{A}{1 + \frac{1}{n}\sum s_j}. \\ p &= \frac{A}{1 + \frac{1}{n}\sum s_j}, \quad \frac{\partial p}{\partial s_i} = \frac{-p}{n(1 + \frac{1}{n}\sum s_j)}, \\ q_i &= \frac{As_i}{n(1 + \frac{1}{n}\sum s_j)}, \quad \frac{\partial q_i}{\partial s_i} = \left(\frac{q_i}{s_i}\right)\frac{1 + \frac{1}{n}\sum_{j \neq i} s_j}{(1 + \frac{1}{n}\sum s_j)}. \end{aligned}$$

Differentiate the profit function $\pi_i(\boldsymbol{s}) = q_i(\boldsymbol{s})p(\boldsymbol{s}) - C(q_i)$ partially w.r.t. s_i to give

$$\begin{aligned} \frac{\partial \pi_i}{\partial s_i} &= (p - \mathrm{MC})\frac{\partial q_i}{\partial s_i} + q_i\frac{\partial p}{\partial s_i}, \\ \frac{s_i}{q_i}\frac{\partial \pi_i}{\partial s_i} &= \frac{(p - ncq_i)(1 + \frac{1}{n}\sum_{j \neq i} s_j) - \frac{1}{n}ps_i}{1 + \frac{1}{n}\sum s_j}. \end{aligned}$$

The f.o.c. is

$$\begin{aligned} p(1 + \frac{1}{n}\sum_{j \neq i} s_j - \frac{1}{n}s_i) &= ncq_i(1 + \frac{1}{n}\sum_{j \neq i} s_j), \\ q_i &= \frac{1 + \frac{1}{n}\sum_{j \neq i} s_j - \frac{1}{n}s_i}{nc(1 + \frac{1}{n}\sum_{j \neq i} s_j)}p = \frac{1}{n}s_i p. \end{aligned}$$

There is a comparable equation for each q_j so we have n equations in the n unknowns, $s_1, \ , s_n$.

Proof of Proposition 3 *With a constant number of firms, if a firm's output under the Nash slope equilibrium is less than under the Nash mark-up output, then the price will be higher. The sign of the difference between the Nash markup-up output and the Nash slope output is given by*

$$\begin{aligned} \mathrm{Sign}(q_m - q_s) &= \mathrm{Sign}(\frac{1}{q_s} - \frac{1}{q_m}) \propto \frac{n(1+s)}{s} - \frac{n(1+c)^2 - 1}{1 + c - \frac{1}{n}}, \\ &\propto n(1+c)(1 - sc) - 1, \\ &\propto n^2(1+c)^2(1 - (1 - \varphi)^{\frac{1}{2}}) - 2(n-1), \end{aligned}$$

where $\theta = n(1+c)\sqrt{1-\varphi}$ and $\varphi = \frac{4(n-1)}{n^2(1+c)^2}$. *But by expansion,* $(1-(1-\varphi)^{\frac{1}{2}}) \equiv \gamma > \frac{1}{2}\varphi = \frac{2(n-1)}{n^2(1+c)^2}$, *so* $\mathrm{Sign}(q_m - q_s) > 0$.

The other part requires us to show that $\mathrm{Sign}(q_s - q_o) > 0$.

$$
\begin{aligned}
\mathrm{Sign}(\frac{1}{q_o} - \frac{1}{q_s}) &\propto 1 + n(1+c) - n - \frac{n}{s}, \\
&\propto s(1+nc) - n \\
&\propto (n - 2 - nc + \theta)(1+nc) - 2nc(n-1) \\
&= n(1-c) - n^2c(1+c) - 2 + (1+nc)n(1+c)(1-\varphi)^{\frac{1}{2}} \\
&= 2(n-1) - (1+nc)n(1+c)\gamma \\
&= n - 1 - \varepsilon > 0, \quad \text{where } \varepsilon = \frac{(\gamma - \varphi/2)(1+nc)n^2(1+c)}{2(n-1)}.
\end{aligned}
$$

Thus in the case of a symmetric oligopoly it is possible to rank the equilibrium prices $p_o > p_s > p_m$. QED

Cournot deviations

The residual demand, Q_r, facing the deviant at price p is

$$
\begin{aligned}
Q_r &= Q - (n-1)q_j = A - (1 + \frac{rs}{n})p = q_d, \quad r \equiv n-1, && (18) \\
p &= \frac{A - q_d}{1 + \frac{rs}{n}}, \quad \frac{\partial p}{\partial q_d} = -\frac{1}{1 + \frac{rs}{n}}. && (19)
\end{aligned}
$$

The deviant chooses q_d to maximize profit $\pi_d = q_d p(q_d) - C(q_d)$, for which the f.o.c. is

$$
\begin{aligned}
p &= \mathrm{MC} - q_d \frac{\partial p}{\partial q_d} = q_d(nc + \frac{1}{1 + \frac{rs}{n}}), \\
q_d &= \frac{p(1 + \frac{rs}{n})}{1 + nc(1 + \frac{rs}{n})} = A - (1 + \frac{rs}{n})p.
\end{aligned}
$$

These can be solved to give (14).

Proof of Proposition 4 *The sign of the difference between the deviant and the other firms' output levels for the slope mark-up equilibrium is*

$$
\begin{aligned}
\mathrm{Sign}(q_d - q_s) &= \mathrm{Sign}(\frac{1}{q_s} - \frac{1}{q_j}) = \frac{n(1+s)}{s} - (2 + nc(1 + \frac{rs}{n})), \\
&= n + (n - 2 - nc)s - (n-1)cs^2 = 0,
\end{aligned}
$$

from (11). The slope-Nash equilibrium is thus robust to Cournot deviations. QED

Proof of Proposition 5 *The market clearing conditions give q_d and price as functions of the deviant's mark-up m:*

$$\begin{aligned}
ncq_d &= p - m = nc(A - (1 + \frac{rs}{n})p, \\
p &= \frac{m + Anc}{1 + c(n + rs)}, \quad \frac{\partial p}{\partial m} = \frac{1}{1 + c(n + rs)}, \\
ncq_d &= p - m, \quad ncq_d = \frac{Anc - mc(n + rs)}{1 + c(n + rs)}, \\
nc\frac{\partial q_d}{\partial m} &= -\frac{c(n + rs)}{1 + c(n + rs)}.
\end{aligned}$$

The deviant chooses his mark-up m to maximize profit, for which the f.o.c. is

$$\begin{aligned}
nc(p - \mathrm{MC})\frac{-\partial q_d}{\partial m} &= ncq_d\frac{\partial p}{\partial m}, \\
\frac{mc(n + rs)}{1 + c(n + rs)} &= \frac{ncq_d}{1 + c(n + rs)} = \frac{Anc - mc(n + rs)}{(1 + c(n + rs))^2} \\
m &= \frac{An}{(n + rs)(2 + c(n + rs))} \\
q_d &= \frac{A}{2 + c(n + rs)}.
\end{aligned}$$

This is exactly the same output that the deviant would choose under the slope mark-up Nash equilibrium. QED.