

# Stochastic Equilibrium Models for Generation Capacity Expansion

EPRG Working Paper 1019

Cambridge Working Paper in Economics 1041

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Capacity expansion models are as old as the optimal dispatch models but the transition from optimization to equilibrium models has not yet taken place. The early optimization models of capacity expansion go back to the late fifties when the industry was still regulated (Morlat and Bessière, 1971). The problem was first formulated as a linear program but further developments quickly followed suit and extensions covered all types of optimization techniques. Capacity expansion, which was initially seen as a true planning exercise was easily reinterpreted in terms of equilibrium in a competitive energy economy in the early seventies after the first energy crisis. The power industry of the seventies was still regulated on a cost plus basis that largely protected it from risk. Deterministic models were thus satisfactory in the situation of the time. Restructuring removed that protection at the same time that various new policies and external events dramatically increased the risk surrounding the electricity sector. This emergence of risk in the investment process strongly suggests to move the analysis from a deterministic to a stochastic environment. The question is thus to transpose former optimization capacity expansion models to stochastic equilibrium models. This extension is the subject of this paper.

The first analysis of a capacity expansion problem in terms of a stochastic equilibrium capacity expansion model in the energy area is probably found in Haurie et al. (1988). The model deals with gas developments and was formulated as an open loop Cournot equilibrium under demand uncertainty. This model could be converted to an optimization model that was later used in Gürkan et al. (1999) to illustrate the method of "Sample Path" since elaborated by several authors. Lin and Fukushima (2009) recently reviewed different models of stochastic equilibrium, among them the one used



by Gürkan et al. (1999) in their application of sample path to the investments in gas production. This model is stated as a stochastic variational inequality problem; we adopt the closely related formulation of stochastic complementarity problems as the modeling paradigm of the investment problem throughout this paper.

This paper begins by introducing a very simple and standard two-stage version of a stochastic optimization capacity expansion model as could have been constructed in the regulated environment. We adopt a standard stochastic programming approach and present the model in terms of its first and second stages. We then immediately reformulate this problem in the stochastic equilibrium format that drives the whole paper. Then we discuss the possibilities and limitations of stochastic equilibrium models to account for idiosyncrasies of restructured electricity markets.

The rest of the paper analyses different risk issues encountered in the investment process. The standard approach in investment problems is to reflect risk in the discount rate. The discount rate is normally regulated when the industry operates as a monopoly; this may have raised economic controversies but did not create modeling difficulties as the discount rate is just a single parameter of the model. The problem is quite different in a world where "project finance" drives the capacity expansion process and requires that plants are evaluated on the basis of different discount rates. The CAPM and the APT are the reference theories for finding these discount rates. Expositions of these theories can be found in any textbook of corporate finance and we take them for granted. The adoption of a project finance approach therefore requires the stochastic equilibrium model to accommodate plant specific discount rates while maintaining the interpretation of a competitive economy that is the justification of the model. A first treatment of the question is given in the form of a fixed point formulation. Then we adopt an alternative, probably more rigorous but also less usual representation of risk. Starting again from a CAPM based formulation it assumes that the different risks affecting plants can be taken care of by modifying the payoff of the different plants using a linear stochastic discount rate. Discounting is then conducted at the risk free rate but with risk adjusted cash flows computed with CAPM based stochastic discount rates. Section 5 considers an alternative version of the risk neutral discounting where the adjustment to the cash flow is derived from risk functions. Risk functions were initially developed by Artzner et al. (1999) and have been recently cast in an optimization context (see the book by Shapiro et al., 2009 for a comprehensive treatment). We extend this view to an equilibrium context to construct alternative adjustments of the cash flows of the



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plants. Finally, we provide a simplified but realistic illustration of these notions.

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Publication  
Financial Support

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September 2010  
GDF SUEZ

