

# Generalized Nash Equilibrium and Market Coupling in the European power system

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The restructuring of the European electricity market is a long process. The integration of various national markets through the so called "Market Coupling" approach is currently the most advanced market design in Europe.

Market coupling essentially relies on an energy market and leaves it to the Transmission System Operators (TSOs hereafter) to take care of most of the constraints appearing in the electricity system by a mix of market and quantitative constructs. The result is what economists call an incomplete market where several constraints are not priced. We take up a particular question of market coupling namely the removal of congestion through counter-trading. This problem has been encountered in many jurisdictions outside of Central Western Europe and hence is of general interest. We then look at the problem of the organization of counter-trading by different system operators through the glasses of Generalized Nash Equilibrium (GNE), which provides a natural context for modeling incomplete markets. Generalized Nash Equilibria are related to Quasi-Variational Inequality (QVI) models for which computational advances have recently been proposed. QVI problems are extensions of Variational Inequality (VI) problems. They differ by both their mathematical properties and economic interpretations. The paper implicitly uses the VI and QVI concepts by respectively referring to the Nash Equilibrium (NE) and Generalized Nash Equilibrium (GNE) problems.

While a Nash Equilibrium describes an equilibrium between agents interacting through their payoffs, a Generalized Nash Equilibrium involves agents that interact both at the level of their payoffs, but also through their strategy sets: the action of an agent can influence the payoff of another agent, but it can also change the set of actions that this agent

can undertake. The idea of using Generalized Nash Equilibrium in electricity transmission controlled by several operators is quite natural: because of Kirchhoff's laws, the actions of one operator influences the set of possible actions of another operator. A transmission system operated by different operators is thus naturally described by a Generalized Nash Equilibrium.

We apply Nabetani, Tseng and Fukushima's algorithm (Nabetani et al. 2009) for Quasi-Variational Inequality problems to a model of market coupling associated to different degrees of coordination in counter-trading. We discuss the economic and mathematical insights provided by the application of the notion of Generalized Nash Equilibrium to that problem.

Starting from a reference scenario where energy and transmission markets are fully integrated into a nodal system, we then consider market coupling where energy and transmission are partially separated (implicit auction). We then assume different organizations of counter-trading. We first suppose that TSOs can resort to any counter-trading resource in the market whether in their jurisdictions or outside. We find that the un-discriminatory access to the same set of counter-trading resources "completes the market" and hence makes it efficient. We also find that this situation reproduces the results of the reference nodal scenario. Any restriction to the internal market of counter-trading resources degrades the situation. A first degradation happens if operators can only resort in a limited way to counter-trading resources outside of their jurisdiction. The situation can be improved by creating a market of transmission services at the counter-trading level, but full efficiency will only be restored in very particular cases. The last case is the one where the market of counter-trading resources is fully segmented. Efficiency is further deteriorated even though the introduction of a common market of transmission resources can again help.

We conduct the whole analysis on a simple six nodes region model, but the results are general. Specifically, the recourse to the Nabetani, Tseng and Fukushima's algorithm only requires solving an optimal power flow problem. This is now a standard model, which shows that the analysis can be conducted for any real world problem.

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