Price Cap Regulation and Investment Incentives Under Demand Uncertainty

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Abstract. We study the effect of price cap regulation on investment in new capacity in an oligopolistic (Cournot) industry, using a continuous time model with stochastic demand. A price cap has two mutually competing effects on investment under demand uncertainty: it makes the option of deferring investment very valuable, but it also reduces the interest of strategic underinvestment to raise prices. We show that there exists an optimal price cap that maximizes investment incentives. Just as in the case of deterministic demand, the optimal price cap is the clearing price of the competitive market. However, unlike the deterministic case, we show that such a price cap does not restore the competitive equilibrium; there is still under-investment. Sensitivity analyses and Monte Carlo simulations show that the efficiency of price cap regulation depends critically on demand volatility and that errors in the choice of the price cap can have detrimental consequences on investment and average prices. The model insights are discussed in the light of the electricity industry.

Keywords: Real options, stochastic games, price cap regulation, electricity markets

JEL code: C73, D92, L51, L94.

1. Introduction

The liberalization wave that swiped the electricity industry in the 1980s and 90s has dramatically changed the structure of the industry. While transmission and distribution remain regulated monopolies, generation has become a competitive industry in which prices are set in a wholesale market. In the early years of liberalization, the focus of academic research and regulatory scrutiny concentrated mainly on short-term market efficiency and

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competitiveness. As the first countries to liberalize have now reached the end of the first investment cycle, much attention is being paid to assessing the long-term dynamic performance of the industry.

One highly polemical issue in both the US and Europe is whether the liberalized electricity industry can deliver adequate investment to maintain security of supply. This entails a variety of issues related to electricity markets design and regulation. Many industry experts and academics defend the view that the idiosyncrasies of the electricity industry (i.e. concentrated markets and remaining non-market mechanisms such as price caps) are likely to result in delayed or under-investment (e.g. de Vries, 2004, Stoft, 2002).[1] In particular, a variety of price control mechanisms related to technical dispatch constraints or market power mitigation procedures are suspected to have a detrimental impact on investment incentives, particularly for peaking units which earn most of their revenues at periods of high prices (Stoft, 2002, Joskow, 2003, Joskow and Tirole, 2006).

This paper studies the impact on investment incentives of price caps in wholesale electricity markets. The use of such price cap in electricity markets is justified theoretically by physical and engineering constraints, which might prevent the market to clear in times of scarcity.[2] The theory of "value of lost load pricing" states that in a perfectly competitive market, a price cap set at the Value of Lost Load (VOLL) results in a socially optimal level of investment in generating capacity, with an optimal duration of power shortages (see e.g. Stoft, 2002). The difficulty with this approach lies in the determination of VOLL, and hence the adequate level of the price cap. Estimates suggest that the value of lost load is some two orders of magnitude higher than regular electricity prices, but they vary widely across different consumers and with the outage duration (Willis and Garrod, 1997, Roques et al., 2005).[3] Moreover, regulators have introduced price caps in many

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[1]Joskow (2003) concludes from his study of the New England electricity market:

> "I think that there are good reasons to believe that spot market prices for energy and operating reserves alone, [...] are unlikely to provide adequate incentives to achieve generating capacity levels that match consumer's preferences for reliability. A variety of market and institutional imperfections contribute to this problem."

[2]Electricity cannot be stored and both supply and demand are very inelastic close to the system's maximum capacity, such that in times of scarcity, when all available power stations are running at full capacity and cannot satisfy demand, the system operator might have to shed load to preserve the system from collapsing. As most consumers are not involved in real-time price-setting, the system operator clears the market at their estimated "value of lost load" (VOLL), which constitutes a de facto price cap.

[3]The former British Pool market was based on an estimate of VOLL at 2000£/MWh in 1990; the Australian market cap at VOLL was set at a similar level to the British one initially (AUD 5000 per MWh), but several actual black-outs lead to an increase of the

electricity markets as part of local market power mitigation procedures at a level well below VOLL. In the U.S. East coast wholesale electricity markets, for instance, market power mitigation procedures cap prices at $1,000/MWh, well below VOLL, thereby raising questions as regard to the impact on investment incentives of such low price caps.

The model presented in this paper concentrates on the level and timing of investment in electricity markets, although it is applicable to any industry characterized by oligopolistic (Cournot) competition, stochastic demand, and irreversible investment. Phillips and Mason (1996) detail for instance a variety of markets which are subject to price controls in the US, such as prices for intrastate delivery, automobile insurance premiums paid to California drivers, or the federally mandated price caps on medical services for Medicare and Medicaid patients. The model insights are also relevant to the broader debate about the impact of price cap regulation under uncertainty on investment in other utilities industries (telecoms, water). We introduce a continuous time model of irreversible investment in an oligopolistic industry with stochastic demand, and study the impact of a price cap on installed capacity and average prices in the long term. Although the model remains highly stylized to allow analytical tractability, it offers several useful insights and words of caution to regulators of such industries.[4]

Price cap regulation was introduced in the UK in the mid 1980s to regulate the newly privatized utilities industry.[5] Cowan (2002) assesses the practice of price-cap regulation for utilities and concludes that while the experience of price-cap regulation has generally been favorable, there remains concern about whether investment can be promoted under such a system of regulation. Although the impact of price cap regulation on efficiency in the short term have attracted a large literature, the effects of a price cap under uncertainty and on investment incentives in the long term have attracted less research.

We contribute to this issue by determining the optimal investment strategy for firms in a regulated Cournot oligopoly exhibiting stochastic demand and irreversible investment. We find that price cap regulation affects investment decisions in two mutually competing ways. On the one hand, it provides a disincentive for investment as it caps potential upside profits

VOLL price cap to AUD 10000 per MWh in order to encourage investment in peaking units.

[4]In the following sections we discuss where our model results need to be interpreted with caution because of the model stylised nature as compared to the great complexity of electricity markets. Such interludes can be omitted by the reader not interested in electricity markets.

[5]See e.g. Varian, 1992, chap. 14, and Laffont and Tirolle, 1993, for a theoretical presentation of price cap regulation.

while leaving potential downside losses unchanged, making the *option to defer* investment very valuable. On the other hand, the presence of a price cap provides an incentive for new investment because it reduces the incentive for firms to leverage their *market power* by strategically underinvesting in order to increase prices. Since firms cannot increase their profits by increasing prices when the price cap is binding, profit maximizing firms can only boost their profit by investing in new capacity in order to increase production. We find that for low price caps, the *option to defer* effect dominates, while for high price caps, close to the unregulated industry entry price, the *market power* mitigation effect dominates.

We find the optimal price cap, that is the price cap which maximizes investment (ignoring allocation aspects). We demonstrate that this optimal price cap corresponds to the perfect competition entry price, similarly to models with deterministic demand. However, unlike deterministic models, setting the price cap at the competitive level does not realize the competitive investment outcome, as there is still underinvestment compared to the competitive market. Besides, we find that the efficiency of price cap regulation depends critically on the volatility of demand. Sensitivity analyses and simulations suggest that not recognizing the option value effects arising out of uncertainty in demand when determining the optimal level of a price cap can have a significant negative impact on investment. We show for instance that a price cap set at a conventionally optimal level without taking into account demand uncertainty can actually be counter-productive in an industry characterized by relatively highly volatile demand, as it may reduce investment and increase prices. We also discuss the practical relevance of these insights in the perspective about the current debate on market design and price caps in electricity markets.

The rest of the paper is organized as follows. In section 2 we present a literature review. Section 3 introduces the model and the Nash-Cournot equilibrium solution in the absence of price cap. Section 4 introduces a price cap and investigates its impact on investment. We demonstrate that there exists an optimal price cap and find a closed form solution for it. We study the efficiency of price cap regulation and complement the results by Monte-Carlo simulation and sensitivity analyses in section 5. Finally, section 6 presents a summary of our conclusions, main findings and suggestions for further research.

2. Literature review

The theory of price cap regulation has been developed following Littlechild (1983)'s original recommendation for British Telecom's privatization and regulation. Littlechild envisaged that the price cap could eventually wither away as competition came in, and thus the point of the price cap was to "hold the fort" until competition arrives. It has taken longer than expected for utilities industries to become competitive, and price cap regulation has gained popularity in many countries. There is now an extensive literature focussing on theoretical studies of price cap regulation, and on comparisons with other forms of price regulation (see e.g. Beesley and Littlechild, 1989, Laffont and Tirole, 1993, and Laffont and Tirole, 2001). The literature generally focusses on the effect of price cap regulation on a monopoly; Molho (1995) gives a simple diagrammatic exposition of price ceilings effects under the cases of Cournot oligopoly.

Cowan (2002) points out that although price cap regulation appears to be successful in its main aim of establishing incentives within the regulatory period for cost efficiency, there remain questions as regards to its efficiency to induce appropriate investment in the long term, and in particular in the presence of uncertainty. Most of the literature uses static models, and thereby is limited to assess the dynamic impact of various kinds of uncertainties. In an attempt to model the effect of uncertainty, Earle et al. (2006) present a one time-period model of Cournot competition with uncertain demand. They show that price cap regulation in the presence of uncertainty might fail to increase production and therefore fail to increase consumer welfare. Biglaiser and Riordan (2000) study the dynamics of price regulation for an industry adjusting to exogenous technological change. Pint (1992) uses a stochastic cost model to compare price-cap versus rate-of-return regulation, focusing on differences in the timing of hearings and the amount of cost information collected.

A second strand of price cap regulation models taking a more intertemporal point of view has developed more recently. These models solve the optimal entry problem of firms under uncertainty using real options type of arguments. The first such model was presented in Dixit (1991) and Dixit and Pindyck (1994), where they study the impact of price control in a perfectly competitive market. They find that such regulatory interventions are uniformly detrimental as they introduce a disincentive for investment. Dobbs (2004) uses a similar model to study price cap regulation of monopolies. Our model bridges the gap between these two models and studies the case of oligopolistic (Cournot) competition in a similar setting. As the number of firms is increased to infinity, we retrieve Dixit and Pindyck's (1994) results, and when the number of firms is equal to one, we retrieve Dobbs' (2004) results.

The third strand of literature relevant to our model is the recent strategic real options literature. These models (Baldursson, 1998, Grenadier, 2002) combine real options arguments with differential games in order to model investment in oligopolistic industries. Such models rely on simplifying the calculation of the Nash equilibrium from a fixed point to a single agent maximization, building on the seminal paper of Leahy (1993). Our work makes use of this result in order to find the Nash equilibrium solution in the presence of price caps.

Turning to the literature on price caps in wholesale electricity markets, Stoft (2002) and Fraser (2003) provide an overview of the theoretical justifications and practical issues related to the use of a price cap in electricity markets. Stoft (2003) studies the impact of a price cap and a capacity subsidy on investment incentives in a static two period model. Joskow and Tirole (2006) explore the impact of a wholesale price cap below the competitive price level and find that it creates a shortage of peaking capacity in the long run when there is market power in the supply of peaking capacity. Grobman and Carey (2001) run simulations of investment in an electricity market with a price cap. Their results show that the long run effects of a price cap on investment and spot prices differ significantly based on market structure.

3. The model

In this section we lay out the model.

3.1. **Model assumptions.**

3.1.1. *Demand.*

Assumption 1. *Price is determined endogenously by the aggregate inverse demand function which takes the isoelastic form:*

$$P(t) = X(t)Q(t)^{-\frac{1}{\gamma}} \tag{1}$$

were $X(t)$ is an industry wide stochastic shock, $Q(t)$ is the aggregate quantity supplied to the market and γ is the elasticity parameter.

The use of such a constant elasticity demand function is common in real options models (e.g. Dixit and Pindyck, 1994, Grenadier, 2002, Dobbs, 2004), as it simplifies the search for closed form solutions. Furthermore such demand specification has one degree of freedom (the elasticity constant γ) which allows for some sensitivity analysis on this critical parameter.

Although this assumption might seem to be rather abstract and restrictive, we argue otherwise. In any industry, and especially in electricity markets, it is important to distinguish demand variability from demand uncertainty. The former corresponds to usual daily and seasonal demand fluctuations which are easy to forecast, while demand uncertainty is due to unexpected events (e.g. plant or transmission line breakdowns) and to weather unpredictability. In this perspective, the demand function used in the model should be interpreted as the “residual demand" or “net demand". That is, the demand that generators who are not engaged in forward contracts face in the market. Therefore, the uncertainty in demand encompasses both the effects of aggregate demand shocks (unforecasted changes in demand at short notice due to changes in weather, e.g. wind or temperature) and supply shocks (sudden breakdown of a “must run" base load plant that had its output contracted forward).

Assumption 2. *The stochastic shock $X(t)$ is assumed to follow a Geometric Brownian Motion (GBM) given by the equation:*

$$dX = X\mu dt + X\sigma dz \tag{2}$$

where μ is the drift of the demand process and σ is the instantaneous standard deviation of the Wiener process dz[6].

This specification implies that $X(t)$ is lognormally distributed. The GBM is a fairly general process whose use is widespread in the Real Options literature even though concerns are frequently voiced about its shortcomings (for example its tails are too flat). However, the focus of this paper is on long-term investment, and Metcalf and Hassett(1995) show that using a mean reverting process instead of GMB has little impact on cumulative investment in a similar real options application.

3.1.2. *Supply.* There are N agents active in the industry, all producing the same homogenous good, (for example electricity) using the same technology of production. We restrict the analysis to N symmetric firms and assume that there is no entry. The model therefore represents a market in which firms have been successful at raising barriers to entry, through for instance

[6]This can be extended to more general Ito processes.

vertical integration.[7] All N agents sell their production in a frictionless market at a single node.[8]

Each of the N agents produces an amount $q_i(t)$ at time t, such that the total quantity produced at time t is:

$$Q(t) = \sum_{i}^{N} q_i(t) \tag{3}$$

3.2. **The game.** The timing of the game is as follows: agents produce at full capacity and sell their output in a frictionless wholesale market. Demand is revealed and the market clearing price is determined through equation (1). Each agent receives his payoff and then decides whether to expand capacity at a fixed and irreversible cost of C per unit or wait.[9] We assume capacity is infinitely divisible and becomes available instantaneously. We will relax this last assumption in the fifth section. Investment in new capacity is assumed to be completely irreversible, which is equivalent to $q_i(t)$ being a non-decreasing function of time.[10]

This game is repeated in continuous time.

Assumption 3. *Normalizing variable and fixed costs to zero, the profit $\pi_i(t)$ that each producer makes at time t is defined by:*

$$\pi_i(q_i(t), Q_{-i}(t), X(t)) = X(t)Q(t)^{-1/\gamma} q_i(t) \tag{4}$$

where $Q_{-i}(t)$ is the aggregate production of all firms but firm i at time t.

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[7]In the light of the recent European electricity industry, it seems credible that the industry will eventually be dominated by an oligopoly of 5 to 7 major vertically integrated companies.

[8]The electricity industry is more complicated than that. We abstract from transmission constraints and network effects which would render the model intractable and are not the focus of this paper. Moreover, we do not model the impact of forward contracts between produces and retailers and assume that generators' output is entirely sold in a wholesale market. The issue of how forward contracts impact on generators's bidding behaviour and profit in spot markets is a problem which has attracted lots of research (see e.g. Allaz and Vila, 1993, Newbery, 1998, Green, 1999, and Murphy and Smeers, 2005), but it is not the focus of this paper. This assumption is justified to keep the model tractable, Murphy and Smeers (2006) for instance make the same assumption, as well as all other models mentioned in the literature review section.

[9]Alternatively we could view this as a two stage game, where agents decide how much to produce up to their capacity limit and then decide whether to increase capacity at an irreversible cost C per unit. The two stage game is reduced to one stage game if $N\gamma > 1$ as shown in Appendix 1.

[10]Investment is irreversible in the sense that firms cannot sell some of their capacity if demand declines. This assumption appears appropriate in electricity markets, which are characterised by large sunk costs.

Under this demand specification, symmetric agents will always produce at full capacity, provided the market is concentrated enough, or demand is elastic enough such that $N\gamma > 1$. This is shown in appendix 1.

For simplicity we assume that there is neither technological progress nor physical asset depreciation[11]. Furthermore we assume that there is no risk of unexpected technical failure[12]. The risk free discount rate is denoted by ρ.

Firms continuously maximize their profit by expanding capacity whenever such strategy is profitable. When they increase capacity, they pay a sunk cost of C per unit. The maximization problem of firm i at time t consists of determining the firm's investment strategy, so that it maximizes its expected profit including the cost of increasing capacity. Each firm therefore faces a sequence of investment opportunities and must determine an exercise strategy. Agents have complete information, are rational and risk neutral.

However there is one crucial difference with standard real options models: firms are no longer price takers. Their investment decisions affect not only their cash flow but also the market clearing price through equation (1). Therefore the optimal investment strategy has to take into account other firms' investment decisions. It is determined as part of a Nash equilibrium, where firms compete à la Cournot. At each time t, firm i will decide whether to increase capacity or not in order to maximize its expected operating profit, taking into account that increasing its own capacity impacts competitors profits and vice versa.

Alternatively we can think of each firm as owning a sequence of call options on the stochastic price of the output. The strike price is the investment cost. However, each firm fully recognizes that the price process is endogenous: The exercise of such options by its competitors will impact its own profits by reducing the market clearing price and vice versa.

Definition 4. *The capacity expansion problem of firm i is an optimal control problem that can be formulated by the following objective functional J, where the expectation operator $\mathbf{E}$ is conditional on the initial state (q_{i0}, Q_{-i0}, X_0):*

[11]Both technological progress and physical depreciation could be incorporated in the model as extra discount rates, see for example Dobbs (2004), Laffont and Tirole (2001). This assumption does not change the qualitative nature of our results and is omitted for simplicity.

[12]Unexpected technical failure could have been modelled by a Poisson process. This would effectively add as an extra discounting term, for example see Dixit and Pindyck, 1994.

$$J(q_{i0}, Q_{-i0}, X_0, q_i(t), Q_{-i}(t), X(t)) = \max_{q_i(t)\in[0,\infty)} \mathbf{E}[\int_{t=0}^{\infty} \pi(q_{i0}, Q_{-i0}, X_0, q_i(t), Q_{-i}(t), X(t))e^{-\rho t}dt - \int_{t=0}^{\infty} Ce^{-\rho t}dq_i(t)] \tag{5}$$

3.3. **Nash equilibrium investment strategies.** We restrict the analysis to symmetric Nash Cournot equilibrium, so that at time t, $q_i(t) = Q(t)/N$, for all firms. We follow Baldursson's (1998) and Grenadier's (2002) simplified approach to derive symmetric Nash-Cournot equilibrium strategies. Baldursson (1998) and Grenadier (2002) build on the seminal paper by Leahy (1993) to demonstrate that the Nash equilibrium investment strategy coincides with the investment strategy of a myopic firm ignoring competitive actions.[13] The importance of this simplification is that it reduces the search for a Nash equilibrium from finding the fixed point of an N dimensional system to the standard real options evaluation problem for a single firm.

We can now use Grenadier's (2002) simplified approach to equilibrium to find the Nash Cournot equilibrium investment strategies of firms for the oligopolistic industry.

Proposition 5. *The marginal value of a firm m_i for the symmetric Nash Cournot equilibrium investment strategy of firm i is given by the following differential equation:*

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 m_i}{\partial^2 P} + \mu P \frac{\partial m_i}{\partial P} - \rho_i + \frac{\partial \pi}{\partial q_i} = 0 \tag{6}$$

with

$$\frac{\partial \pi}{\partial q_i} = P\frac{N\gamma - 1}{N\gamma} \tag{7}$$

and the free boundary conditions[14]*:*
Value matching condition at the investment trigger P^*

$$m_i(P^*, Q_{-i}, q_i) = C \tag{8}$$

Smooth pasting condition at the investment trigger P^*

$$\frac{\partial m_i}{\partial P}(P^*, Q_{-i}, q_i) = 0 \tag{9}$$

[13]Baldursson (1998) and Grenadier (2002) show that besides the monopoly and perfectly industry cases, it is also possible to solve the oligopoly case as a single agent optimisation problem. The procedure is just to pretend that the industry is perfectly competitive, maximising a "fictitious" objective function, using an "artificial" demand function.

[14]The smooth pasting conditions in this continuous time model are akin to the first order necessary conditions for value maximisation in a static optimisation model.

Proof. See Grenadier (2002). □

Proposition 6. *The investment price trigger P^* is given by:*

$$P^* = C(\rho - \mu)\frac{\beta_1}{(\beta_1 - 1)}\frac{N\gamma}{(N\gamma - 1)} \tag{10}$$

where β_1 and β_2 are the roots of the second degree characteristic equation corresponding to equation (6). The investment trigger is a decreasing function of the number of firms N in the industry.

Proof. See Appendix 2. □

The first two terms are familiar. $P = C(\rho - \mu)$ represents the investment price trigger in a competitive industry without uncertainty, (see e.g. Laffont and Tirole (2001), p.151). This price is the zero NPV point in the sense that if there is no uncertainty, a firm increasing capacity infinitesimally every time the price hits this points (which happens continuously) will be making a zero economic profit. The term $\frac{\beta_1}{(\beta_1-1)} > 1$ is often referred to as the option value multiplier. It is the classic real options result that in the presence of uncertainty, firms should not invest as soon as the NPV is positive but they should wait until is positive enough to make the possibility of not recouping their irreversible investment costs remote. Therefore the investment price trigger in a competitive industry under uncertainty is given by

$$P^*_{N=\infty} = C(\rho - \mu)\frac{\beta_1}{(\beta_1 - 1)} \tag{11}$$

(see e.g. Dixit and Pindyck, 1994).

The term $\alpha = \frac{N\gamma}{(N\gamma-1)} > 1$ can be interpreted as a market power mark-up. That is, the investment price trigger for the oligopoly P^* is equal to the competitive investment entry price trigger $P^*_{N=\infty}$ multiplied by the oligopoly mark-up α. Figure 1 illustrates the impact of market concentration on the investment trigger in an oligopolistic industry and a perfectly competitive industry with and without price cap. The investment price trigger is a decreasing function of the number of firms, and since the oligopoly mark-up $\alpha > 1$, firms in the oligopolistic industry only add capacity when prices reach value values that are higher than would be necessary for capacity increase under perfect competition. Prices are therefore uniformly higher under oligopolistic competition, while installed capacity is less.

Moreover, in the limit of a perfect competition (as N goes to infinity), α tends towards 1 and the investment price trigger P^* of equation (10) tends towards $P^*_{N=\infty} = C(\rho - \mu)\frac{\beta_1}{(\beta_1-1)}$, which corresponds to the investment price trigger in a perfectly competitive industry (see Dixit and Pindyck, 1994). Moreover, when $N = 1$, $\alpha = \frac{\gamma}{\gamma - 1}$ which corresponds to the monopoly mark-up as in Dobbs (2004).

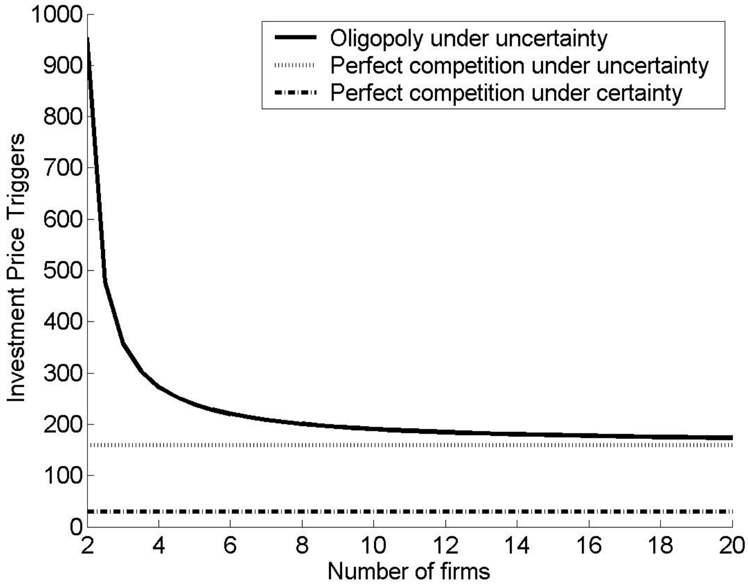

Figure 1 - Price Triggers vs. Market Concentration
$C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08$

4. Impact of price cap regulation

Let us now assume that prices are capped at a predetermined level $\bar{P}$ by the regulator.[15]

4.1. **Nash-Cournot equilibrium with a price cap.** In this section, P represents the hypothetical market-clearing or "shadow" price, which is monotonically related to the pressure of demand through equation (1). When $P \leq \bar{P}$ the shadow price coincides with the actual market price, otherwise the market price is the price cap $\bar{P}$.

[15] Two types of price caps need to be distinguished in electricity markets, namely *retail* and *wholesale* market price caps. This stylised model concentrates on the impact of *wholesale* price caps (see e.g. Stoft, 2002 for a technical discussion of the impact of retail and wholesale price caps in electricity markets). Wholesale price caps constitute a widespread tool used by regulators to mitigate market power exercise in electricity markets, but are believed to have important side effects on investment. In PJM and most of the US East Coast wholesale markets, there is for instance a 1000\$/MWh bidding price cap. Fraser (2003) highlights for instance the detrimental impact of price caps on investment that caused temporary capacity shortages and price increases in the late 1990s in Australia and in Canada (Alberta and Ontario). The price cap in this model can also be interpreted more broadly as any kind of regulatory rule that prevents prices from moving freely up when the market is tight. Such regulatory rules and engineering constraints on wholesale markets are detailed for instance in Stoft (2002), Joskow (2003), and Joskow and Tirole (2006).

Definition 7. *The profit* π_i *each producer makes at time* t *is given by:*

$$\pi_i(q_i, Q_{-i}, X) = \min\left\{XQ^{-1/\gamma}q_i, \bar{P}q_i\right\} \tag{12}$$

where Q_{-i} *is the aggregate production of all firms but firm* i *at time* t.

We denote by $\bar{P}^*$ the shadow investment price trigger on which it is optimal for firms to invest in more capacity when a price cap $\bar{P}$ is implemented. A price cap higher than the regulated industry investment price trigger is simply irrelevant, as voluntary investment decisions will always generate enough capacity to keep the price below the price cap. In other words, necessarily $\bar{P} \leq \bar{P}^*$ because $\bar{P}^*$ is a reflecting boundary for the price process. When the price is at the ceiling $\bar{P}$, the price cap is binding and excess demand is rationed in such a way that generators do not capture any of the scarcity rent.

Solving now the stochastic optimal control problem of equation (5) using (12) is a bit more complicated than the problem of the previous section. We now have to solve a differential equation in two different price regimes: binding and not binding price caps. The details of the proof are presented in Appendix 3. We present below the main result:

Proposition 8. *When the regulator caps prices at* $\bar{P}$*, the shadow investment price trigger* $\bar{P}^*$ *is given by:*

$$\bar{P}^* = [\lambda(C - \frac{\bar{P}}{\rho})\bar{P}^{(\beta_2 - 1)}]^{1/\beta_2} \tag{13}$$

with λ *defined as:*

$$\frac{1}{\lambda} = \frac{(\beta_1 - 1)}{\beta_1}\frac{1}{\alpha(\rho - \mu)} - \frac{1}{\rho} \tag{14}$$

where α *is the elasticity mark-up:*

$$\alpha = \frac{N\gamma}{(N\gamma - 1)} > 1 \tag{15}$$

Proof. See Appendix 3. □

4.2. **Optimal price cap.** Let us now search for the optimal level of the price cap $\bar{P}_{opt}$, that is the level at which a regulator aiming to minimize the shadow investment price trigger should set the price cap at.[16]

[16]Note that we optimise for investment and ignore allocation aspects, which can in practice be an important determinant of the choice of a price cap (e.g. regulators can be tempted to introduce a price cap to prevent excess profits for extended periods, or to protect consumers from sustained high or volatile prices).

Proposition 9. *The optimal price cap $\bar{P}_{opt}$ is given by the following expression:*

$$\bar{P}_{opt} = P^*_{N=\infty} = C\left(\rho - \mu\right)\frac{\beta_1}{(\beta_1 - 1)} \tag{16}$$

Proof. See appendix 5. □

In other words, under uncertainty, similarly to the deterministic case, the optimal level of the price cap is equal to the competitive industry investment trigger price. Equation (16) indicates that the optimal price cap in this intertemporal model does not depend on the market concentration, but does depend on the volatility of demand and on the discount rate.

Plugging in the optimal price cap of equation (16) to the shadow investment price trigger equation (13) we obtain:

$$\bar{P}^*(\bar{P}_{opt}) = P^*_{N=\infty}\frac{N\gamma}{(N\gamma - 1)}(1 - \frac{1}{N\gamma}\frac{\beta_1 - 1}{\beta_1}\frac{\lambda}{\rho - \mu})^{1/\beta_2} \tag{17}$$

Since $\bar{P}_{opt}$ is the optimal price cap, the shadow investment price trigger of equation (17) is the lowest possible price trigger that the regulator can induce for the oligopolistic market. Note that as the number of firm increases the price trigger approaches the competitive market investment price trigger from above. For finite values of N this price trigger is always higher than the competitive level.

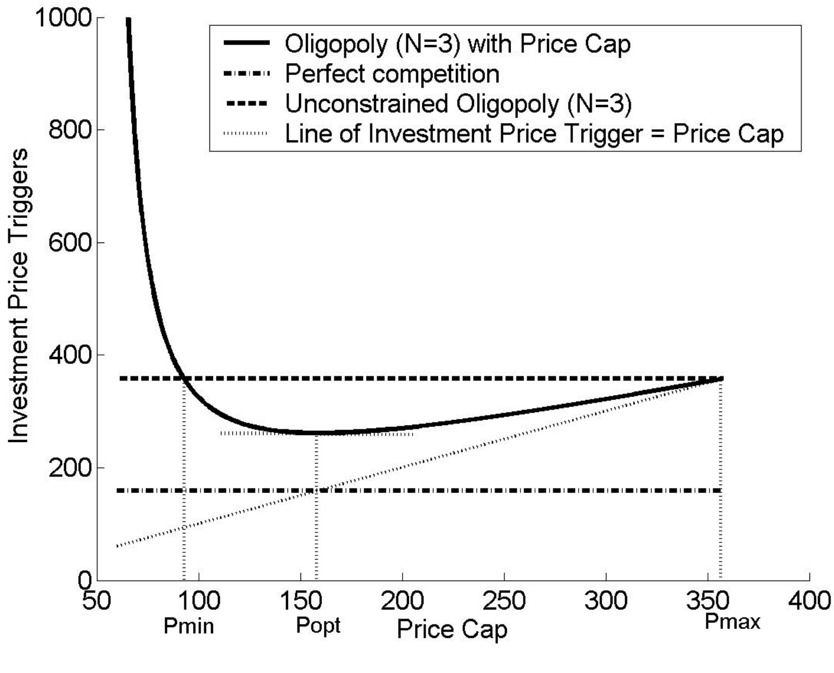

Figure 2 - Investment Price Trigger vs. Price Cap, $C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 3.$

Figure 2 illustrates the point that the optimal price cap does not depend on the market concentration. What is also observed is that there is a range over which the price cap has a beneficial effect by lowering the shadow investment price trigger. We examine this region with the following proposition:

Proposition 10. *There exists an interval* $(\bar{P}_{\min}, P^*)$ *over which the introduction of a price cap* $\bar{P} \in (\bar{P}_{\min}, P^*)$ *lowers the shadow investment price trigger compared to the unregulated investment price trigger (i.e.* $\bar{P}^* < P^*$*).*

Proof. See Appendix 4 □

Figure 2 shows that the relationship between the shadow investment price trigger $\bar{P}^*$ and the level of the price cap $\bar{P}$ is not monotonic. This result contrasts with Dixit (1991) and Dixit and Pindyck (1994)'s findings that under perfect competition, the shadow investment price trigger is a decreasing function of the level of the price cap.

Note that $\bar{P}_{opt} \in (\bar{P}_{\min}, P^*)$. For $\bar{P} < \bar{P}_{opt}$, the shadow investment price trigger $\bar{P}^*$ is a decreasing function of the level of the price cap $\bar{P}$, and tends towards infinity as $\bar{P}$ is lowered towards variable costs (which we have set to zero), while for $\bar{P} \in (\bar{P}_{opt}, P^*)$, the shadow investment price trigger $\bar{P}^*$ is an increasing function of the level of the price cap $\bar{P}$.

The intuition for this result is that under oligopolistic competition, the price cap has an impact not only on the *option value of waiting* characterizing irreversible investments under uncertainty, but also reduces the benefits of *strategic underinvestment*. These two effects work in opposite directions as regards to investment incentives.

- Impact of a price cap on the *option value of waiting (deferring investment)*

On the one hand, a price cap has a negative impact on the option value of waiting associated with demand uncertainty, as it caps potential upside profits while leaving unchanged potential downside losses, thereby providing a disincentive to investment. A price cap reduces the likelihood of future high profits, so that investors need to be confident that the pressure of demand will stay high for longer (and hence the price will be equal to the price cap) than when there is no price cap in order to commit to new investment. Hence, a tighter price cap implies that a greater current pressure of demand is needed to bring about investment. As Dixit (1991) explains in his model of perfect competition,

> "As the imposed ceiling is lowered toward the long-run average cost, the critical shadow price that induces new investment goes to infinity. In other words, if the imposed price

cap is so low that at this point the return on capital is only just above the normal rate, then investors want to be assured that this state of affairs will last almost forever before they will commit irreversible capital."

- Impact of a price cap on the *benefits of strategic underinvestment*

As seen before, under symmetric oligopolistic (Cournot) competition without price cap, the value maximizing strategy of a firm is to reduce capacity through strategic underinvestment as compared to the perfect competition case, in order to increase prices. But when there is a price cap, in the case where it is binding, increasing capacity in a Nash Cournot game does not lead to a reduction in price, hence providing an incentive to increase investment. In other words, a price cap reduces the benefits for firms to underinvest. Indeed, when the price cap is binding, firms have an incentive to invest as the increased capacity will not reduce price. A tighter price cap should therefore reduce the shadow investment price trigger $\bar{P}^*$ and thereby induce greater investment and lower prices.

A price cap has therefore a *dual impact* on the shadow investment price trigger in an oligopolistic industry, which explains the two regimes observed on Figure 2.

For $\bar{P} \in (\bar{P}_{opt}, P^*)$, the positive effect of the price cap on *strategic underinvestment* dominates the negative impact of the price cap on the *option value of waiting*, so that the shadow investment price trigger $\bar{P}^*$ is an increasing function of the level of the price cap $\bar{P}$.

On the contrary, for a price cap lower than the competitive entry price $\bar{P}_{opt}$, the impact of the price cap on the *option value of waiting* dominates, such that the shadow investment price trigger $\bar{P}^*$ is a decreasing function of the level of the price cap $\bar{P}$.

Moreover, the implementation of a price cap at a level lower than $\bar{P}_{\min}$ would be counterproductive, as it raises the shadow investment price trigger above the unregulated oligopolistic investment price trigger. As the price cap is lowered to zero, the shadow investment price trigger tends towards infinity, as in the case of perfect competition.[17]

[17]These results are consistent with Dobbs (2004), who studies the impact of price cap on a monopoly under demand uncertainty.

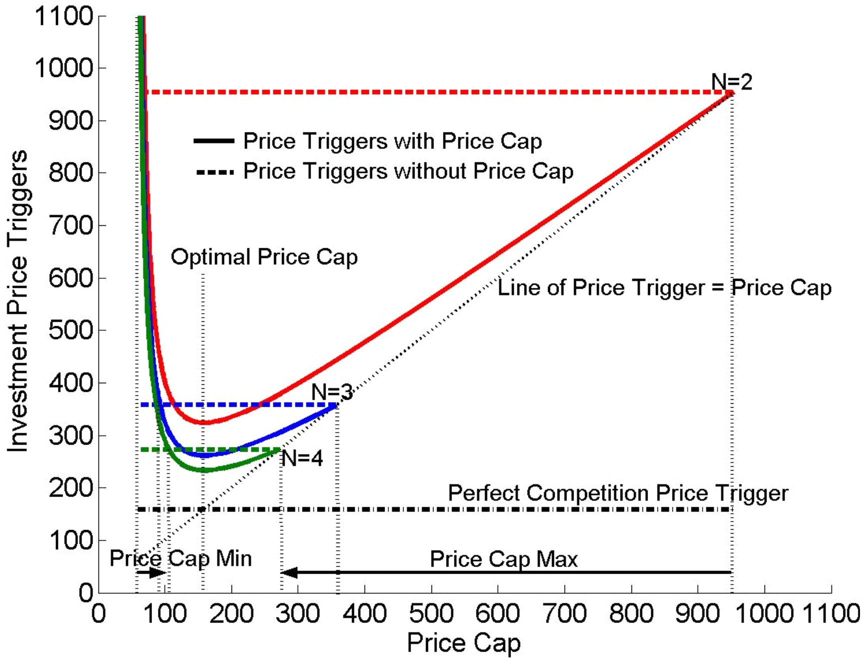

Figure 3 - Investment price triggers vs. price cap for different degrees of market concentration,
$C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, .N = 2, 3, 4.$

Figure 3 illustrates the point that the optimal price cap does not depend on the market concentration, but that the bounds of the interval $(\bar{P}_{\min}, \bar{P}_{\max} = P^*)$ do. The higher the industry concentration (the lower N), the lower $\bar{P}_{\min}$, and the larger P^*. This is a fairly intuitive result: the higher the industry concentration, the larger the interval over which the introduction of a price cap is beneficial and lowers the shadow investment price trigger as compared to the oligopolistic industry investment trigger without price cap. This generalizes Dobbs' (2004) similar results obtained in the case of a monopoly to the case of a Cournot oligopoly.

Figure 4 shows the impact of demand volatility on the optimal price cap $\bar{P}_{opt}$. It indicates that the optimal price cap $\bar{P}_{opt}$depends critically on the level of demand volatility in the market, and suggests that regulators should pay attention to demand uncertainty when deciding on the level of the price cap. This finding is particularly relevant to the debate on investment incentives in electricity markets, as it suggests that a price cap set at the long run marginal cost of the most expensive peaking unit might be significantly too low to induce adequate investment, due to the high demand uncertainty characterizing the electricity industry, and in particular peaking units.

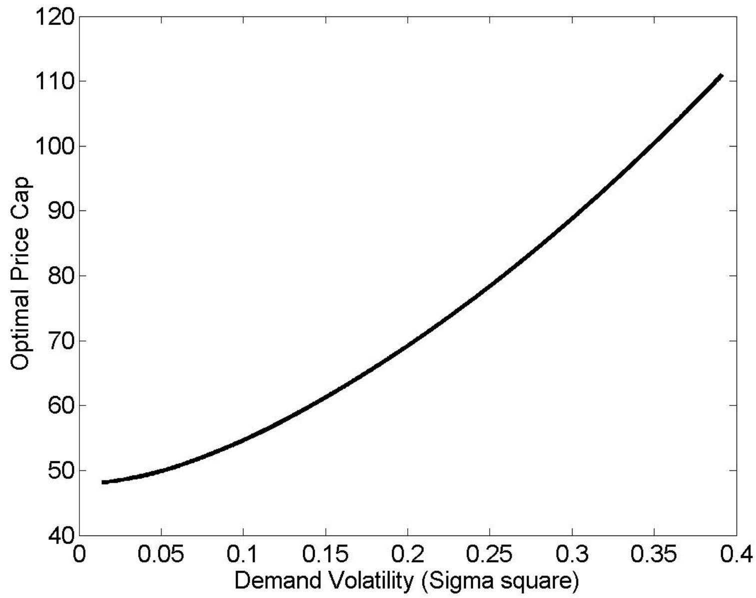

Figure 4 - Optimal price cap vs. demand volatility,
$C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 3.$

5. Sensitivity analysis and simulations

In this section we investigate the efficiency of price cap regulation and we look into the potential impact of not taking into account demand uncertainty when setting a price cap. We finish the section with a simulation of the long term effect of price cap regulation on investment in new capacity and average price.

5.1. **Price Cap efficiency.** In section 4 we found that contrary to the results of the static models, price cap regulation cannot restore the perfectly competitive equilibrium. Figure 2 and 3 showed that even with the optimal price cap, there will be under-investment compared to the perfect competition case and firms will enjoy positive rents. In this subsection, we investigate how close price cap regulation can bring the market to the competitive outcome.

Definition 11. *We define the price cap efficiency coefficient e by*

$$e = \frac{P^* - \bar{P}^*}{P^* - P^*_{N=\infty}}$$

where P^* is the investment price trigger for the unregulated industry, $\bar{P}^*$ is the shadow investment price trigger when the regulator caps prices at $\bar{P}_{opt}$, and $P^*_{N=\infty} = \bar{P}_{opt} = C(\rho - \mu)\frac{\beta_1}{(\beta_1 - 1)}$ is the investment price trigger of the unregulated competitive industry.

The price cap efficiency coefficient e takes the value of 1 if price cap regulation successfully retrieves the competitive outcome and it is zero if, at best, it price cap regulation is irrelevant.[18]

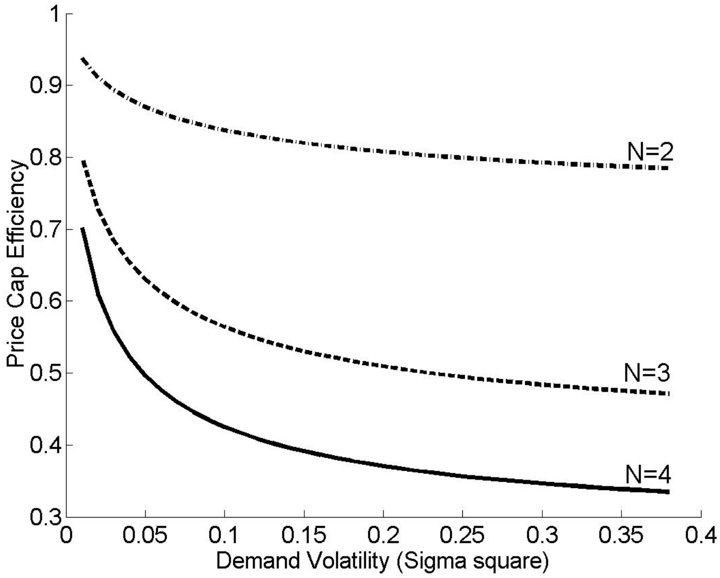

Figure 5 - Price cap efficiency vs. demand volatility,
$C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 2, 3, 4.$

Figure 5 confirms the intuition that the more concentrated the market, the more efficient a price cap is. Moreover, figure 5 shows that the efficiency of a price cap depends critically on demand uncertainty, as high demand uncertainty reduces dramatically the efficiency of price cap regulation. This is intuitively explained by the option value to defer investment, as explained before: uncertainty (measured here by demand volatility) makes the option to defer investment in new capacity valuable. A price cap interferes destructively with the upside potential without reducing the potential downside losses, thereby making it necessary for firms to wait for even higher demand before committing investment. Therefore increased uncertainty reduces the efficiency of price cap regulation because it amplifies the value of postponing investment.

5.2. **Misspecification of the price cap due to demand uncertainty.** In this subsection we examine the effect of not (or incorrectly) taking into account demand uncertainty when setting the level of price cap regulation. Let us suppose that the regulator does not take into account that demand is uncertain when choosing the price cap level. Figure 6 shows the percentage

[18] An interesting extension left for further research would be to study the impact of a price cap on the market efficiency, i.e. on a measure of social welfare such as the sum of firm profits and consumer surpluses. A proper evaluation of the latter would require a more detailed study of quantity rationing and explicit modelling of consumer surplus.

error on the optimal price cap made by treating demand as deterministic. For instance, a price cap set at the optimal level when demand is deterministic ($\sigma^2 = 0$) results to a 39% underestimation of the optimal price cap when the actual demand volatility is $\sigma^2 = 0.01$, and a 67% under-estimation when the actual demand volatility is $\sigma^2 = 0.1$. For relatively low actual demand volatilities, such an underestimated price cap would not be dire, it would still lower the shadow investment price trigger as compared to the unregulated oligopoly, and therefore increase investment. However for higher demand volatility, the price cap would be counter productive in the sense that it would increase the shadow investment price trigger as compared to the unregulated market, and therefore lead to underinvestment.

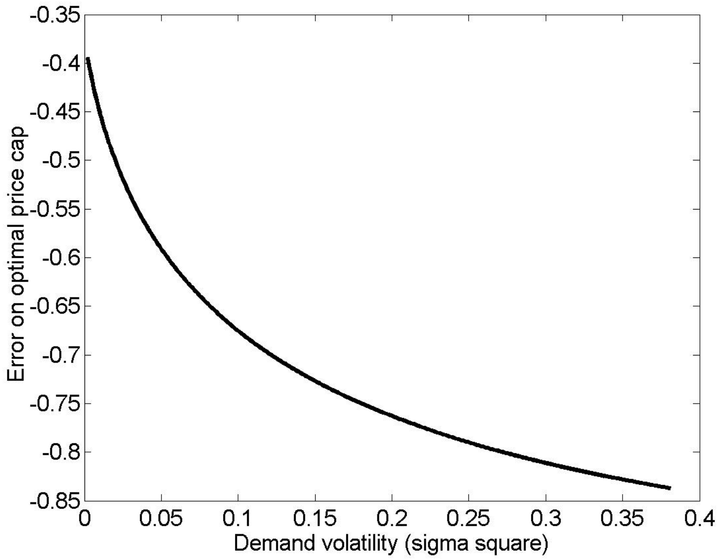

Figure 6 - Error on optimal price cap when not taking into account demand volatility, $C = 600, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 3$.

This has important implications for regulated industries where demand volatility is difficult to measure and is subject to structural breaks or macroeconomic shocks. The lesson for regulators is that it is better to *over estimate* volatility rather than underestimate it. Significant underestimation has the potential to make regulatory intervention counter productive by slowing down investment beyond the level of the unregulated market. On the contrary, significant over estimation will reduce the efficiency of price cap regulation, but it will not slow down investment.

5.3. **Long term effect of price cap regulation.** In this section we aim to provide some insights on the magnitude of the investment delays caused either by the exercise of market power, or by a counter productive use of a price cap by the regulator (i.e. a price cap set too low). We simulate several realizations of the stochastic demand, and observe the corresponding capacity expansion and price paths[19].

5.3.1. *Simulation parameters.* The model is implemented numerically using typical parameter values for electricity markets. We do not pretend to offer a precise characterization of investment in electricity markets, due to the model's stylized nature. Rather, the aim of this section is to provide some insights on the order of magnitude of the quantitative impact on the delays and under-investment identified in the previous sections. Table 1 summarizes the main model variables and the default simulation parameters. Initial capacity is normalized to one, as we are interested in comparing the relative level of investments. It should be noted that the results presented in this section were found to be qualitatively robust over a wide range of parameters.

Base case parameters	Value	Unit
Fixed investment costs[20]	$C = 600$	US$/kW
Volatility	$\sigma^2 = 0.3$	p.a.
Demand growth ($\mu < \rho$)	$\mu = 0.03$	p.a.
Price elasticity	$\gamma = 0.6$	p.a.
Risk free discount rate	$\rho = 0.08$	%
Number of firms ($N > 1/\gamma$)	$N = 3$	
Initial production quantity	$Q_0 = 1$	Normalized
Optimal Price cap	$\bar{P}_{opt} = 158.94$	US$
Initial inverse demand	$X_0 Q_0^{-1/\gamma} = 160$	MW

Table 1 - Base case simulation parameters

5.3.2. *Market evolution:* Figure 7 shows a typical demand and price realization together with the evolution of capacity in three cases: perfect competition, oligopoly without price cap, and oligopoly with an optimal price cap. The figure illustrates that although the regulated oligopoly is investing more in new capacity than the unregulated oligopoly at any time, it still does not invest as much as the competitive industry.

[19]Numerical solutions and simulations were done using MATLAB. The code is available from the authors on request.

[20]Recent estimates from the US Department of Energy suggest that the capital cost for a new plant range from US$ 400/KW for an open cycle gas turbine, US$ 600 for a combined-cycle gas turbine, US$ 1200/KW for a coal plant, to US$ 2000/KW for a nuclear plant (DOE 2004).

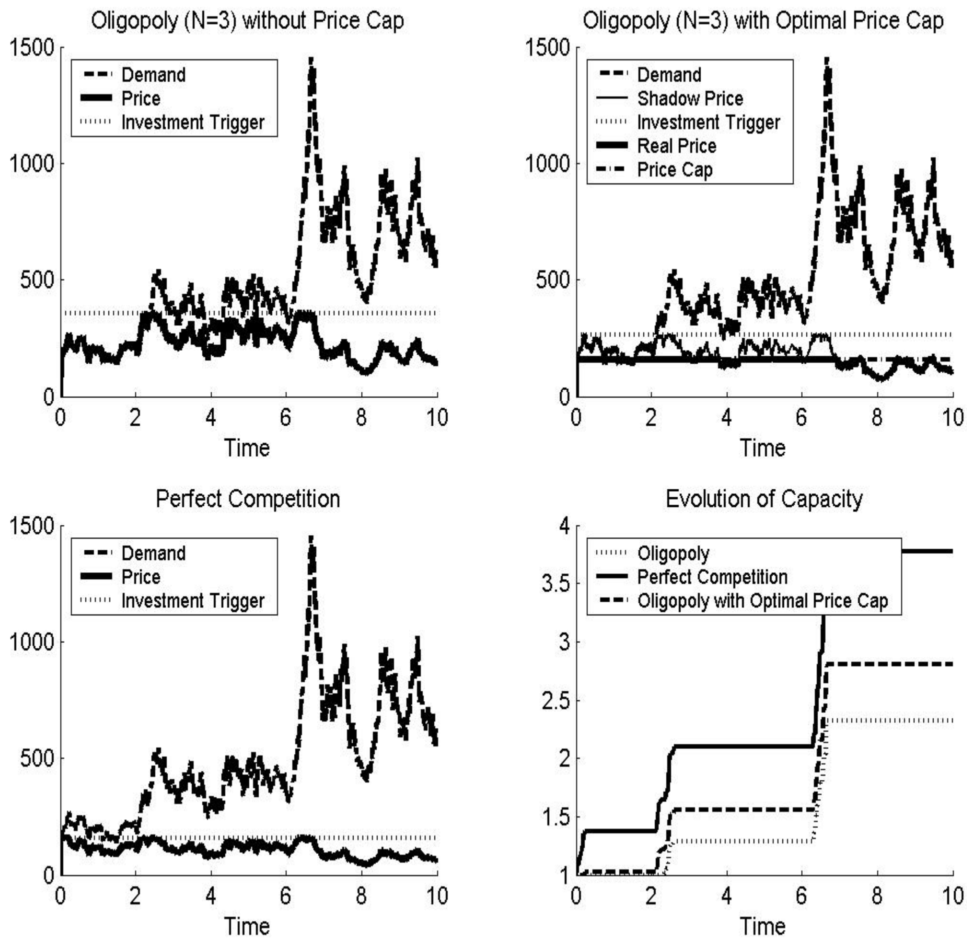

Figure 7 - Price and capacity evolutions for one typical demand realisation, $C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 3, X_0 = 160$.

5.4. **Simulation.** Figure 7 shows the evolution of price, and installed capacity for one demand realization. In this section, we use a Monte Carlo simulation to gain some insight on the long term effect of price cap regulation on both investment in new capacity and prices.

5.4.1. *Impact of price cap on installed capacity.* We compute the average installed capacity after 10 years over 10,000 demand realizations in the case of unregulated oligopoly, regulated oligopoly and perfectly competitive industry and calculate the following ratios:

- $\frac{Q_{Olig}}{Q_{comp}}$ represents the ratio of the average installed capacity after 10 years of an unregulated oligopoly to that of the perfectly competitive industry.
- $\frac{Q_{OPC}}{Q_{comp}}$ represents the ratio of the average installed capacity after 10 years of a regulated oligopoly to that of the perfectly competitive industry.

Table 2 shows the extend of underinvestment caused by market power for varying values of the base-case parameters. The results of the simulation suggest that the impact of imperfect competition on installed capacity can be quite significant. Both the unregulated as well as the regulated oligopolies install on average only 69% and 78% respectively of the perfectly competitive industry capacity after 10 years with the base-case parameters. Table 2 shows also that variations of the investment cost, growth and volatility of demand, price elasticity, and market concentration significantly impact both the extend of underinvestment, and the efficiency of price cap regulation.

Investment cost			Volatility			Load growth			Price elasticity			Discount rate			Number of firms		
K	$\frac{Q_{Olig}}{Q_{comp}}$	$\frac{Q_{OPC}}{Q_{comp}}$	σ^2	$\frac{Q_{Olig}}{Q_{comp}}$	$\frac{Q_{OPC}}{Q_{comp}}$	m	$\frac{Q_{Olig}}{Q_{comp}}$	$\frac{Q_{OPC}}{Q_{comp}}$	γ	$\frac{Q_{Olig}}{Q_{comp}}$	$\frac{Q_{OPC}}{Q_{comp}}$	ρ	$\frac{Q_{Olig}}{Q_{comp}}$	$\frac{Q_{OPC}}{Q_{comp}}$	N	$\frac{Q_{Olig}}{Q_{comp}}$	$\frac{Q_{OPC}}{Q_{comp}}$
300	0.62	0.74	**0.001**	0.61	0.97	**-0.03**	0.76	0.82	**0.35**	0.71	0.82	**0.04**	0.65	0.74	**2**	0.56	0.71
400	0.64	0.75	**0.01**	0.61	0.87	**0.00**	0.72	0.80	**0.4**	0.70	0.81	**0.06**	0.67	0.76	**3**	0.69	0.78
500	0.66	0.76	**0.1**	0.63	0.77	**0.01**	0.71	0.79	**0.5**	0.69	0.79	**0.07**	0.68	0.77	**4**	0.76	0.82
600	0.69	0.78	**0.3**	0.69	0.78	**0.03**	0.69	0.78	**0.6**	0.69	0.78	**0.08**	0.69	0.78	**5**	0.81	0.85
700	0.72	0.80	**0.5**	0.74	0.81	**0.04**	0.68	0.77	**1**	0.69	0.75	**0.1**	0.71	0.79	**7**	0.86	0.88
800	0.74	0.81	**0.7**	0.76	0.83	**0.05**	0.67	0.77	**1.5**	0.69	0.73	**0.15**	0.75	0.83	**10**	0.91	0.92
900	0.76	0.83	**1**	0.80	0.85	**0.07**	0.66	0.76	**2**	0.69	0.73	**0.2**	0.78	0.85	**20**	0.95	0.96

Table 2 - Average installed capacity after 10 years (expressed as % of competitive market capacity)

5.4.2. *Impact of price cap on long term average price.* Similarly, we run a Monte-Carlo simulation to compute the average price markup after 10 years over 10,000 demand realizations in both a non-regulated oligopolistic industry and a regulated oligopolistic industry with an optimal price cap as compared to the competitive price, and compute the following ratios:

- $\frac{P_{Olig}}{P_{comp}}$ represents the ratio of the average price after 10 years in a non-regulated oligopolistic industry and in the perfectly competitive industry.
- $\frac{P_{OPC}}{P_{comp}}$ represents the ratio of the average price after 10 years in a regulated oligopolistic industry with an optimal price cap and in the perfectly competitive industry.

Table 3 shows the mark-up over the competitive price for varying values of the base-case parameters introduced in Table 11. The average price in the non-regulated oligopolistic industry represents on average 193% of the average competitive price after 10 years with the base-case parameters, and the average price in the regulated industry represents on average 139% of the competitive price. Table 3 shows also that variations of the investment cost, volatility of demand, load growth, price elasticity, and market concentration significantly affect the mark-up in both the non-regulated and the regulated oligopolistic industries.

Investment cost			Volatility			Load growth			Price elasticity			Discount rate			Number of firms		
K	$\frac{P_{Olig}}{P_{comp}}$	$\frac{P_{OPC}}{P_{comp}}$	σ^2	$\frac{P_{Olig}}{P_{comp}}$	$\frac{P_{OPC}}{P_{comp}}$	m	$\frac{P_{Olig}}{P_{comp}}$	$\frac{P_{OPC}}{P_{comp}}$	γ	$\frac{P_{Olig}}{P_{comp}}$	$\frac{P_{OPC}}{P_{comp}}$	ρ	$\frac{P_{Olig}}{P_{comp}}$	$\frac{P_{OPC}}{P_{comp}}$	N	$\frac{P_{Olig}}{P_{comp}}$	$\frac{P_{OPC}}{P_{comp}}$
300	2.22	1.49	**0.001**	2.25	1.01	**-0.03**	1.73	1.35	**0.35**	4.91	1.55	**0.04**	2.08	1.47	**2**	3.30	1.53
400	2.13	1.46	**0.01**	2.25	1.11	**0**	1.83	1.38	**0.4**	3.31	1.52	**0.06**	2.00	1.43	**3**	1.93	1.39
500	2.02	1.43	**0.1**	2.18	1.37	**0.01**	1.88	1.38	**0.5**	2.31	1.45	**0.07**	1.96	1.41	**4**	1.59	1.31
600	1.93	1.39	**0.3**	1.93	1.39	**0.03**	1.93	1.39	**0.6**	1.93	1.39	**0.08**	1.93	1.39	**5**	1.43	1.26
700	1.85	1.35	**0.5**	1.82	1.36	**0.04**	1.96	1.39	**1**	1.43	1.26	**0.1**	1.87	1.36	**7**	1.28	1.19
800	1.79	1.32	**0.7**	1.81	1.34	**0.05**	1.98	1.41	**1.5**	1.26	1.18	**0.15**	1.77	1.30	**10**	1.18	1.14
900	1.75	1.31	**1**	1.80	1.34	**0.07**	2.03	1.39	**2**	1.18	1.14	**0.2**	1.69	1.27	**20**	1.08	1.07

Table 3 - Average markup over competitive price after 10 years

6. Conclusions

This paper studies the effect of price cap regulation on investment in new capacity in a continuous time model of an oligopolistic (Cournot) industry with stochastic demand. Following the methodology developed by Grenadier (2002) we solve the symmetric game by finding the equilibrium shadow investment price trigger (the shadow price level that will trigger investment in new capacity) for the regulated oligopoly.

We find that a price cap has two competing effects on investment incentives. On the one hand, it makes the *option to defer investment* more valuable because it cuts upside potential profits without reducing potential downside losses, therefore slowing down investment. On the other hand, it reduces the value of *strategic underinvestment*; since firms cannot induce an increase in price, the only way to increase profits is to increase capacity. Therefore, a price cap has an ambiguous effect on investment under demand uncertainty. We find that for a price cap lower than the competitive entry price, the impact of the price cap on the option value of waiting dominates. Conversely, the negative impact of the price cap on the benefits of strategic underinvestment dominates for a price cap higher than the competitive entry price.

Moreover, we find that similarly to deterministic models, the optimal price cap is the investment price trigger of the competitive industry. However, in contrast to the results of deterministic models, we show that such a price cap does not restore the competitive equilibrium. There is still underinvestment and firms capture positive rents. Besides, our Cournot oligopoly model bridges the gap between the results of previous models of monopoly price cap regulation (Dobbs, 2004) and price control under perfect competition (Dixit and Pindyck, 1994) under demand uncertainty.

Such findings are relevant to the current debate about the impact of uncertainty on the efficiency of price cap regulation for utilities industries, particularly with regards to investment incentives. Our results underline the importance for regulators to take into account the option value of waiting arising out of uncertainty in demand. We show that the efficiency of price cap regulation depends critically on the level of demand volatility. We study the effect of a price cap misspecification and find that underestimating demand volatility when setting the price cap might have counterproductive results by slowing down investment as compared to the non-regulated oligopoly equilibrium. Overestimating volatility is not so dire as the worst it can do is render regulatory intervention irrelevant. We also studied the long term installed capacity and average price for regulated and unregulated markets using Monte Carlo simulation. Our results confirm the intuition that on average, sensible price cap regulation increases capacity and reduces average prices, but highlight the potentially large detrimental consequences of a misestimation of the price cap.

Although stylized, this model provided several valuable insights for the design and regulation of electricity markets. Our model suggests that regulators should pay great attention to demand uncertainty when deciding on the level of the price cap. In particular, our results show that a price cap set at the long run marginal cost of the most expensive peaking unit will be significantly too low to induce adequate investment, due to the high demand uncertainty characterizing the electricity industry, and in particular peaking units. This gives some weight to the claim that electricity markets are likely to see delayed or under-investment, in particular in peaking units, because of the relatively low level price caps that are used as part of market power mitigation procedures in most of the US. Besides, as electricity demand volatility is particularly difficult to estimate and is subject to frequent structural breaks, it appears easy for a regulator to underestimate the correct level of a price cap, which might render regulatory intervention counterproductive.

To conclude on a broader note, our findings echo Cowan (2002) 's concern that price cap regulation might be more efficient in establishing incentives for operating and cost efficiency than in inducing appropriate investment incentives. As pointed out by Wolak (2005), the regulatory risk, i.e. the probability that the firm will earn very high profits or negative profits, is much more significant with high-powered price regulation schemes (e.g. price cap regulation) than with other forms of price regulation such as cost-of-service regulation. Our results show indeed that price cap regulation under uncertainty is very sensitive to small errors in the estimation of the optimal price cap, while *ex post* regulation schemes such as cost-of-service regulation do not face this problem. Cowan (2002) states that "price-cap regulation may be more appropriate for industries without substantial investment requirements where there is excess capacity than for expanding industries with large investment plans." Our findings give some weight to the argument that a regulatory price-setting process that balances the risk of regulatory failure against the greater incentives for efficient behavior that pure price-cap plans might be better suited for industries with significant investment needs and subject to important demand or technological progress uncertainty.

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7. Appendices

7.1. **Appendix 1.**

Lemma 12. *Profit maximising producers produce at full capacity provided that $N\gamma > 1$.*

Proof. The profit of firm i is given by equation (4). The marginal profit resulting from a marginal increase in capacity is given by

$$\frac{\partial \pi}{\partial q} = XQ^{-1/\gamma} - 1/\gamma XQ^{-\frac{1}{\gamma}-1}q \tag{18}$$

Given that $q = \frac{Q}{N}$, a little algebra gives

$$\frac{\partial \pi}{\partial q} = XQ^{-1/\gamma}\frac{N\gamma - 1}{N\gamma} = P\frac{N\gamma - 1}{N\gamma} \tag{19}$$

Since we $\gamma > \frac{1}{N}$, $\frac{\partial \pi}{\partial q} > 0$ which demonstrates that producers produce always at full capacity. □

7.2. **Appendix 2.**

Let β_1 and β_1 be the roots of the 2nd degree polynomial characteristic equation of the differential equation (6):

$$\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - \rho = 0 \tag{20}$$

The two roots are:

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}} > 1 \tag{21}$$

and

(22) $$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}} < 0$$

The general solution of the differential equation (6) takes the following form

(23) $$m(P(Q), Q_{-i}, q) = H_0 P^{\beta_1} + H_1 P^{\beta_2} + AP + B$$

where H_0,H_1,A and B are constants. $A_0 P + B_0$ is the particular integral of the differential equation, with $A = \frac{N\gamma - 1}{N\gamma(\rho - \mu)}$ and $B = 0$.

Since $\beta_2 < 0$, $H_1 = 0$ otherwise the term $H_1 P^{\beta_2}$ would tend to infinity as P approached zero.

Using now the two boundary conditions (8) and (9) yields after a little algebra the analytical expression of the investment price trigger P^* given in equation (10) and H_0 is given by

(24) $$H_0 = \frac{N\gamma - 1}{N\gamma(\mu - \rho)} \frac{1}{\beta_1} [C \frac{N\gamma(\mu - \rho)\beta_1}{(N\gamma - 1)(1 - \beta_1)}]^{1 - \beta_1}.$$

7.3. **Appendix 3.** Solving the stochastic optimal control problem of equation (5) using (12) requires to distinguish two different regimes depending on demand:

(1) Non-binding Price cap

If $P \leq \bar{P}$, the price cap is not binding and we can use the results of the previous section. The marginal value m_i of firm i takes the following form:

(25) $$m_i(P, Q_{-i}, q_i) = H_1 P^{\beta_1} + \frac{N\gamma - 1}{N\gamma(\rho - \mu)} P$$

(2) Binding Price cap

If $P \geq \bar{P}$, new investment is made only when the pressure of demand rises to a critical level $\bar{P}^*$ at which the shadow price exceeds the imposed price cap $\bar{P}$ at which the actual price is stuck. The solution of equation (7) becomes:

(26) $$m_i(P, Q_{-i}, q_i) = H_2 P^{\beta_1} + H_3 P^{\beta_2} + \frac{\bar{P}}{\rho}$$

with $\beta_1 > 1$ and $\beta_2 < 0$ given by equations (21,22).

The marginal value satisfies the two free boundary conditions:

- Value matching condition at the shadow investment price trigger $\bar{P}^*$

(27) $$m(\bar{P}^*, Q_{-i}, q_i) = C$$

- Smooth pasting condition at the shadow investment price trigger $\bar{P}^*$

$$\frac{\partial m_i}{\partial P}(\bar{P}^*, Q_{-i}, q_i) = 0 \tag{28}$$

Bringing now the two cases together, continuity of value and fist derivative at the price cap $\bar{P}$ give two additional boundary conditions:

$$m_i(\bar{P}^{(-)}, Q_-i, q_i) = m_i(\bar{P}^{(+)}, Q_-i, q_i) \tag{29}$$

$$\frac{\partial m_i}{\partial P}\left(\bar{P}^{(-)}, Q_-i, q_i\right) = \frac{\partial m_i}{\partial P}(\bar{P}^{(+)}, , Q_-i, q_i) \tag{30}$$

where the notation $\bar{P}^{(-)}$ and $\bar{P}^{(+)}$ refer respectively to the limit of the function evaluated below and above the price cap $\bar{P}$

The system of four equations (27), (28), (29), and (30) with four unknowns $(H_1, H_2, H_3, \bar{P}^*)$ defines the symmetric Nash Cournot equilibrium investment strategies of a firm when prices are capped at $\bar{P}$. An analytical expression of these four equations is given below by equations (31), (32), (33) and (34):

$$H_2\bar{P}^{*\beta_1} + H_3\bar{P}^{*\beta_2} + \frac{\bar{P}}{\rho} = C \tag{31}$$

$$H_2\beta_1\bar{P}^{*\beta_{1-1}} + H_3\beta_2\bar{P}^{*\beta_{2-1}} = 0 \tag{32}$$

$$H_1\bar{P}^{\beta_1} + \frac{N\gamma - 1}{N\gamma(\rho-\mu)}\bar{P} = H_2\bar{P}^{\beta_1} + H_3\bar{P}^{\beta_2} + \frac{\bar{P}}{\rho} \tag{33}$$

$$H_1\beta_1 P^{\beta_1 - 1} + \frac{N\gamma - 1}{N\gamma(\rho-\mu)} = H_2\beta_1\bar{P}^{\beta_1 - 1} + H_3\beta_2\bar{P}^{\beta_2 - 1} \tag{34}$$

This system is non linear but can be solved analytically. We provide a sketch of the calculations and leave the intermediary steps to the interested reader.

Subtracting $\frac{\bar{P}^*}{\beta_1}$.(32) from (31) to eliminate H_2 yields

$$H_3 = \bar{P}^{*(-\beta_2)}(C - \frac{\bar{P}}{\rho})\frac{\beta_1}{\beta_1 - \beta_2} \tag{35}$$

Subtracting $\frac{\bar{P}}{\beta_1}$(34) from (33) to eliminate H_2 yields

$$H_3 = \bar{P}^{(1-\beta_2)}\frac{[\frac{(\beta_1-1)}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}]}{\beta_1 - \beta_2} \tag{36}$$

where we introduce $\alpha = \frac{N\gamma}{(N\gamma-1)}$ to simplify notation.

Equating (35) and (36) to eliminate H_3 gives

$$\bar{P}^{*\beta_2} = \frac{\beta_1}{\frac{(\beta_1-1)}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}}(C - \frac{\bar{P}}{\rho})\bar{P}^{(\beta_2-1)} \tag{37}$$

which upon rearrangement yields the following analytical solution for $\bar{P}^*$

$$\bar{P}^* = [\lambda(C - \frac{\bar{P}}{\rho})\bar{P}^{(\beta_2-1)}]^{1/\beta_2} \tag{38}$$

with λ defined as

$$\frac{1}{\lambda} = \frac{\beta_1 - 1}{\beta_1}\frac{1}{\alpha(\rho-\mu)} - \frac{1}{\rho} \tag{39}$$

7.4. **Appendix 4.** In this appendix we prove that there exists an interval $(\bar{P}_{\min}, \bar{P}_{\max})$ over which the introduction of a price cap lowers the shadow investment price trigger as compared to the unregulated oligopolistic industry investment trigger (i.e. $\bar{P}^* \leq P^*$). We will also show that the upper limit $P_{\max}$ is equal to the investment trigger without a price cap P^*.

Define $\Delta = \bar{P}^* - P^*$ the difference between the industry investment price trigger with and without price cap at $\bar{P}$. To demonstrate proposition (10), it is sufficient to prove that $\Delta \leq 0$ over the interval $[\bar{P}_{\min}, \bar{P}_{\max}]$.

From (13) and (10) we have

$$\Delta = [\lambda(\frac{\bar{P}}{\rho} - C)\bar{P}^{(\beta_2-1)}]^{1/\beta_2} - P^* \tag{40}$$

$\Delta \leq 0$ is equivalent to

$$[\lambda(\frac{\bar{P}}{\rho} - C)\bar{P}^{(\beta_2-1)}]^{1/\beta_2} \leq P^* \tag{41}$$

and since $\beta_2 < 0$

$$\frac{\lambda}{\rho}\bar{P}^{\beta_2} - \lambda C\bar{P}^{(\beta_2-1)} - P^{*\beta_2} \geq 0 \tag{42}$$

It can be shown graphically that this equation will have two solutions. It is not difficult to show that the investment price trigger without price cap P^* is the larger of the two solutions. This is intuitively what one should expect since a price cap higher than the competitive price trigger is irrelevant. The other solution is not easy to find analytically.

7.5. **Appendix 5.** From (13) we have

$$\bar{P}^{*\beta_2} = \lambda(\frac{\bar{P}}{\rho})^{\beta_2} - \lambda C\bar{P}^{(\beta_2-1)} \tag{43}$$

Differentiating this expression relatively to the price cap $\bar{P}$ we obtain

$$\frac{\partial(\bar{P}^{*\beta_2})}{\partial\bar{P}} = \beta_2\frac{\partial\bar{P}^*}{\partial\bar{P}}\bar{P}^{*(\beta_2-1)} = \frac{\lambda}{\rho}\beta_2\bar{P}^{(\beta_2-1)} - \lambda C(\beta_2-1)\bar{P}^{(\beta_2-2)} \tag{44}$$

The first order condition of optimality $\frac{\partial\bar{P}^*}{\partial\bar{P}} = 0$ implies

$$\frac{\lambda}{\rho}\beta_2\bar{P}^{(\beta_2-1)} = \lambda C(\beta_2-1)\bar{P}^{(\beta_2-2)} \tag{45}$$

and the optimal level of the price cap $\bar{P}_{opt}$ is therefore given by the following expression

$$\bar{P}_{opt} = \frac{\rho C(\beta_2-1)}{\beta_2} \tag{46}$$

The second order condition at $\bar{P}_{opt}$ can be shown to be $\frac{\partial\bar{P}^*}{\partial\bar{P}} > 0$ which implies that $\bar{P}_{opt}$ is indeed a minimum.

In the rest of the appendix we demonstrate that the expression (46) is equivalent to (11), the investment price trigger in a competitive industry.

β_1 and β_2 are the two roots of the characteristic equation of (6) and therefore satisfy the following two relations:

$$\beta_1 + \beta_2 = 1 - \frac{2\mu}{\sigma^2} \tag{47}$$

and

$$\beta_1\beta_2 = -\frac{2\rho}{\sigma^2} \tag{48}$$

From (47) and (48) we obtain

$$\rho - \mu = -\frac{\sigma^2}{2}[\beta_1\beta_2 - (\beta_1+\beta_2) + 1] \tag{49}$$

Using again (48) to replace $-\frac{\sigma^2}{2}$ by $\frac{\rho}{\beta_1\beta_2}$ and rearranging inside the brackets yields

$$\rho - \mu = \frac{\rho}{\beta_1\beta_2}[(\beta_2-1)(\beta_1-1)] \tag{50}$$

and therefore the optimal level of the price cap is equal to the investment price trigger in the competitive industry without price cap:

$$\bar{P}_{opt} = \frac{\rho C(\beta_2-1)}{\beta_2} = C\,(\rho-\mu)\,\frac{\beta_1}{(\beta_1-1)} = P^*_{N=\infty} \tag{51}$$