Market Equilibrium & Gaming Models for Electricity Policy Analysis & Policy Design

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Questions Addressed by Strategic Market Models

What might be the effect of policies concerning…

- ***Generation structure*** *(mergers, ownership, distributed resources, entry…)*
- ***Transmission investment*** *(new lines …)*
- ***Market rules***
  - *Transmission pricing (taxes, congestion, counterflows, zonal …)*
  - *Access (retail load, generators, arbitragers …)*
  - *Environmental markets (green certs., CO2 trading …)*

…upon…

- ***Economic efficiency*** *(allocative & productive efficiency)*
- ***Income distribution*** *(TSO revenues, profits, consumer surplus)*
- ***Emissions***

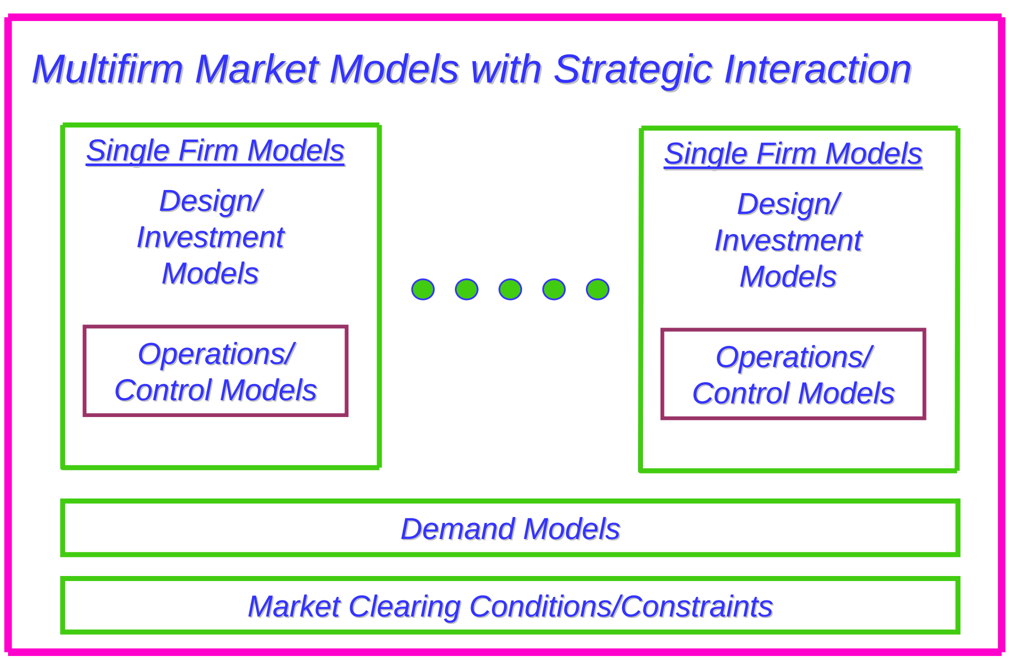
…considering generator strategic behavior?

- ***Bidding***
- ***Capacity withdrawal***
- ***Manipulation of transmission*** *(deliberate congestion, decongestion)*

Course Outline

- I. Bottom-up Models of Markets: Philosophy
- II. Review of Operations & Planning Models
  - A. Dispatch
  - B. Generation mix
  - C. Linearized DC load flow
- III. Perfect Competition Market Models
  - A. Equivalency Result: Samuelson's Principle
  - B. General Equilibrium Model
- IV. Strategic Market Models
  - A. Basic concepts
  - B. Simple Nash-Cournot example
  - C. Transmission-Constrained Cournot model
  - D. Advanced Models
- V. Conclusions

Overview of Lecture



I. "Process" or "Bottom-Up" Analysis: Company & Market Models

- What are bottom-up/engineering-economic models? And how can they be used for policy analysis?

$\triangleq$ Explicit representation & optimization of individual elements and processes based on physical relationships

Process Optimization Models

Elements:

- Decision variables. E.g.,
  - <u>Design:</u> MW of new combustion turbine capacity
  - <u>Operation:</u> MWh generation from existing coal units
- Objective(s). E.g.,
  - Maximize profit or minimize total cost
- Constraints. E.g.,
  - Σ Generation = Demand
  - Respect generation & transmission capacity limits
  - Comply with environmental regulations
  - Invest in sufficient capacity to maintain reliability

Traditional uses:

- Evaluate investments under alternative scenarios (e.g., demand, fuel prices) (3-40 yrs)
- Operations Planning (8 hrs - 5 yrs)
- Real time operations (<1 second - 1 hr)

Bottom-Up/Process Models vs. Top-Down Models

- Bottom-up models simulate investment & operating decisions by an individual firm..
  - E.g., capacity expansion, production costing models
  - Individual firm models can be assembled into market models
- Top-down models start with an aggregate market representation (e.g., supply curve for power, rather than outputs of individual plants).
  - Often consider interactions of multiple markets
  - E.g., National energy models

Functions of Process Model: Firm Level Decisions

Real time operations:

- *__Automatic protection__ (<1 second): auto. generator control (AGC) methods to protect equipment, prevent service interruptions. (Responsibility of:* Independent System Operator ISO*)*
- *__Dispatch__ (1-10 minutes): optimization programs (convex) min. fuel cost, s.t. voltage, frequency constraints (*ISO or generating companies GENCOs*)*

Operations Planning:

- *__Unit commitment__ (8-168 hours). Integer NLPs choose which generators to be on line to min. cost, s.t. "operating reserve" constraints (*ISO *or* GENCOs*)*
- *__Maintenance & production scheduling__ (1-5 yrs): schedule fuel deliveries & storage and maintenance outages (*GENCOs*)*

Firm Decisions Made Using Process Models, Continued

Investment Planning

- *__Demand-side planning__ (3-15 yrs): implement programs to modify loads to lower energy costs (*consumer, energy services cos. ESCOs, distribution cos. DISCOs*)*
- *__Transmission & distribution planning__ (5-15 yrs): add circuits to maintain reliability and minimize costs/ environmental effects (*Regional Transmission Organization RTO)
- *__Resource planning__ (10 - 40 yrs): define most profitable mix of supply sources and D-S programs using LP, DP, and risk analysis methods for projected prices, demands, fuel prices (*GENCOs*)*

Pricing Decisions

- *__Bidding__ (1 day - 5 yrs): optimize offers to provide power, subject to fuel and power price risks (*suppliers*)*
- *__Market clearing price determination__ (0.5- 168 hours): maximize social surplus/match offers (*Power Exchange PX, marketers*)*

Emerging Uses of Process Models

- Profit maximization rather than cost minimization guides firm's decisions
- Market simulation:
  - Use model of firm's decisions to simulate market. Paul Samuelson:

    MAX {consumer + producer surplus}

    ⇔ Marginal Cost Supply = Marg. Benefit Consumption

    ⇔ Competitive market outcome

    Other formulations for imperfect markets
  - Price forecasts
  - Effects of environmental policies on market outcomes (costs, prices, emissions & impacts, income distribution)
  - Effects of market design & structure upon market outcomes

Advantages of Process Models for Policy Analysis & Market Design

Explicitness:
- Model changes in technology, policies by altering:
  - decision variables
  - objective function coefficients
  - constraints
- assumptions can be laid bare

Descriptive uses:
- Texture! Detailed impacts of policy
- Costs, emission, technology choices, market prices, consumer welfare

Normative:
- identify better solutions through use of optimizatio
- show tradeoffs among policy objectives

II.A Operations Model:
1. System Dispatch "Linear Program"

- Basic model
  - Cost minimization, pure thermal system, deterministic

In words:
- Choose level of operation of each generator (decision variable),
- …to minimize total system cost (objective)
- …subject to load, capacity limit (constraints)

Decision variable:

y_{it} = megawatt [MW] output of generating unit i (i=1,..,I) during period t (t=1,..,T)

Coefficients:

CY_{it} = variable operating cost [$/MWh] for y_{it}

X_i = MW capacity of generating unit i.

$LOAD_t$ = MW demand to be met in period t

H_t = length of period t [hours/yr]. (Note: in pure thermal system, periods do not need to be sequential)

Operations LP

MIN Variable Cost = $\Sigma_{i,t} H_t CY_{it} y_{it}$

subject to constraints:

Meet load:

$\Sigma_i y_{it} = LOAD_t \qquad \forall t$

Generation no more than capacity:

$y_{it} \leq X_i \qquad \forall i,t$

Nonnegativity:

$y_{it} \geq 0 \qquad \forall i,t$

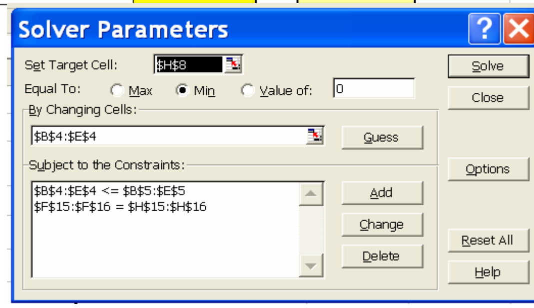
This is a “Linear Program” (i.e., objective, constraints are linear in decision variables)

Operations LP Exercise

- Two generation types
  - A: Peak: 800 MW, MC = $70/MWh
  - B: Baseload: 1500 MW, MC = $25/MWh
- Load
  - Pk: Peak: 2200 MW, 760 hours/yr
  - OP: Offpeak: 1300 MW, 8000 hours/yr
- Assignment:
  - Write down LP
  - What is best solution (by inspection?)

EXCEL Solver Model for Cost Minimization

	A	B	C	D	E	F	G	H	I	J
1	**Operations Linear Program**									
2	***Decision Variables***									
3	Name	$q_{A,Pk}$	$q_{B,Pk}$	$q_{A,OP}$	$q_{B,OP}$					
4	**Value** q	700	1500	0	1300					
5	Capacity $Q_{i,max}$	800	1500	800	1500					
6	$CM_{i,t}$ $/MWh	70	25	70	25					
7	Hours/yr	760	760	8000	8000	**Objective "Variable Cost"**				
8	Obj f() term	37240000	28500000	0	260000000			325740000		
9										
10	***Constraints***									
11	**Constraint Coefficients (Left Side)**									
12	Load Peak	1	1							
13	Load Offpeak			1	1					
14	**Constraint Coefficients times Value**					**Left Hand Side**		**Right Hand Side**	Dual $/yr	$/MWh
15	Load Peak	700	1500	0	0	2200	=	2200	53200	70
16	Load Offpeak	0	0	0	1300	1300	=	1300	200000	25



Operating Model Formulation, Continued: Complications

- Other objectives
  - Max Profit? Min Emissions?
- Energy storage
  - Pumped storage, batteries, hydropower
- Explicitly stochastic
  - Usual assumption: forced outages are random and independent
- Transmission constraints
- Commitment variables
  - E.g., start-up costs
- Cogeneration

II.A.2. Unit Commitment: A Mixed Integer Program

- Disregard forced outages & fuels; assume:
  - u_{it} = *1 if unit i is committed in t (0 o.w.)*
  - *CU_i = fixed running cost of i if committed*
  - *MR_i = "must run" (minimum MW) if committed*
  - *Periods t =1,..,T are consecutive, and H_t=1*
  - *RR_i = Max allowed hourly change in output*

- MIN $\Sigma_{i,t} CY_{it}\, y_{it} + \Sigma_{i,t} CU_i\, u_{it}$

  s.t. $\Sigma_i y_{it} = LOAD_t \quad \forall t$

  $MR_i u_{it} \le y_i \le X_i u_{it} \quad \forall i,t$

  $-RR_i \le (y_{it} - y_{i,t-1}) \le RR_i \quad \forall i,t$

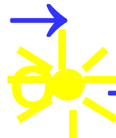
  $y_{it} \ge 0 \quad \forall i,t;\ u_{it} \in \{0,1\} \quad \forall i,t$

II.A.3. Using Operating Models to Assess NO_x Regulation

(Leppitsch & Hobbs, IEEE Trans. Power Systems, 1996)

- NO_x: an ozone precursor

  $N_2 + O_2$ + heat → NO_x

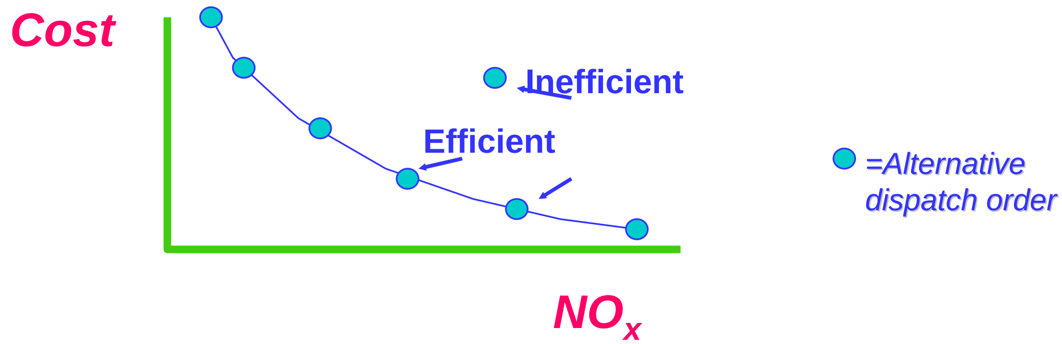
  NO_x + VOC +  → O_3

- Power plants emit ~1/3 of anthropogenic NO_x in USA

- Policy question: How effectively can NO_x limits be met by changed operations ("emissions dispatch")?

Framework

We want less cost and less NO_x



How To Generate Alternatives

Solve the following model for alternative levels of the regulatory constraint:

MIN $\Sigma_i\, CY_i\, y_i$

s.t. 1. $MR_i \le y_i \le X_i$ (note nonzero LB)

2. $\Sigma_i\, y_i \ge$ LOAD (MW)

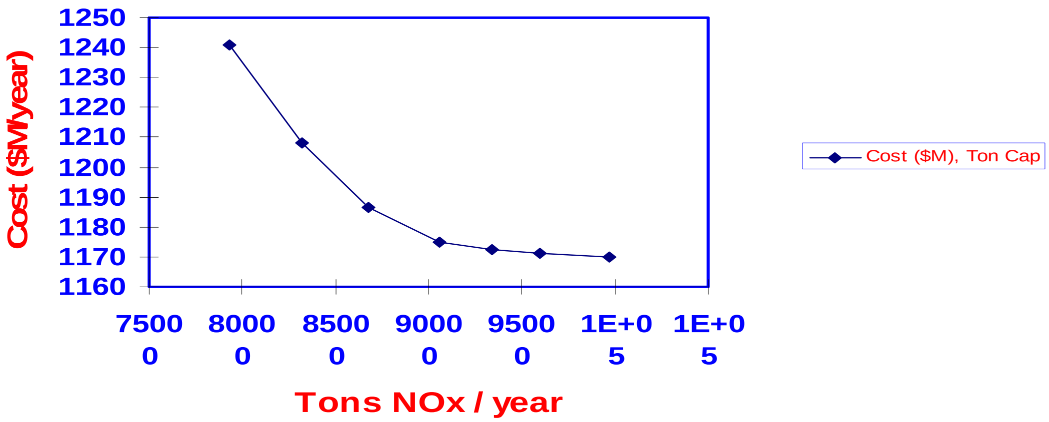
3. $\Sigma_i\, E_i\, y_i \le$ MASS CAP (tons)

Note: MR, X, LOAD vary (used a stochastic programming method: probabilistic production costing with side constraints)

- Data: 11,400 MW peak and 12,050 MW of capacity, mostly gas and some coal. Most of capacity has same fuel cost/MBTU. Plant emission rates vary by order of magnitude (0.06 - 0.50 lb/MBTU)

Cost-Emissions Tradeoffs

- The cost of reducing emissions by 20% is \$70M (a 5% increase in fuel cost).



II.B.1. Deterministic Investment Analysis: LP Snap Shot Analysis

Let generation capacity x_i [MW] now be a variable, with (annualized) cost = CX_i [\$/MW/yr]

$$\text{MIN} \quad \Sigma_{i,t} H_t CY_{it}\, y_{it} + \Sigma_i CX_i\, x_i$$

$$\text{s.t. } \Sigma_i y_{it} = LOAD_t \qquad \forall t$$

$$\Sigma_f y_{it} - x_i \le 0 \qquad \forall i,t$$

$$\Sigma_i x_i \ge LOAD_1 (1+M) \qquad \text{(“reserve margin” constraint)}$$

$$y_{it} \ge 0 \qquad \forall i,t$$

$$x_i \ge 0 \qquad \forall i$$

Some Complications

- Dynamics (timing of investment)
- Plants available only in certain sizes
- Retrofit of pollution control equipment
- Construction of transmission lines
- “Demand-side management” investments
- Uncertain future (demands, fuel prices)
- Other objectives (profit)

Planning LP Exercise

- Two generation types
  - A: Peak: 800 MW, MC = $70/MWh
    - Operating Cost = $70/MWh
    - Capital Cost = $70,000 / MW/yr
  - B: Baseload:
    - Operating Cost = $25/MWh
    - Capital Cost = $120,000 / MW/yr
- Load
  - Peak: 2200 MW, 760 hours/yr
  - Offpeak: 1300 MW, 8000 hours/yr
  - Reserve Margin: 15%
- Assignment:
  - Write down LP
  - What is best solution (by inspection?)

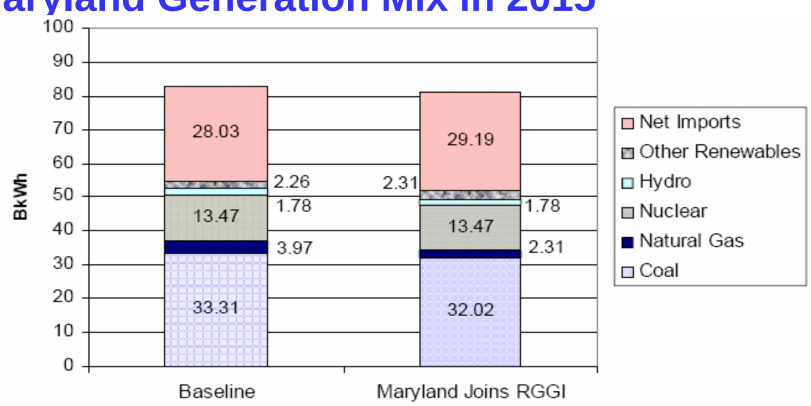
Example Capacity Expansion Analysis: Costs of Maryland Joining the Regional Greenhouse Gas Initiative

- RGGI: CO_2 trading program for generators in Northeastern US states
- Maryland Healthy Air Act (2006): Requires a study of the reliability and cost impacts of Maryland joining RGGI
- Also: what are the emissions effects? What is the effect of CO_2 "leakage"? How is this affected by market power?
- Tools:
  - Haiku (Resources for the Future competitive market model—includes capacity expansion)
  - JHU market power model

Maryland Joining the Regional Greenhouse Gas Initiative: Findings

Report, University of Maryland, January 2006 (www.cier.umd.edu)

- Emissions modestly lower
  - -10% for Maryland; -4% for RGGI
  - Some offset by leakage
- Electricity demand decreases due to DSM programs, consumers save money
- Generator profits drop, but few plant closures
- Maryland Generation Mix in 2015



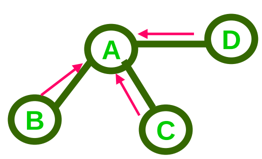
II.C Including Transmission: or Why Power Transport is Not Like Hauling Apples in a Cart

- Review of “Laws”
- Weird implications
- Calculating “Load Flow”

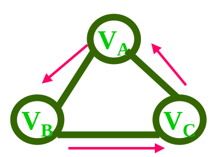
Review of DC Circuit Laws

- **Ohm’s Law:**
  - $V_A - V_B = I_{AB} * R_{AB}$
  - Voltage difference proportional to current * resistance

- **Kirchhoff’s Current Law:**
  - No net current inflow to a node
  - $\Sigma_j \; I_{Aj} = 0$



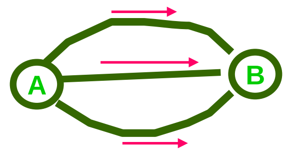
- **Kirchhoff’s Voltage Law:**
  - Sum of voltage differences around any loop = 0
  - $(V_A - V_B) + (V_B - V_C) + (V_C - V_A) = 0$
  - Sub in Ohm’s Law: $I_{AB}*R_{AB} + I_{BC}*R_{BC} + I_{CA}*R_{CA} = 0$



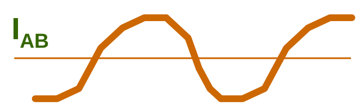
- **Losses:**
  - $L_{AB} = I_{AB}^2 R_{AB}$
  - Doubling the current implies *four times* the losses

Implications of Laws

- **Use laws to calculate flows**
  - If you know power generation and consumption at “bus” except the “swing bus”, then ...
  - The “load flow” (currents in each line, voltages at each are uniquely determined by Kirchhoff’s two laws!
  - This is the “load flow” problem
- **Some odd byproducts of laws:**
  - Can’t “route” flow
  - Parallel flows
  - Transmission paths (e.g., choosing which German-Dutch interface to buy) are a fiction
  - What you do affects everyone else
  - Adding a line can *worsen* transmission capacity of system



AC Load Flow is More Complex

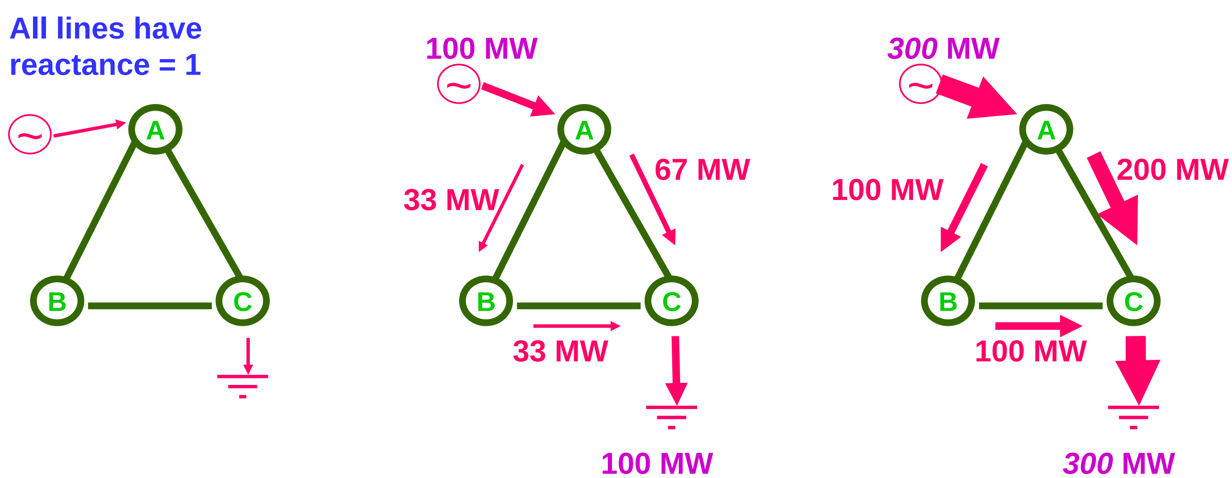


- Voltage at each bus is sinusoidal (with RMS amplitude and phase angle), as are line currents
- “Reactive” (vs. “real” power) a result of “reactance” (capacitance and inductance)
- This is the power stored and released in magnetic fields of capacitors and inductors as the current changes direction
- Although reactive power doesn’t do useful work, it causes resistance losses & uses up capacity

“DC” Linearization of AC load flow

- **Assumptions**
  - Assume reactance >> resistance
  - Voltage amplitude same at all buses
  - Changes in voltage angles θ_A-θ_B from one end of a line to another is small
- **Results:**
  - Power flow t_{AB} proportional to:
    - current I_{AB}
    - difference in voltage angle θ_A-θ_B
  - Analogies to Kirchhoff’s Laws:
    - Current law at A: $\Sigma_i\, y_{iA} = \Sigma_{\text{neighboring m}}\ t_{Am} + \text{LOAD}_A$
    - Voltage law: t_{AB}*R_{AB} + t_{BC}*R_{BC} + t_{CA}*R_{CA} = 0
  - Given power injections at each bus, flows are unique

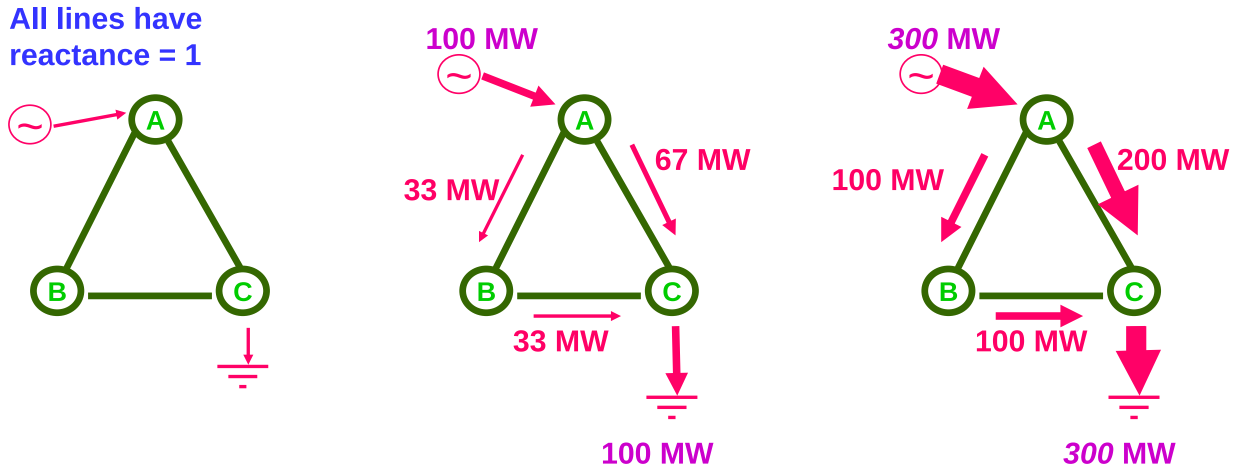
Example of “DC” Load Flow



<u>Kirchhoff’s Current Law at C:</u>
+33 + 67 - 100 = 0
<u>Kirchhoff’s Voltage Law:</u>
1*33 + 1*33 + 1*(-67) = 0

Proportionality!

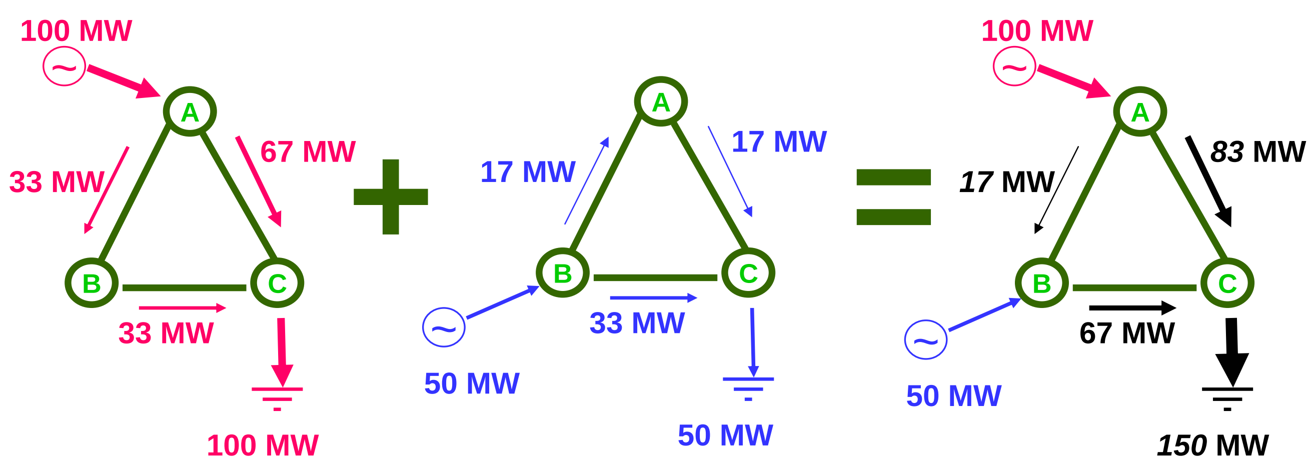
Proportionality means "Power Transmission Distribution Factors" can be used to calculate flows



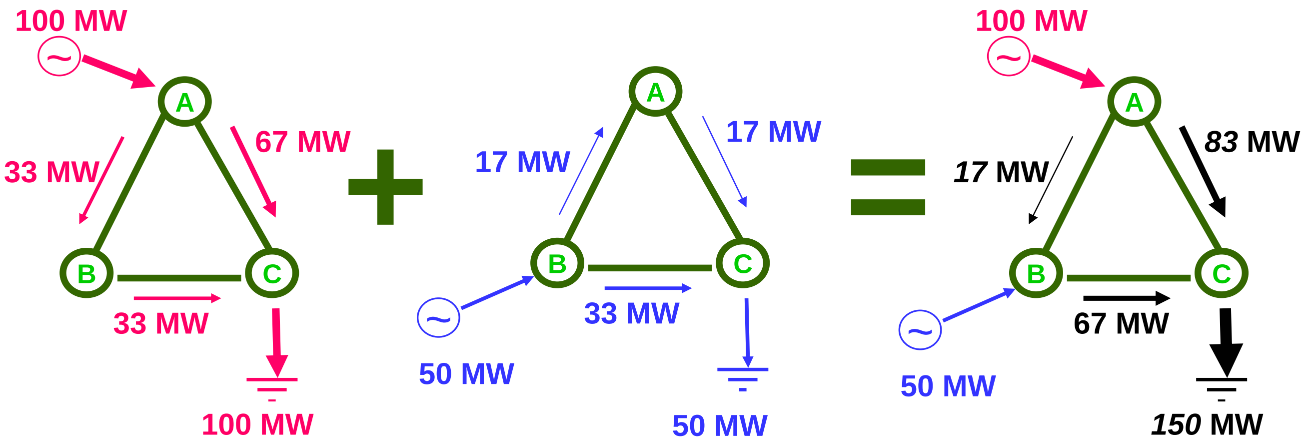
PTDF$_{mn,jk}$ = the MW flowing from *j* to *k*, if 1 MW is injected at *m* and 1 MW is removed at *n*

E.g., *PTDF*$_{AC,AB}$ = 0.33

Principle of Superposition



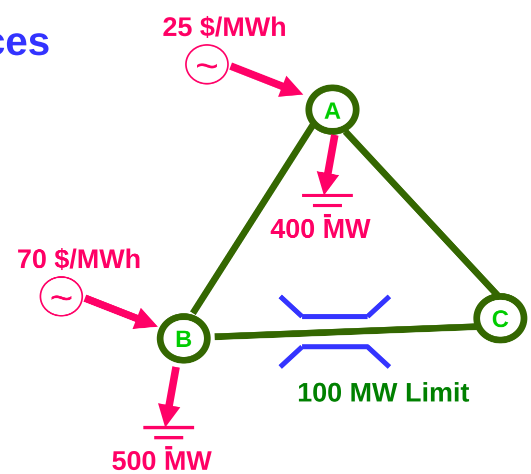
Using PTDFs to Calculate Total Flow



Total flow from B to C = $PTDF_{AC,BC}$*100 + $PTDF_{BC,BC}$*50
= 0.33*100 + 0.67*50 = 67 MW

Exercise in Transmission Modeling

- **Assumptions**
  - Triangle network, equal reactances
    - Line from A to C: 100 MW limit
  - Two plants:
    - A: MC = 25 $/MWh
    - B: MC = 70 $/MWh
  - Load:
    - A: 400 MW
    - B: 500 MW



- **What's the optimal dispatch?**
- **What's the marginal cost of meeting an increase of 1 MW of load at A; at B; at C?**

Linearized Transmission Constraints in Operations LP

y_{imt} = MW from plant *i*, at node *m*, during *t*

z_{mt} = Net MW injection at node *m*, during t

MIN Variable Cost = $\Sigma_m \Sigma_{i,t} H_t CY_{im} y_{imt}$

subject to:

Net Injection: $\Sigma_i y_{imt}$ - $LOAD_{tm} = z_{mt}$ $\forall$t,m

Injection Balance: $\Sigma_m z_{mt} = 0$ $\forall$t

GenCap: $y_{imt} \leq X_{im}$ $\forall$i,m,t

Transmission: $T_{k-} \leq [\Sigma_m PTDF_{mk} z_{mt}] \leq T_{k+}$ $\forall$k,t

$y_{imt} \geq 0$ $\forall$i,m,t

Linearized Transmission Constraints in Operations LP: Exercise Example

MIN Variable Cost = $25y_A + 70y_B$

subject to:

Net Injection: $y_A - 400 = z_A$

$y_B - 500 = z_B$

Injection Balance: $z_A + z_B = 0$

Transmission: $-100 \leq [\, 0.33z_A + 0.0\, z_B \,] \leq +100$

Nonnegativity:

Note: In calculating PTDFs, I assume that all injections "sink" at node B

- E.g., injection z_A at A is assumed to be accompanied by an equal withdrawal $-z_A$ at B

III. Mathematical Programming Models of Perfectly Competitive Energy Markets

A. An Equivalency Result

- Definition of pure competition market equilibrium:
  - Each player maximizes their profit, subject to fixed prices (no market power)
  - Market clears (supply = demand)
- Assemble:
  - "First order" optimization conditions for players
  - Market clearing

  This yields set of simultaneous equations that can be solved for a market equilibrium
- Same set of equations are first order conditions for a single optimization model (MAX net social welfare)
  - MAX (Area under demand curves)-(Cost)
  - Results in intersection of demand + supply curves
- Widely used in energy policy analysis

Applications of the Pure Competition Equivalency Principle

- MARKAL: Used by Intl. Energy Agency countries for analyzing national energy policy, especially CO_2 policies
- US Project Independence Evaluation System (PIES) & successors (W. Hogan, "Energy Policy Models for Project Independence," Computers and Operations Research, 2, 251-271, 1975; F. Murphy and S. Shaw, "The Evolution of Energy Modeling at the Federal Energy Administration and the Energy Information Administration," Interfaces, 25, 173-193, 1995.)
- US Natl. Energy Modeling System (C. Andrews, ed., Regulating Regional Power Systems, Quorum Press, 1995, Ch. 12, M.J. Hutzler, "Top-Down: The National Energy Modeling System".)
- ICF Coal and Electric Utility Model (http://www.epa.gov/capi/capi/frcst.html)
  - Acid rain, Clear Skies, Clean Air Interstate Rule
- POEMS (http://www.retailenergy.com/articles/cecasum.htm)
  - Economic & environmental benefits of US restructuring
- Some of these modified to model imperfect competition (price regulation, market power)

B. Equilibrium Model Formulations

Meet the FOC'ers:

"First Order Conditions" for Optimization

Let an optimization problem be:

$$\max_{\{X\}} F(X)$$

$$\text{s.t.: } G(X) \le 0$$

$$X \ge 0$$

Assume F(X) smooth/concave, G(X) smooth/convex.

A solution {X,λ} to the KKT ("Karush-Kuhn-Tucker") conditions below is optimal for the above problem, and vice versa. "KKTs necessary & sufficient for optimality."

$$X \left\{ \begin{array}{l} X \ge 0; \quad \partial F/\partial X - \lambda\, \partial G/\partial X \le 0; \\ X(\partial F/\partial X - \lambda\, \partial G/\partial X) = 0 \end{array} \right.$$

$$\lambda \left\{ \begin{array}{l} \lambda \ge 0; \quad G(X) \le 0; \\ \lambda\, G(X) = 0 \end{array} \right.$$

"Perp" Notation for the FOC'ers:

Let an optimization problem be:

$$\max_{\{X\}} F(X)$$

$$\text{s.t.: } G(X) \le 0$$

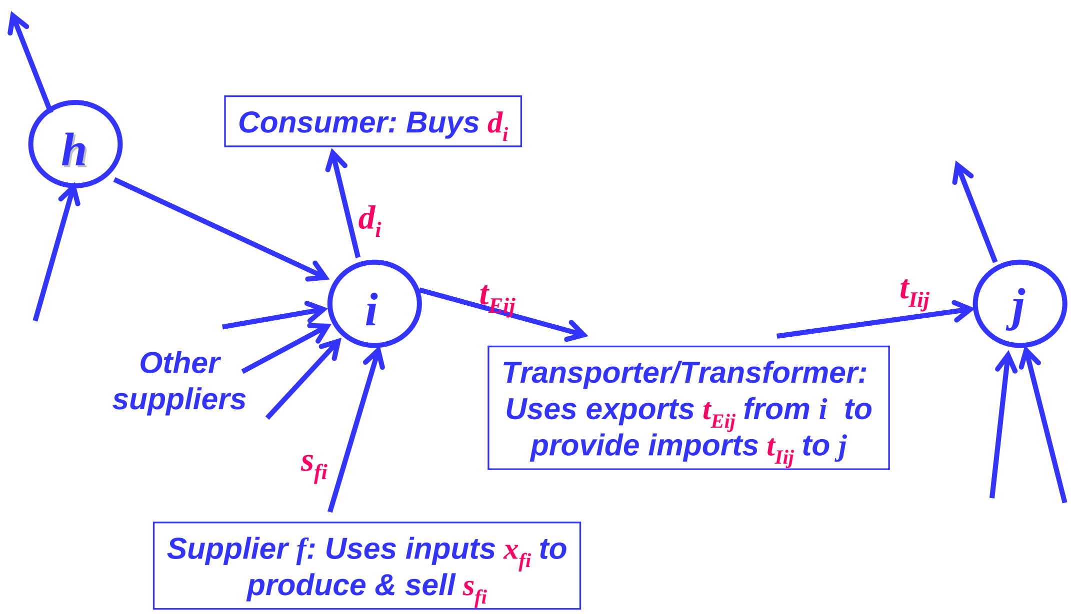
$$X \ge 0$$

The KKT's, written in "perp" notation, are:

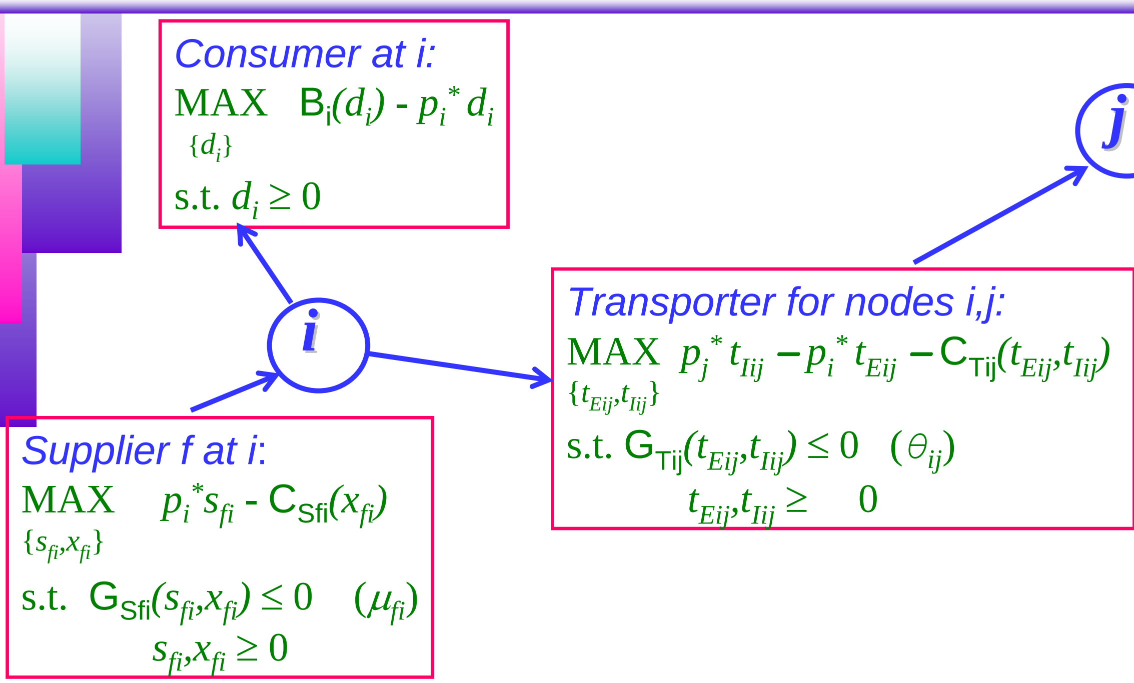
$$X \left\{ \quad 0 \le X \perp \partial F/\partial X - \lambda\, \partial G/\partial X \le 0 \right.$$

$$\lambda \left\{ \quad 0 \le \lambda \perp G(X) \le 0 \right.$$

Notation: Each node i is a separate commodity (type, location, timing)



Players' Profit Maximization Problems



Players' Profit Maximization Problems

Consumer FOCs/KKTs at i :
--one for each decision variable
--one for each constraint

Transporter FOCs/KKTs for i,j:
--one for each decision variable
--one for each constraint

Supplier FOCs/KKTs at i
--one for each decision variable
--one for each constraint

i j

KKTs for All Players in Market Game + Market Clearing Condition

Consumer FOCs/KKTs , $\forall i$:
--one for each decision variable
--one for each constraint

Supplier FOCs/KKTs, $\forall i$
--one for each decision variable
--one for each constraint

Transporter/Transformer KKTs, $\forall ij$
--one for each decision variable
--one for each constraint

Market Clearing, $\forall$ i:

$$p_i^*: \left[\Sigma_f s_{fi} + \Sigma_{j \in I(i)} t_{Iji} - \Sigma_{j \in E(i)} t_{Eij} - d_i \right] = 0$$

N conditions & N unknowns!

An Optimization Model for Simulating a Commodity Market

MAX (Value of Consumption) - (Production, Transport Cost)

$$\max_{\{d_i, s_{fi}, x_{fi}, t_{Eij}, t_{Iij}\}} \sum_i B_i(d_i) - \sum_{fi} C_{Gi}(x_{fi}) - \sum_{ij} C_{Tij}(t_{Eij}, t_{Iij})$$

s.t. Production Functions for each firm:

$$G_{Si}(s_i, x_i) \le 0, \forall i$$

$$G_{Tij}(t_{Eij}, t_{Iij}) \le 0, \forall ij$$

Market Clearing for each commodity:

$$\sum_f s_{fi} + \sum_{j \in I(i)} t_{Iji} - \sum_{j \in E(i)} t_{Eij} - d_i = 0, \forall i$$

…and the usual nonnegativity conditions

Its FOC conditions = market equilibrium conditions for the purely competitive commodities market! So:

- a single NLP can simulate a market
- a purely competitive market maximizes social surplus

An Optimization Model for Simulating a Competitive Energy Market

MAX Social Surplus = $\Sigma_t B(d_t)$ — $\Sigma_{i,t} H_t CY_{it} y_{it}$

subject to constraints:

Meet load:

$-\Sigma_i y_{it} + d_t = 0 \qquad \forall t$

Generation no more than capacity:

$y_{it} \le X_i \qquad \forall i,t$

Nonnegativity:

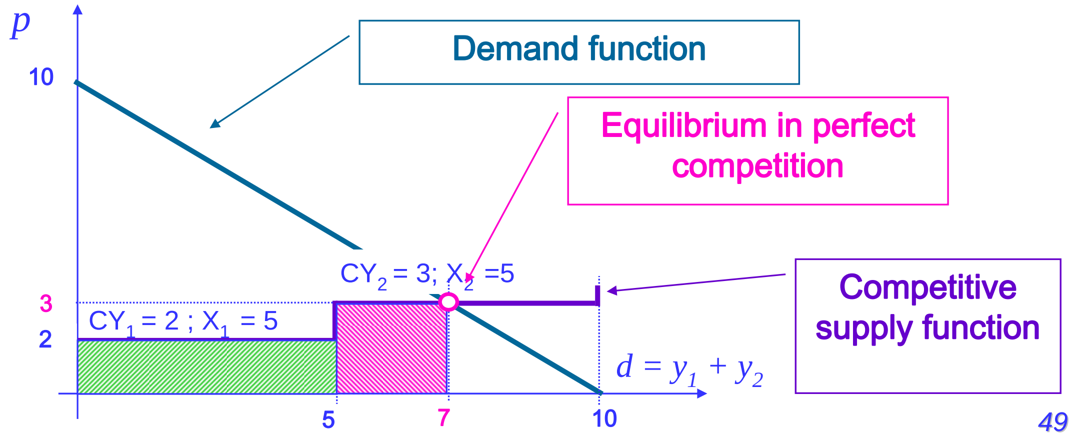
$y_{it} \ge 0 \qquad \forall i,t$

This is a "Quadratic Program" (i.e., objective, constraints are either linear or quadratic in decision variables)

Competitive Model Example

Perfect competition

- Company 1: $CY_1 = 2, \quad X_1 = 5$
- Company 2: $CY_2 = 3, \quad X_2 = 5$
- Demand: $p = 10 - d = 10 - (y_1 + y_2)$



Competitive Model Example

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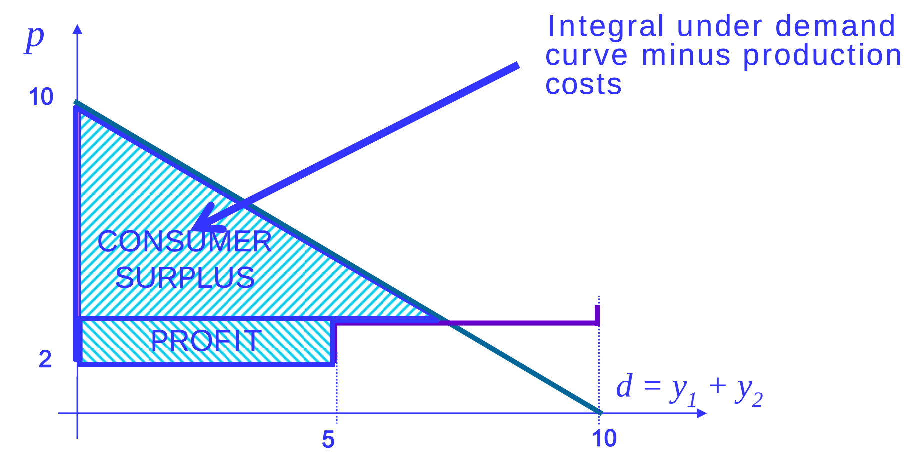
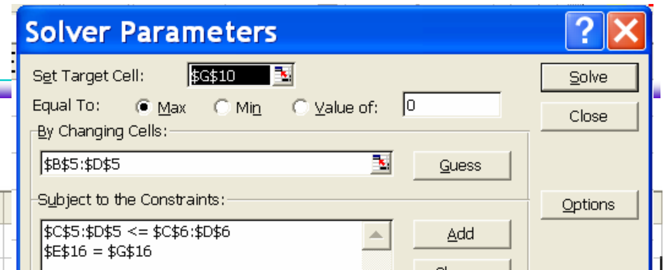


Excel Solver Perfect Competition Model



	A	B	C	D	E	F	G	H
1	**Market Simulation for Competitive Market**							
2	***Decision Variables***							
3		Demand	Operations Variables					
4	Name	d	y_A	y_B				
5	**Value of d.v.**	7	5	2				
6	Capacity X_i	n.a.	5	5				
7	CY_i $/MWh		2	3				
8	Demand Price Intercept	10						
9	Demand Slope	-1					**MAX 'Social Surplus'**	
10	Obj f() term	45.5	-10	-6			29.5	
11								
12	***Other Constraints***							
13	*Constraint Coefficients (Left Side)*							**P = MC =**
14	Load constraint	1	-1	-1				**Dual Price**
15	*Constraint Coefficients times Value*				**LHS "G(X)"**		**RHS "B"**	**$/MWh**
16	Load constraint	7	-5	-2	0	=	0	3

General Procedure for Building Equilibrium Models

- Not all equilibrium problems can be formulated as optimization problems
  - Complementarity models are more general
    - Some but not all complementarity equilibrium problems have an equivalent optimization problem
    - But all convex optimization problems have an equivalent equilibrium (KKT) problem
- Five steps:
  1. Formulate optimization submodel for each market party
  2. Derive KKTs for each party's submodel
  3. Create a complementarity problem consisting of those conditions for all parties plus market clearing
     - Should be as many conditions (either perp or equality) as variables. As check, associate one variable with each condition
     - Types of complementarity problems include linear/nonlinear, nonmixed/mixed (without or with equality conditions, each with a matching unrestricted variable)
  4. Analyze resulting problem for existence, uniqueness, other properties
  5. Parameterize & solve

III.C. Commodity Modeling Exercise

1. Draw a diagram representing the following market structure:
   - Two electricity companies in California
     - *Use two commodities as inputs:*
       1. *NO_x emissions allowances*
       2. *Natural gas*
     - *Sell power in offpeak and peak electricity markets*
   - Supply of NO_x emission allowances auctioned by EPA
   - Natural gas produced by companies in Texas, and piped to California
2. Write an optimization problem that gives an equivalent solution
3. Homework: Write optimization problem for each party & derive a complementarity problem (in very general terms) that would represent a competitive equilibrium
   - Assume all parties are 'price takers'

IV. Strategic Market Modeling: Oligopoly
A. Concepts

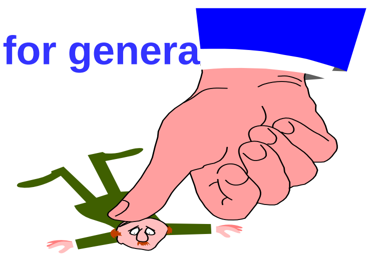
- Oligopoly or imperfect competition is the most representative market structure in real electric power markets
  - Small number of large generating firms.
- Imperfect market analysis and modelling is more complex
  - Each generator must bear in mind the interdependence between its decisions and the decisions of all other agents
  - This strategic interdependence varies with the time horizon of the decisions to be made

Market Power = Ability to manipulate prices persistently to one's advantage, independently of the actions of others

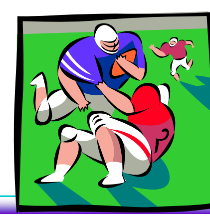
Generators: The ability to raise prices above marginal cost by restricting output

Generators may be able to exercise market power because of:

- economies of scale
- large existing firms
- transmission costs, constraints
- siting constraints, long lead time for genera construction
- dumb market designs



Types of Games



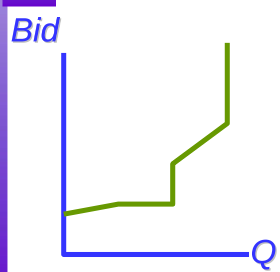
- <u>Noncooperative Games (Symmetric):</u> Each player has same "strategic variable"
  - Each player implicitly assumes that other players won't react.
  - "Nash Equilibrium": no player believes it can do better by a unilateral move
- Let $\pi_i(X_i, X_{-i})$ = i's profit, a function of i's strategy X_i and everyone else's strategy X_{-i}
- Nash equilibrium $\{X_i^*, X_{-i}^*\}$ occurs if:

$$\pi_i(X_i^*, X_{-i}^*) \geq \pi_i(X_i, X_{-i}^*)$$

for all feasible X_i, and for all i

Types of Games (Continued)

- <u>Examples of Nash Games:</u>
  - <u>Bertrand (Game in Prices)</u>. Implicit: You believe that market prices won`t be affected by your actions, so by cutting prices, you gain sales at expense of competitors.
    - COMPETITIVE COMMODITY MODEL!
  - <u>Cournot (Game in Quantities)</u>: Implicit: You believe that if you change your output, your competitors will maintain sales by cutting or raising their prices
  - <u>Supply function (Game in Bid Schedule)</u>: Implicit: You believe that competitors won't alter supply functions they bid

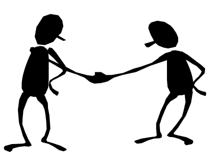


Digression: History Quiz

What was the profession of John Nash's father?

Types of Games (Continued)

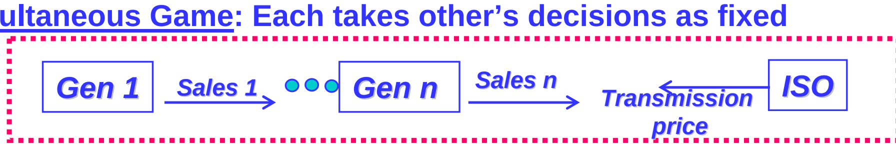
- Noncooperative Game (Asymmetric/Leader-Follower): Leader knows how followers will react.
  - E.g.: strategic generators anticipate:
    - how a passive ISO prices transmission
    - competitive fringe of small generators, consumers
  - "Stackelberg Equilibrium"
  - Multiple leaders possible:
    - Several large generators competiting a la Nash with each other, but each anticipating reaction of ISO (transmission pricing) and fringe generators (outputs)



- Cooperative Game (Exchangable Utility/Collusion): Max joint profit.
  - E.g., competitors match your changes in prices or output

Three General Generator-Transmission Games

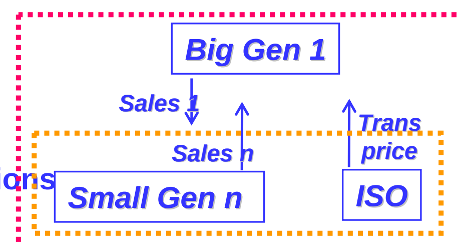
Simultaneous Game: Each takes other's decisions as fixed



Sequential Game:

--Single Leader anticipates Follower's reactions

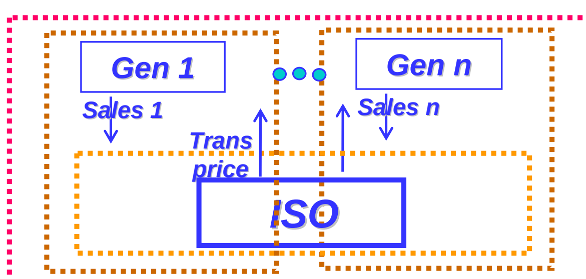
--Follower takes leader's decisions as fixed



Multiple Leader-Follower:

--Each leader anticipates follower's reactions

--Each leader takes other's decisions as fixed



B. Computation Methods for Nash (Simultaneous) Games
Simple Example

1. Payoff Matrix: Enumerate all combinations of player strategies; look for stable equilibrium
2. Iteration/Diagonalization/Alternate Play/Gauss-Seidel: Simulate player reactions to each other until no player wants to change
3. Direct Solution of Equilibrium Conditions: Collect FOCs/KKTs for all players; add market clearing conditions; solve resulting system of conditions directly
   – Usually involves complementarity conditions
4. Equivalent Optimization: May exist a single optimization model that gives same solution ("Hashimoto")

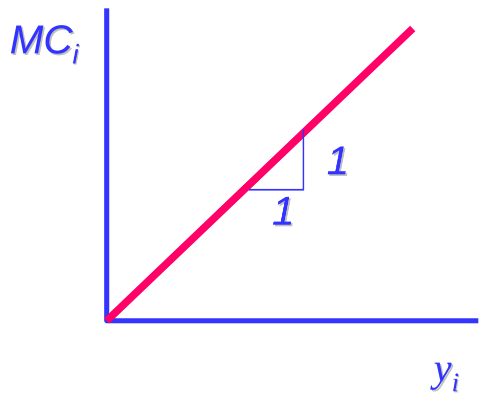
From *Econometrica*, 1933, courtesy of Claire Friedland and George J. Stigler.

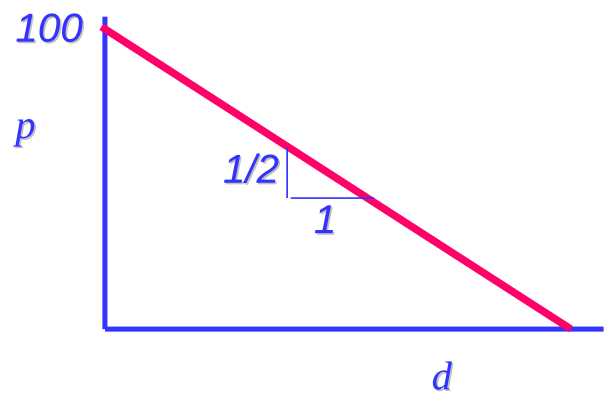
Strategic Modeling Exercise

- Two Cournot generators (competing on quantity)
  - Sell output in ISO day ahead market
    - Strategic variables is quantity bid
  - "Locational marginal pricing" – "first price auction" -- market clearing price
  - Equivalent to bilateral contracting with efficient arbitrage
- Solve example with 4 methods
- Variant: "Pay as Bid"
  - Strategic variable is price bid
  - No single price; if cut price, you might sell more, but at a lower price
  - Also try to solve with payoff matrix

Strategic Modeling Exercise: Cournot/Quantity Model

- Each firm i's marginal cost = y_i , i = A,B (Total cost = 0.5 y_i^2)
- Demand function: p = 100 - d/2 [$/MWh]





Strategic Modeling Exercise: Cournot/Quantity Equilibrium

- Firm A: $MAX\ \pi_A(y_A,y_B) = P(y_A+y_B)y_A - C_A(y_A)$
  $= (100 - 0.5(y_A+y_B))y_A - 0.5y_A^2$
  $s.t.\ y_A \geq 0$
  - KKTs: $0 \leq y_A \perp P + P'y_A - MC_A \leq 0$
    or $0 \leq y_A \perp (100 - y_A - 0.5y_B) - y_A \leq 0$
- Firm B: $MAX\ \pi_B(y_A,y_B)$
  $= MAX\ (100 - 0.5(y_A+y_B))y_B - 0.5y_B^2$
  $s.t.\ y_B \geq 0$
  - KKTs: $0 \leq y_B \perp (100 - y_A - 0.5y_B) - y_B \leq 0$
- Market clearing: $d = y_A + y_B$
- The market participant's KKTs + market clearing form a complementarity problem

Method 1:

Find cell such that π_A is highest in column (Firm A maximizes its profit given y_B) and π_B is highest in row (Firm B maximizes its profit given y_A). In the below table, ***Bold italics*** represents Firm A's best response to y_B, while **Bold** represents Firm B's best response to y_A. The format of the table is:

$y_A \setminus y_B$:	B's y
A's y	π_A / π_B

$y_A \setminus y_B$:	30	32	34	36	38	40	42	44	46	48	50
30	1650 / 1650	1620 / 1696	1590 / 1734	1560 / 1764	1530 / 1786	1500 / 1800	1470 / **1806**	1440 / 1804	1410 / 1794	1380 / 1776	1350 / 1750
32	1696 / 1620	1664 / 1664	1632 / 1700	1600 / 1728	1568 / 1748	1536 / 1760	1504 / **1764**	1472 / 1760	1440 / 1748	1408 / 1728	1376 / 1700
34	1734 / 1590	1700 / 1632	1666 / 1666	1632 / 1692	1598 / 1710	1564 / 1720	1530 / **1722**	1496 / 1716	1462 / 1702	1428 / 1680	1394 / 1650
36	1764 / 1560	1728 / 1600	1692 / 1632	1656 / 1656	1620 / 1672	1584 / **1680**	1548 / **1680**	1512 / 1672	1476 / 1656	1440 / 1632	1404 / 1600
38	1786 / 1530	1748 / 1568	1710 / 1598	1672 / 1620	1634 / 1634	1596 / **1640**	1558 / 1638	***1520*** / 1628	***1482*** / 1610	***1444*** / 1584	***1406*** / 1550
40	1800 / 1500	1760 / 1536	1720 / 1564	***1680*** / 1584	***1640*** / 1596	***1600*** / **1600**	***1560*** / 1596	***1520*** / 1584	1480 / 1564	1440 / 1536	1400 / 1500
42	***1806*** / 1470	***1764*** / 1504	***1722*** / 1530	***1680*** / 1548	1638 / 1558	1596 / **1560**	1554 / 1554	1512 / 1540	1470 / 1518	1428 / 1488	1386 / 1450
44	1804 / 1440	1760 / 1472	1716 / 1496	1672 / 1512	1628 / **1520**	1584 / **1520**	1540 / 1512	1496 / 1496	1452 / 1472	1408 / 1440	1364 / 1400
46	1794 / 1410	1748 / 1440	1702 / 1462	1656 / 1476	1610 / **1482**	1564 / 1480	1518 / 1470	1472 / 1452	1426 / 1426	1380 / 1392	1334 / 1350
48	1776 / 1380	1728 / 1408	1680 / 1428	1632 / 1440	1584 / **1444**	1536 / 1440	1488 / 1428	1440 / 1408	1392 / 1380	1344 / 1344	1296 / 1300
50	1750 / 1350	1700 / 1376	1650 / 1394	1600 / 1404	1550 / **1406**	1500 / 1400	1450 / 1386	1400 / 1364	1350 / 1334	1300 / 1296	1250 / 1250

Method 2: Diagonalization/Iteration Method

- **Optimal reaction of Firm A to y_B is found by maximizing $\pi_A(y_A,y_B)$ w.r.t. y_A. The resulting KKT condition that defines the optimal response y_A is:**
  - ***$0 \le y_A \perp d\pi_A(y_A,y_B)/dy_A \le 0$, or:***
  - ***$0 \le y_A \perp (100 - y_A - 0.5y_B) - y_A \le 0$***
- **If the optimal $y_A > 0$, then $y_A = 50 - y_B/4$ is the optimal reaction. A similar development given B's optimal reaction to y_A as $y_B = 50 - y_A/4$.**
- **Tennis anyone?**

Iteration #	y_A	y_B
0		70 ***= initial point***
1	32.5	
2		41.875
3	39.531	
4		40.117
5	39.971	
6		40.007
7	39.998	
8		40.0005

Method 3: Mixed Linear Complementarity Problem Statement

- Mixed LCP statement: *Find* $\{y_A, y_B, d\}$ *such that the following conditions are satisfied:*
  - *Firm A:* $0 \le y_A \perp (100 - y_A - 0.5y_B) - y_A \le 0$
  - *Firm B:* $0 \le y_B \perp (100 - 0.5y_A - y_B) - y_B \le 0$
  - *Market clearing:* $d = y_A + y_B$
- Mixed LCP: Has equalities as well as complementarity conditions
- Well-formulated problem will have equal number of variables and conditions
- Could use PATH to solve this problem
  - Lemke's algorithm
  - Iteratively linearizes for NCP
  - Solution: $y_A = y_B = 40$ *MW;* $d = 80$ *MW;* $P(y_A+y_B) = 60$ *\$/MWh*

Method 4: Single Equivalent Optimization Problem (Hashimoto 1985)

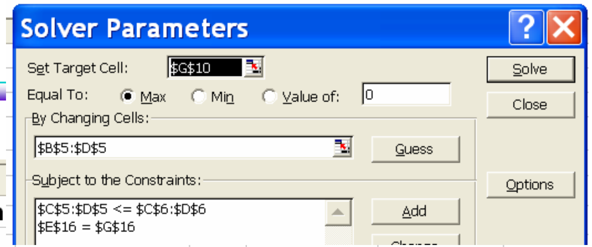
- Consider the following MP:

$$MAX \int_0^d (100-q/2)dq - \underline{(y_A^2/4 + y_B^2/4)} - C_A(y_A) - C_B(y_B)$$

$$\text{s.t. } d - y_A - y_B = 0; \qquad y_A, y_B \ge 0$$

- First term: integral of demand curve. If the underlined term was omitted, this would be the standard welfare max (perfect competition) model.
  - Underlined term modifies customer value term (integral) so that the derivative of {integral + underscored term} w.r.t. y_f is the marginal revenue (MR) for a Cournot firm f rather than price.
- KKT conditions =equilibrium conditions (Method 3)
- But it is not always possible to define a single optimization problem whose KKTs match the equilibrium conditions of a hypothetical market

Method 4: Excel Solver Cournot Model



	A	B	C	D	E	F	G	H
1	**Nash-Cournot Market Simulation: Quadratic Program**							
2	***Decision Variables***							
3		Demand	Operations Variables			***Note: Actual***		
4	Name	*d*	y_1	y_2		***Social Surplus=***		
5	**Value of d.v.**	80	40	40			**4800**	
6	Capacity X_i	n.a.	99999	99999				
7	$C_i(y_i)$ $/MWh		$=0.5*(y_A)^2$	$=0.5*(y_B)^2$				
8	Demand Price Intercept	100					**MAX ' Modified**	
9	Demand Slope	-0.5					**Social Surplus'**	
10	Obj f() term	**6400**	-800	-800	**-800**		4000	
11		***Demand curve integral***			***Hashimoto term***			
12	***Other Constraints***							
13	*Constraint Coefficients (Left Side)*							**Price =**
14	Load constraint	1	-1	-1				**Dual Price**
15	*Constraint Coefficients times Value*				**LHS "G(X)"**		**RHS "B"**	**$/MWh**
16	Load constraint	80	-40	-40	0.00	=	0	60

Example of Nonexistence of Pure Strategy Equilibria

- **Definitions:**
  - **Pure strategy equilibrium:** A firm *i* chooses X_i^* with probability 1
  - **Mixed strategy:** Let the strategy space be discretized $\{X_{ih}, h = 1,..,H\}$. In a mixed strategy, a firm i chooses X_{ih} with probability $P_{ih} < 1$. The strategy can be designated as the vector $\underline{P_i}$
    - Can also define mixed strategies using continuous strategy space and probability densities
    - Let $\underline{P_i}^c = \{\underline{P_j}, \forall j \neq i\}$
  - **Mixed strategy equilibrium:** $\{\underline{P_i}^*, \forall i\}$ is mixed strategy Nash Equilibrium iff:

$$\pi_i(\underline{P_i}^*, \underline{P_i}^{c*}) \geq \pi_i(\underline{P_i}, \underline{P_i}^{c*}), \forall i; \ \forall \underline{P_i}: \Sigma_h P_{ih} = 1, P_{ih} \geq 0$$

- By Nash's theorem, a mixed strategy equilibrium always exists (perhaps in degenerate pure strategy form) if strategy space finite.

Another Example But with a Difference: Pay as Bid Case:

"Pay as Bid" Example. Each firm i has 100 MW of capacity and zero MC, and submits a bid BID_i to supply it to the market.

Payoff Matrix method. Find cell such that π_A is highest in column (Firm A bids to maximize its profit given BID_B) and π_B is highest in row (Firm B maximizes its profit given BID_A)

BID_A \ BID_B:	2's Bid
1's Bid	π_1 / π_2

Bold italics represents Firm A's best bid response to BID_B

Bold represents Firm B's best bid response to BID_A

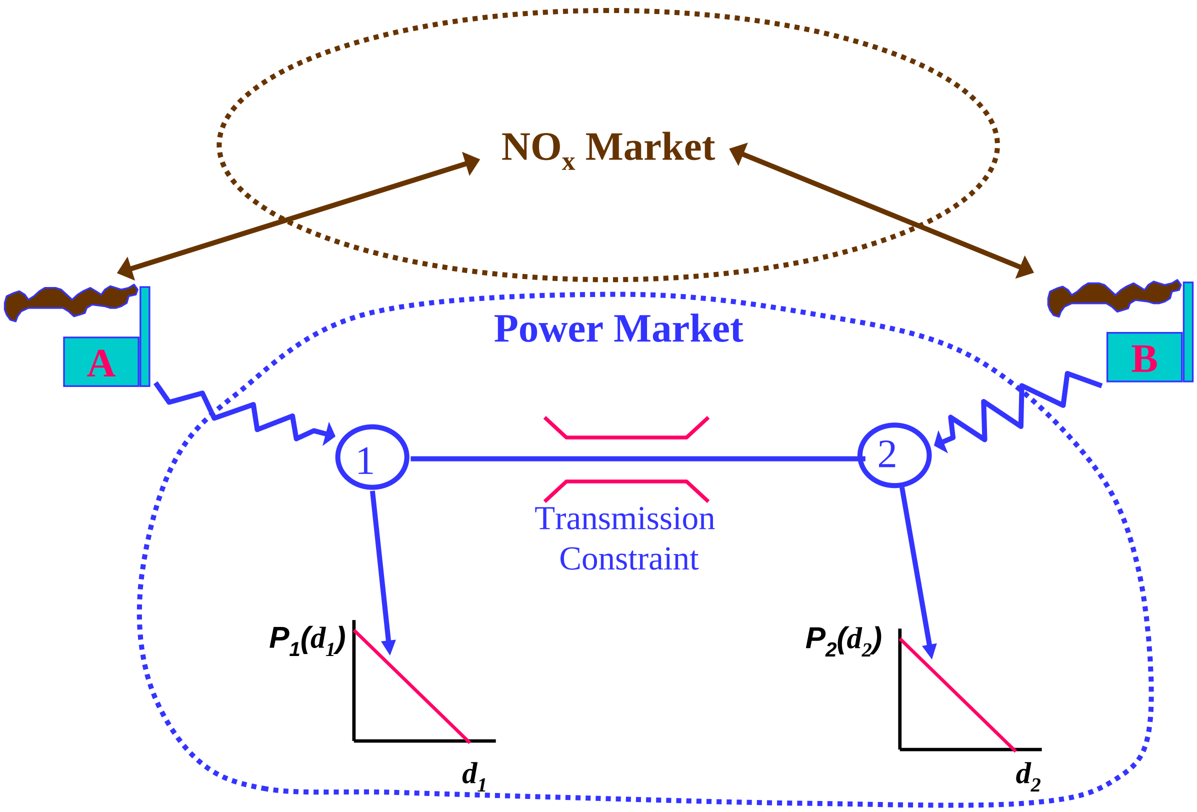
Note that there is no single cell that is the best response by both firms!

BID_A \ BID_B:	11.25	12.5	13.75	15	16.25	17.5	18.75	20	21.25	22.5	23.75	25
11.25	998 / 998	1125 / 938	1125 / 997	1125 / 1050	1125 / 1097	1125 / 1138	1125 / 1172	1125 / 1200	1125 / 1222	1125 / 1238	1125 / 1247	1125 / **1250**
12.5	938 / 1125	1094 / 1094	***1250*** / 997	1250 / 1050	1250 / 1097	1250 / 1138	1250 / 1172	1250 / 1200	1250 / 1222	1250 / 1238	1250 / 1247	1250 / **1250**
13.75	997 / 1125	997 / **1250**	1186 / 1186	***1375*** / 1050	1375 / 1097	1375 / 1138	1375 / 1172	1375 / 1200	1375 / 1222	1375 / 1238	1375 / 1247	1375 / 1250
15	1050 / 1125	1050 / 1250	1050 / **1375**	1275 / 1275	***1500*** / 1097	1500 / 1138	1500 / 1172	1500 / 1200	1500 / 1222	1500 / 1238	1500 / 1247	1500 / 1250
16.25	1097 / 1125	1097 / 1250	1097 / 1375	1097 / **1500**	1361 / 1361	***1625*** / 1138	1625 / 1172	1625 / 1200	1625 / 1222	1625 / 1238	1625 / 1247	1625 / 1250
17.5	1138 / 1125	1138 / 1250	1138 / 1375	1138 / 1500	1138 / **1625**	1444 / 1444	***1750*** / 1172	1750 / 1200	1750 / 1222	1750 / 1238	1750 / 1247	1750 / 1250
18.75	1172 / 1125	1172 / 1250	1172 / 1375	1172 / 1500	1172 / 1625	1172 / **1750**	1523 / 1523	***1875*** / 1200	1875 / 1222	1875 / 1238	1875 / 1247	1875 / 1250
20	1200 / 1125	1200 / 1250	1200 / 1375	1200 / 1500	1200 / 1625	1200 / 1750	1200 / **1875**	1600 / 1600	***2000*** / 1222	2000 / 1238	2000 / 1247	2000 / 1250
21.25	1222 / 1125	1222 / 1250	1222 / 1375	1222 / 1500	1222 / 1625	1222 / 1750	1222 / 1875	1222 / **2000**	1673 / 1673	***2125*** / 1238	2125 / 1247	2125 / 1250
22.5	1238 / 1125	1238 / 1250	1238 / 1375	1238 / 1500	1238 / 1625	1238 / 1750	1238 / 1875	1238 / 2000	1238 / **2125**	1744 / 1744	***2250*** / 1247	2250 / 1250
23.75	1247 / 1125	1247 / 1250	1247 / 1375	1247 / 1500	1247 / 1625	1247 / 1750	1247 / 1875	1247 / 2000	1247 / 2125	1247 / **2250**	1811 / 1811	***2375*** / 1250
25	***1250*** / 1125	***1250*** / 1250	1250 / 1375	1250 / 1500	1250 / 1625	1250 / 1750	1250 / 1875	1250 / 2000	1250 / 2125	1250 / 2250	1250 / **2375**	1875 / 1875

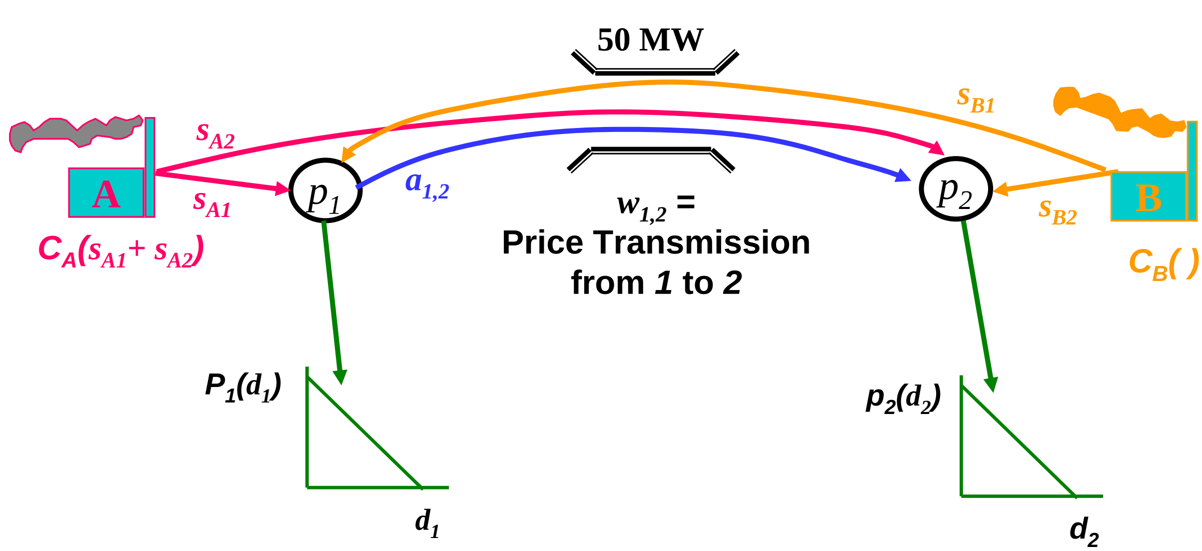
C. A Cournot Transmission-Constrained Model

- **Features:**
  - **Bilateral market (generators sell to customers, buy transmission services from ISO)**
  - **Cournot in power sales**
  - **Generators assume transmission fees fixed; linearized DC load flow formulation**
  - **If there are arbitragers, then same as POOLCO Cournot model**
    - **In which generators sell to "single buyer"**
  - **Mixed LCP formulation: allows for solution of very large problems**

Adding Transmission (and other commodities!)



Simple Example of Model: Generation Duopoly with Arbitrage and Transmission Constraint



Perfect Competition Model

Everyone a price taker w.r.t. nodal energy prices P_1, P_2, and transmission price $W_{1,2}$

Equilibrium problem: Find $\{p_1, p_2, w_{1,2}, s_{A1}, s_{A2}, s_{B1}, s_{B2}, a_{1,2}, y_{1,2}\}$ that simultaneously solve the following problems:

Gen A: *Given* $\{p_1, p_2, w_{1,2}\}$:
$$\underset{\{s_{A1}, s_{A2} \geq 0\}}{MAX} \quad p_1 s_{A1} + p_2 s_{A2} - w_{1,2} s_{A2} - C_A(s_{A1}+s_{A2})$$

Gen B: *Given* $\{p_1, p_2, w_{1,2}\}$:
$$\underset{\{s_{B1}, s_{B2} \geq 0\}}{MAX} \quad p_1 s_{B1} + p_2 s_{B2} + w_{1,2} s_{B1} - C_B(s_{B1}+s_{B2})$$

Consumer 1: *Given* $\{p_1\}$:
$$\underset{\{d_1 \geq 0\}}{MAX} \int_0^{d_1} P_1(x)dx - p_1 d_1$$

Consumer 2: *Given* $\{p_2\}$:
$$\underset{\{d_2 \geq 0\}}{MAX} \int_0^{d_2} P_2(x)dx - p_2 d_2$$

TSO: *Given* $\{w_{1,2}\}$:
$$\underset{\{y_{1,2}\}}{MAX} \quad w_{1,2} y_{1,2}$$
s.t. $-50 \leq y_{1,2} \leq +50$

Arbitrager: *Given* $\{p_1, p_2, w_{1,2}\}$:
$$\underset{\{a_{1,2}\}}{MAX} \quad (p_2 - p_1 - w_{1,2}) a_{1,2}$$

Market clearing:
$$d_1 = -a_{1,2} + s_{A1} + s_{B1}$$
$$d_2 = +a_{1,2} + s_{A2} + s_{B2}$$
$$y_{1,2} = a_{1,2} + s_{A2} - s_{B1}$$

Perfect Competition Model

Derive KKTs for each player's problem; combine with market clearing conditions

Mixed LCP: Find $\{p_1, p_2, w_{1,2}, s_{A1}, s_{A2}, s_{B1}, s_{B2}, a_{1,2}, y_{1,2}, \lambda_{1,2}^+, \lambda_{1,2}^-\}$ that simultaneously solves the following *mixed complementarity problem*:

Gen A:
$$0 \leq s_{A1} \perp (p_1 - C_A') \leq 0$$
$$0 \leq s_{A2} \perp (p_2 - w_{1,2} - C_A') \leq 0$$

Gen B:
$$0 \leq s_{B1} \perp (p_1 + w_{1,2} - C_B') \leq 0$$
$$0 \leq s_{B2} \perp (p_2 - C_B') \leq 0$$

Consumer 1:
$$0 \leq d_1 \perp P_1(d_1) - p_1 \leq 0$$

Consumer 2:
$$0 \leq d_2 \perp P_2(d_2) - p_2 \leq 0$$

TSO:
$$w_{1,2} - \lambda_{1,2}^+ + \lambda_{1,2}^- = 0$$
$$0 \leq \lambda_{1,2}^+ \perp y_{1,2} - 50 \leq 0$$
$$0 \leq \lambda_{1,2}^- \perp -y_{1,2} - 50 \leq 0$$

Arbitrager:
$$p_2 - p_1 - w_{1,2} = 0$$

Market clearing:
$$d_1 = -a_{1,2} + s_{A1} + s_{B1}$$
$$d_2 = +a_{1,2} + s_{A2} + s_{B2}$$
$$y_{1,2} = a_{1,2} + s_{A2} - s_{B1}$$

Under mild conditions, solution (1) exists, (2) is unique

Oligopolistic Generation

Naïve assumption that Generators are Bertrand (price takers) with respect to transmission costs W (e.g., Wei & Smeers, 2000)

Equilibrium Problem: Find $\{w_{1,2}, s_{A1}, s_{A2}, s_{B1}, s_{B2}, a_{1,2}, y_{1,2}\}$ that simultaneously solve the following problems:

Gen A: *Given* $\{w_{1,2}, s_{B1}, s_{B2}, a_{1,2}\}$:

$$\underset{\{s_{A1}, s_{A2} \geq 0;\, d_1, d_2\}}{MAX} \quad P_1(d_1)s_{A1} + P_2(d_2)s_{A2} - w_{1,2}s_{A2} - C_A(s_{A1}+s_{A2})$$

s.t. $d_1 = -a_{1,2} + s_{A1} + s_{B1}$

$d_2 = +a_{1,2} + s_{A2} + s_{B2}$

Gen B: *Given* $\{w_{1,2}, s_{A1}, s_{A2}, a_{1,2}\}$:

$$\underset{\{s_{B1}, s_{B2} \geq 0;\, d_1, d_2\}}{MAX} \quad P_1(d_1)s_{B1} + P_2(d_2)s_{B2} + w_{1,2}s_{B1} - C_B(s_{B1}+s_{B2})$$

s.t. $d_1 = -a_{1,2} + s_{A1} + s_{B1}$

$d_2 = +a_{1,2} + s_{A2} + s_{B2}$

TSO: *Given* $\{w_{1,2}\}$:

$$\underset{\{y_{1,2}\}}{MAX} \quad w_{1,2}y_{1,2}$$

s.t. $-50 \leq y_{1,2} \leq +50$

Arbitrager: *Given* $\{p_1, p_2, w_{1,2}\}$:

$$\underset{\{a_{1,2}\}}{MAX} \quad (p_2 - p_1 - w_{1,2})a_{1,2}$$

Market clearing: $y_{1,2} = a_{1,2} + s_{A2} - s_{B1}$

Oligopolistic Generation Model

Derive KKTs for each player's problem; combine with market clearing conditions. After rearrangement, we get:

Mixed LCP: Find $\{p_1, p_2, w_{1,2}, s_{A1}, s_{A2}, s_{B1}, s_{B2}, a_{1,2}, y_{1,2}, \lambda_{1,2}^+, \lambda_{1,2}^-\}$ that simultaneously solves the following *mixed complementarity problem*:

Gen A:

$0 \leq s_{A1} \perp (P_1 - P_1' s_{A1} - C_A') \leq 0$

$0 \leq s_{A2} \perp (P_2 - P_2' s_{A2} - w_{1,2} - C_A') \leq 0$

Gen B:

$0 \leq s_{B1} \perp (P_1 - P_1' s_{B1} + w_{1,2} - C_B') \leq 0$

$0 \leq s_{B2} \perp (P_2 - P_2' s_{B2} - C_B') \leq 0$

TSO: $w_{1,2} - \lambda_{1,2}^+ + \lambda_{1,2}^- = 0$

$0 \leq \lambda_{1,2}^+ \perp y_{1,2} - 50 \leq 0$

$0 \leq \lambda_{1,2}^- \perp -y_{1,2} - 50 \leq 0$

Arbitrager:

$P_1 - P_2 - w_{1,2} = 0$

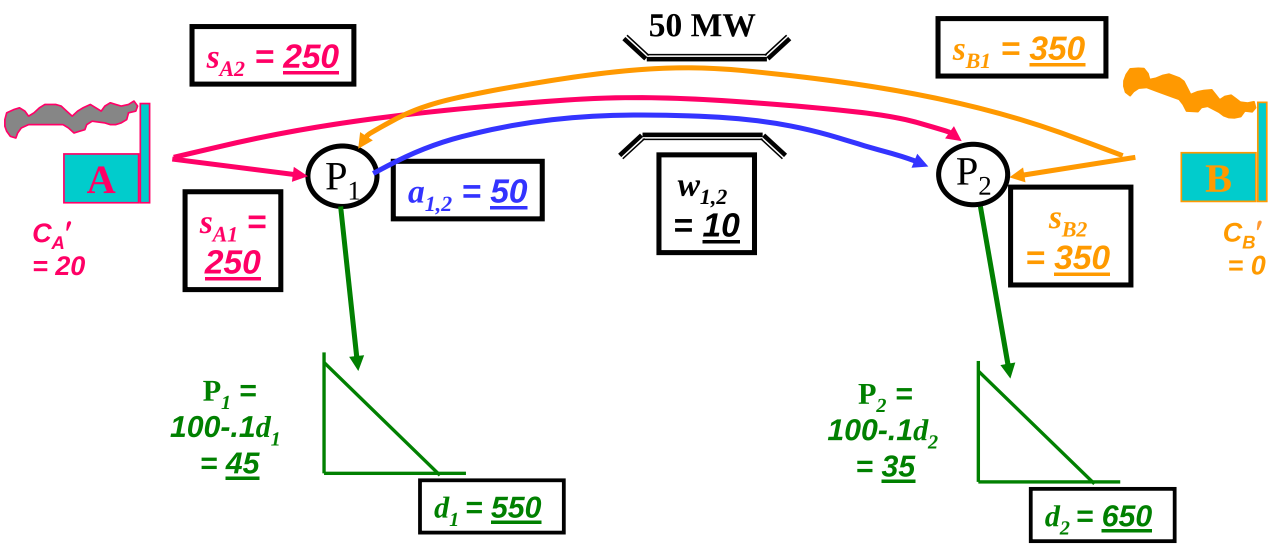
Market clearing: $d_1 = -a_{1,2} + s_{A1} + s_{B1}$

$d_2 = +a_{1,2} + s_{A2} + s_{B2}$

$y_{1,2} = a_{1,2} + s_{A2} - s_{B1}$

Under mild conditions, solution to resulting MCP (1) exists, (2) is unique, and (3) is equivalent to POOLCO Cournot equilibrium

Simple Example of Model: Generation Duopoly with Arbitrage and Transmission Constraint



D. A Large Scale Cournot Bilateral & POOLCO Model

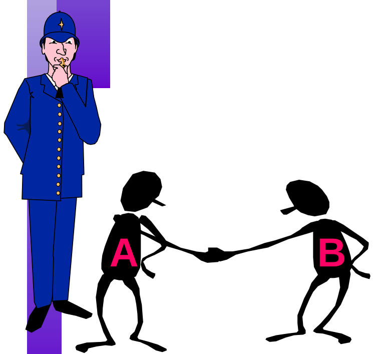
(B.F. Hobbs and U. Helman, "Complementarity-Based Equilibrium Modeling for Electric Power Markets," in D.W. Bunn (ed.), Modeling Prices in Competitive Electricity Markets, J. Wiley, 2004)

Features:

- Bilateral market (generators sell to customers, buy transmission services from ISO)
- Cournot in power sales
- Generators assume transmission fees fixed; linearized DC load flow formulation
- Mixed "linear complementarity" formulation: allows for solution of very large problems

A Large Scale Implementation: Eastern Interconnection Model

- 100 nodes representing control areas and 15 interconnections with ERCOT, WSSC, and Canada
- 829 firms (of which 528 are NUGs)
- 2725 generating plants (in some cases aggregated by prime mover/fuel type/costs); approximately 600,000 MW capacity
- Implemented by FERC staff
  - Spatial market power issues (congestion, addition of transmission constraints)
  - Effects of mergers

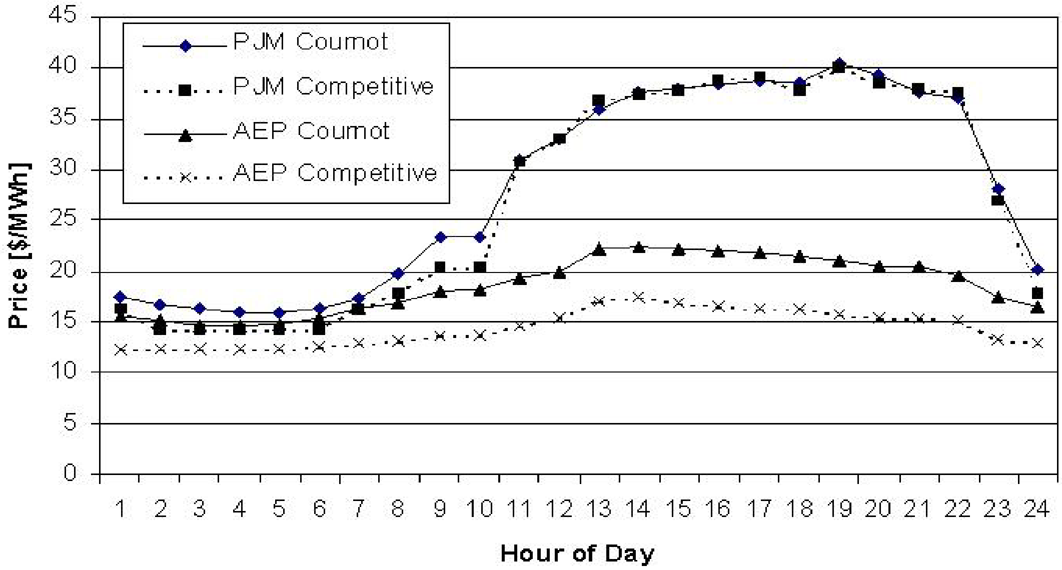


Eastern Interconnection Model

- 814 flowgates, each with PTDFs for each node (most flowgates and PTDFs defined by NERC; a mix of physical and contingency flowgate limits)

- 68 firms represented as Cournot players (with capacity above 1000 MW). Remainder is competitive fringe

Selected Market Power Results



Merger Example

(Firm A at Node A, Firm B at Node B)

	pre-merger	merger
Competition	$22.17	$22.17
Cournot w/ arbitrage	$22.85	$22.86
Difference	3.07%	3.11%
Cournot Price Node A	$18.89	$18.92
Cournot Price Node B	$27.65	$27.66
Profits	A+B=$162,723	AB=$162,400

E. Advanced Models
Desirable Improvements
(thanks to R. Baldick, 2006)

- Improved models of *Physical* system
  - Better representation of technology constraints
  - The economist's "production function"
- Improved models of *Commercial* system
  - Definition of products / markets
  - "Settlement rules": who gets paid what in each market
- Improved models of *Economic* system
  - Agent objectives
  - Agent strategic variables
  - Agent state of knowledge / expectations
  - Agent cooperation

Examples of Improved Models:
Generation

- Better *physical* models
  - Multiple periods and hydro (Bushnell 2002)
  - Capacity additions
    - *Make capacity & energy decisions at same time ("open loop") (Wei, Smeers, 1999)*
    - *Make capacity decisions anticipating effect on energy market ("closed loop") (Murphy, Smeers, 2004)*
  - Emissions permits markets (*)
- Better *commercial* models
  - Locational operating reserves markets (Helman 2002; Bautista et al. 2005)
  - Two-settlement systems: day ahead (perhaps zonal pricing) and real-time (locational) (EPEC!) (Kamat & Oren, 2004)
- Better *economic* models
  - Forward contracts:
    - *Exogenous contracts (Green 2002)*
    - *Endogenous contracts: Two stage models (EPECs!) (Yao, Oren, Adler 2005)*
  - Anticipate supply response of rivals
    - *Include fringe's KKTs in leader's constraint set (MPEC!)(Neuhoff et al.)*
    - *"Conjectured supply response" (Day/Hobbs 2002)*
    - *Inverse problem (estimate conjectural variations) (Garcia-Alcalde et al. 2002)*
  - Tacit collusion (multiperiod "supergames") (Liu, Harrington, Hobbs, Pang, 2005)

Examples of Improved Models: *Transmission*

- Better *physical* models:
  - Linearized DC Load flow model (*)
    - *TSO constraints involve PTDFs*
    - *Quadratic losses*
  - AC Load flow model (Anjos, Bautista et al. 2006)
  - Controllable DC lines, phase shifters in linearized DC load flow (Hobbs et al. 2006)
- Better *commercial* models:
  - Commercial rules results in (economically) imperfect transmission pricing
    - *Path-based models* (Hobbs, Rijkers et al. 2003)
    - *No-netting of flows (use nonnegative flow variables for each direction)* (Hobbs, Rijkers et al. 2003)
    - *Average cost-based tariffs* (Wei and Smeers, 2000)
- Better economic models:
  - Generators anticipate transmission price changes
    - Include TSO KKTs as constraints: MPEC! (e.g., Cardell/Hitt/Hogan 1997; Hobbs/Metzler/Pang 2000; Borenstein/Bushnell/Stoft 2000)
    - Or "conjectured transmission price response" (MCP) (Hobbs/Rijkers 2003)

Example of a Stackelberg (Leader-Follower) Model

- Large supplier as <u>leader</u>, ISO & other suppliers as <u>followers</u> in POOLCO market
- Problem: choose bids B_{Li} to max π_L

  $$\text{MAX } \pi_L = \Sigma_i [P_i y_{Li} - C_i(y_{Li})]$$

  s.t. $0 \leq y_{Li} \leq X_i, \ \forall i$

  KKTs for ISO (depend on B_{Li}'s)

  KKTs for other suppliers (price takers)
- <u>The Challenge:</u> the complementarity conditions in the leader's constraint set render the leader's problem non-convex (i.e., feasible region non-convex)
- Algorithms for math programs with equilibrium constraints (MPECs) and equilibrium programs with equilibrium constraints (EPECs) are improving

EPEC

Sophisticated assumption that Generators are Stackelberg leader with respect to transmission costs w (e.g., Hobbs, Metzler, Pang, 2000)

Equilibrium Problem: Find $\{w_{1,2}, s_{A1}, s_{A2}, s_{B1}, s_{B2}, a_{1,2}, y_{1,2}\}$ that simultaneously solve the following problems:

Gen A: *Given* $\{w_{1,2}, s_{B1}, s_{B2}, a_{1,2}\}$:

$$\max_{\{s_{A1}, s_{A2} \geq 0;\, d_1, d_2\}} P_1(d_1)s_{A1} + P_2(d_2)s_{A2} - w_{1,2}s_{A2} - C_A(s_{A1}+s_{A2})$$

s.t. $d_1 = -a_{1,2} + s_{A1} + s_{B1}$

$d_2 = +a_{1,2} + s_{A2} + s_{B2}$

TSO: $50 \geq y_{1,2} \perp (w_{1,2} - \lambda_{1,2}^{+}) \leq 0$
$-50 \leq y_{1,2} \perp (-w_{1,2} - \lambda_{1,2}^{-}) \leq 0$

Arbitrager: $p_1 - p_2 - w_{1,2} = 0$

Market clearing: $y_{1,2} = a_{1,2} + s_{A2} - s_{B1}$

Gen B: *Given* $\{s_{A1}, s_{A2}\}$:

$$\max \; p_1(d_1)s_{B1} + p_2(d_2)s_{B2} + W_{1,2}s_{B1} - C_B(s_{B1}+s_{B2})$$

s.t. $d_1 = -a_{1,2} + s_{A1} + s_{B1}$

$d_2 = +a_{1,2} + s_{A2} + s_{B2}$

TSO: $50 \geq y_{1,2} \perp (w_{1,2} - \lambda_{1,2}^{+}) \leq 0$
$-50 \leq y_{1,2} \perp (-w_{1,2} - \lambda_{1,2}^{-}) \leq 0$

Arbitrager: $p_1 - p_2 - w_{1,2} = 0$

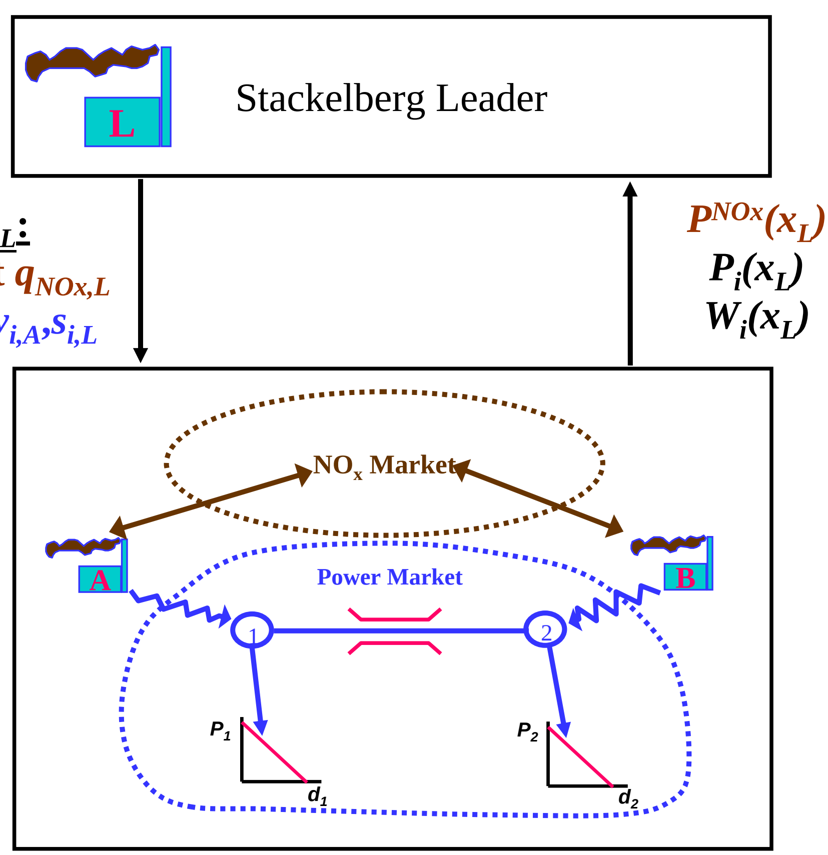
Market clearing: $y_{1,2} = a_{1,2} + s_{A2} - s_{B1}$

Iteration/Diagonalization (Gauss-Seidel) among MPECs often used

Generally: pure strategy solutions may neither exist nor be unique

Stackelberg Analysis

L's decisions x_L:
Allowances bought $q_{NOx,L}$
Energy decisions $y_{i,A}, s_{i,L}$



Chen, Hobbs, Munson, Leyffer, Computational Management Science, 2006, 307-330

Stackelberg Leader's Problem

The firm with a longest position in NOx market and greatest power sales is designated as the leader

q^w = *Stackelberg's NO_x withholding variable [tons]*

$\overline{q}_f^{NO_x}$ = *Firm's available NO_x allowances [tons]*

$$MAX_{s_{if},g_{if},q^w} \sum_i \{[p_i(s_{if} + \sum_{g \neq f} s_{ig}) - W_i]s_{if} - [C_{if}(y_{if}) - W_i y_{if}]$$

$$- p^{NO_x}[E_f^{NO_x} - (\overline{q}_f^{NO_x} - q^w)]\}$$

$$s.t.: \; y_{if} \le CAP_{if}, \forall i$$

$$\sum_i s_{if} = \sum_i y_{if}$$

$$s_{if}, y_{if} \ge 0, \forall i$$

$$0 \le q^W \le \overline{q}_f^{NO_x}$$

$$0 \le p^{NO_x} \perp \sum_f (E_f^{NO_x} - \overline{q}_f^{NO_x}) + q^w \le 0$$

- *Other Producer & TSO KKT Conditions*
- *Market Clearing Conditions*

ISO Optimization Problem
Quadratic Loss Functions

- **ISO's decision variables:**
  - z_i = transmission service from hub to i
  - q_i^{Losses} = make-up loss from node i
  - t_{ij} = flow in arc (i,j)
- **ISO's maximizes the "value of services" :**

$$MAX \quad \pi_{ISO}(t_{ij}, z_i, q_i^{Losses}) = \sum_i (W_i z_i - p_i q_i^{Losses})$$

$$s.t.: z_i - q_i^{Losses} + \sum_{j \in J(i)} (t_{ij} - (1 - L_{ji} t_{ji}) t_{ji}) \le 0, \quad \forall i \qquad \textit{Kirchhoff's Current Law}$$

$$\sum_{(i,j) \in v(k)} R_{ij}(t_{ij} - t_{ji}) = 0, \quad \forall k, (i,j) \in v(k) \qquad \textit{Kirchhoff's Voltage Law}$$

$$\sum_i z_i = 0 \qquad \textit{Services Balance}$$

$$0 \le t_{ij} \le T_{ij}, \quad \forall i, j \qquad T_{ij} = \textit{capacity of line (i,j)}$$

$$q_i^{Losses} \ge 0, \quad \forall i$$

- **Solution allocates transmission to most valuable transactions**
- **Define the model's KKTs (complementarity conditions), one per variable $\underline{x}_{ISO}$**

Model Statistics

- 18,618 variables; 9739 constraints
  - Order of magnitude larger than test problems in R. Fletcher and S. Leyffer, "Numerical Experience with Solving MPECs as NLPs," Univ. of Dundee, 2002
- Solved by PATH and SQP (SNOPT, FILTER) (Thanks to Todd Munson & Sven Leyffer!)
- 9,536 seconds (1.8 MHz Pentium 4)
  - Other MPECs took much less time

Stackelberg Results

Compared to the Cournot Case:

- *Stackelberg leader:*
  - *withholds 5,536 tons of allowances (7.2% of total available)*
  - *… increasing NO_x price from 0 to 1,173 [$/ton]*
- *Output:*
  - *other producers shrink their power sales (87.4→83.5 x10^6 MWh) due to increased NO_x price*
  - *… while the leader expands its output (24.6→28.7 x10^6 MWh)*
- *Profit:*
  - *Stackelberg leader earns more profit (893 → 970 M$)*
  - *… at the expense of other producers (2394 → 2273 M$)*
- *Consumers:*
  - *are only marginally better off with a gain of 14 [M$] in consumer surplus, as power prices are essentially unchanged*

V. Conclusion

- There are practical market models that capture key features of the market:
  - Kirchhoff's current & voltage laws,
  - transmission pricing,
  - generator strategic behavior
- Market features present computational & analytical challenges for the power engineering/O.R. researcher
- Prices can't be predicted precisely because games are repeated, and conjectural variations are fluid and more complex than can be modeled. Models most useful for exploring issues/gaining insight--thus, simpler models preferred
  - MODELS ARE FOR INSIGHT, <u>NOT</u> NUMBERS!!!
- Apply to market structure evaluation, market design, and strategic pricing
- Need comparisons of model results with each other, and with actual experience

Operations LP Answer: Model Formulation

MIN $760(70\, y_{A,Pk} + 25\, y_{B,Pk}) + 8000(70\, y_{A,OP} + 25\, y_{B,OP})$

subject to:

Meet load:

$$y_{A,Pk} + y_{B,Pk} = 2200$$

$$y_{A,OP} + y_{B,OP} = 1300$$

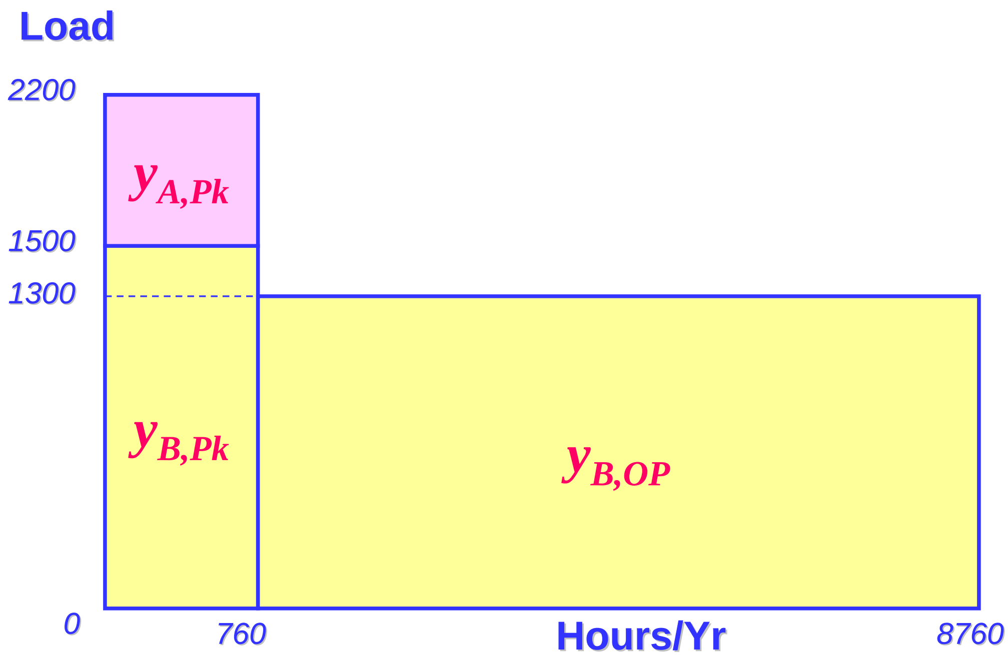
Generation $\leq$ capacity:

$$y_{A,Pk} \leq 800; \; y_{A,OP} \leq 800$$

$$y_{B,Pk} \leq 1500; \; y_{B,OP} \leq 1500$$

Nonnegativity: $y_{A,Pk}, y_{A,OP}, y_{B,Pk}, y_{B,OP} \geq 0$

Operations LP Answer: Load Duration Curve



Planning LP Answer: Model Formulation

MIN $760(70\, y_{A,Pk} + 25\, y_{B,Pk}) + 8000(70\, y_{A,OP} + 25\, y_{B,OP}) + 70{,}000\, x_A + 120{,}000\, x_B$

subject to:

Meet load:

$y_{A,Pk} + y_{B,Pk} = 2200$

$y_{A,OP} + y_{B,OP} = 1300$

Generation $\leq$ capacity:

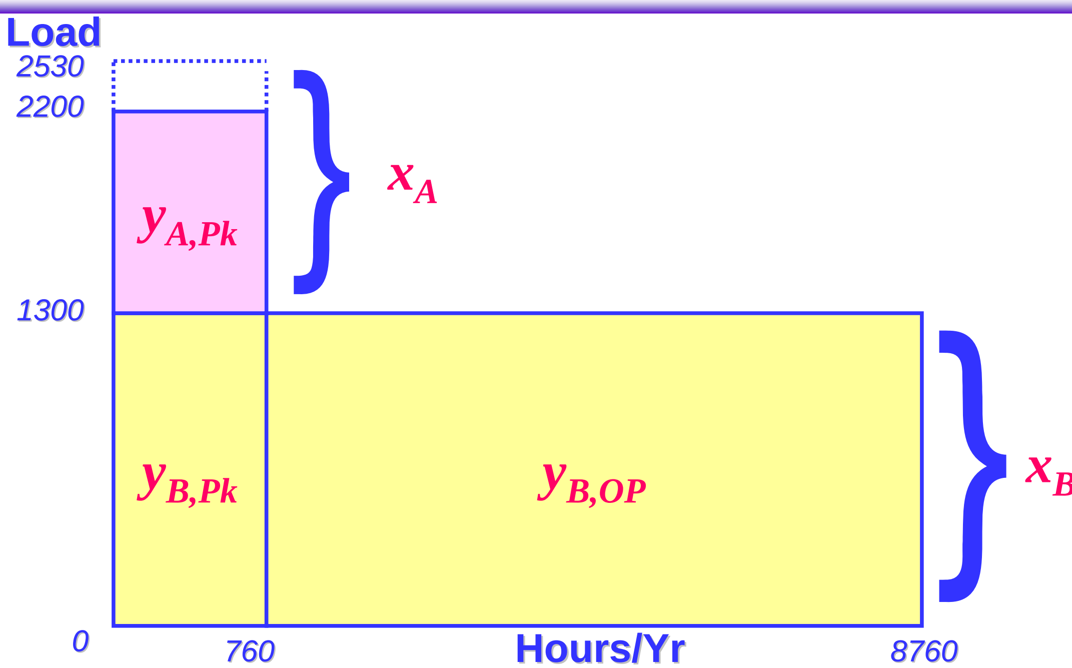
$y_{A,Pk} - x_A \leq 0;\ y_{A,OP} - x_A \leq 0$

$y_{B,Pk} - x_B \leq 0;\ y_{B,OP} - x_B \leq 0$

Reserve: $x_A + x_B \geq 1.15 * 2200$

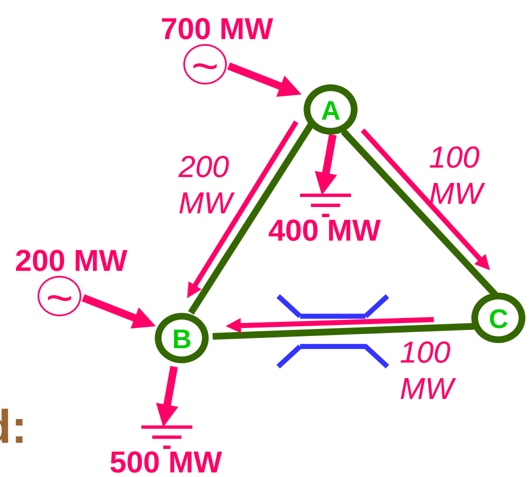
Nonnegativity: $y_{A,Pk},\ y_{A,OP},\ y_{B,Pk},\ y_{B,OP} \geq 0$

Planning LP Answer: Load Duration Curve



Exercise in Transmission Modeling: Answer

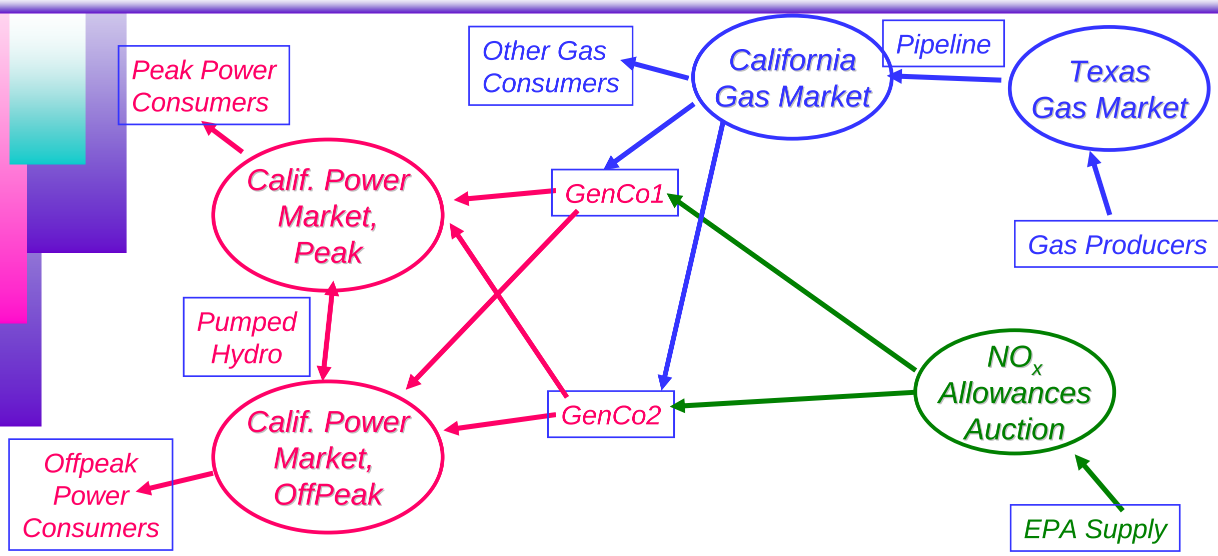
- **Optimal Dispatch**
  - Two plants:
    - A: Meet load at A (400 MW) plus inject maximum amount that transmission limit allows (100 MW/PTDF = 100/.33 = 300 MW)
      = 700 MW
    - B: Serve the load at B not served by A (= 500 MW-300 MW)
      = 200 MW



- **Marginal Costs ("LMP") to Load:**
  - A: The cost of Plant A ($25)
  - B: The cost of Plant B ($70)
  - C: More complex! To bring 1 MW to C, you can back off 1 MW at B and expand 2 MW at A:
    = -$70 + 2*$25 = -$20

Commodity Markets Exercise

(Rectangles are Optimizing Market Parties; Ovals are Markets with Clearing Conditions)



Market Simulation Model: *Max Value to Power & Other Gas Consumers*
minus Costs of power, gas production & transport
s.t. market clearing, production functions for power & gas, capacity limits