Supply Function Equilibria: Step Functions and Continuous Representations

Pär Holmberg, **David Newbery** and Daniel Ralph

OR 50

9-11 September 2008, York

http://www.electricitypolicy.org.uk

Price determination in electricity markets

- Liberalisation creates wholesale markets
  - day-ahead, balancing, over-the-counter, contract ..
- generators submit offers (supply functions)
- agents submit bids for demand
- Market operator clears market at market clearing price

How to model the supply function equilibrium?

Wholesale electricity markets

- Typically uniform price auctions
  - Separate price determined for each period
  - English Pool: offers day-ahead for 48 half-hours
- Generating costs are common knowledge
- Electricity is a homogeneous good
- Few producers => bid strategically
- Many consumers => price-takers

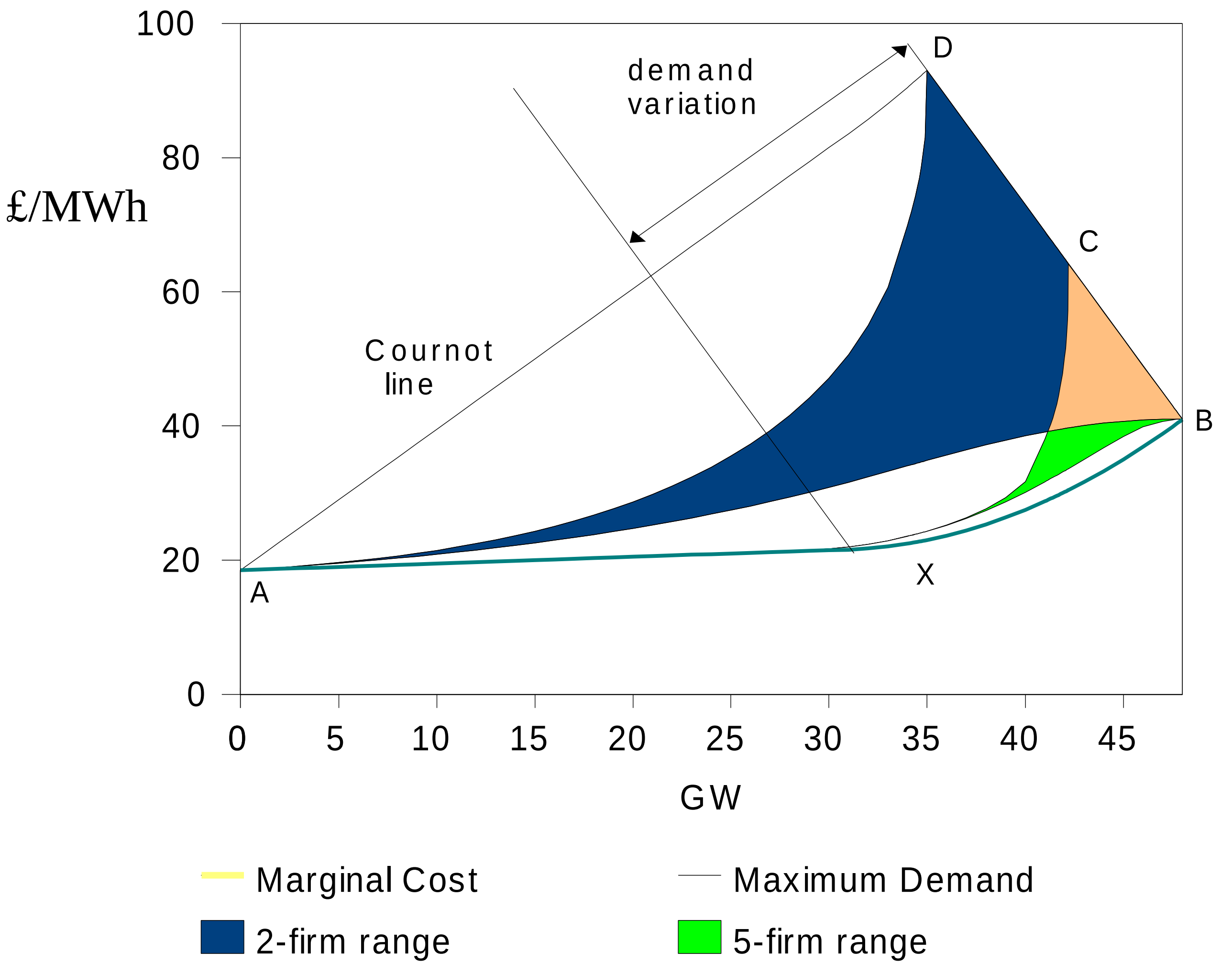
Modelling market power important

Continuous SFE

- Green and Newbery adapt Klemperer and Meyer supply function model for electricity:
  - uncertainty = time varying demand
  - Nash Equil: Given varying demand and competitors' SF, each producer i = 1,…N, chooses its SF $S_i(p)$ to maximise profit at each level of residual demand $D(p,\varepsilon)-\Sigma_j S_j(p)$
- SFE determined by system of DE's:

$$\begin{cases} S_1(p) - \left[S_{-1}'(p) - D'(p)\right]\left\{p - C_1'\left[S_1(p)\right]\right\} = 0 \\ \vdots \\ S_N(p) - \left[S_{-N}'(p) - D'(p)\right]\left\{p - C_N'\left[S_N(p)\right]\right\} = 0 \end{cases}$$

Feasible Supply Functions
Duopoly and Quintopoly

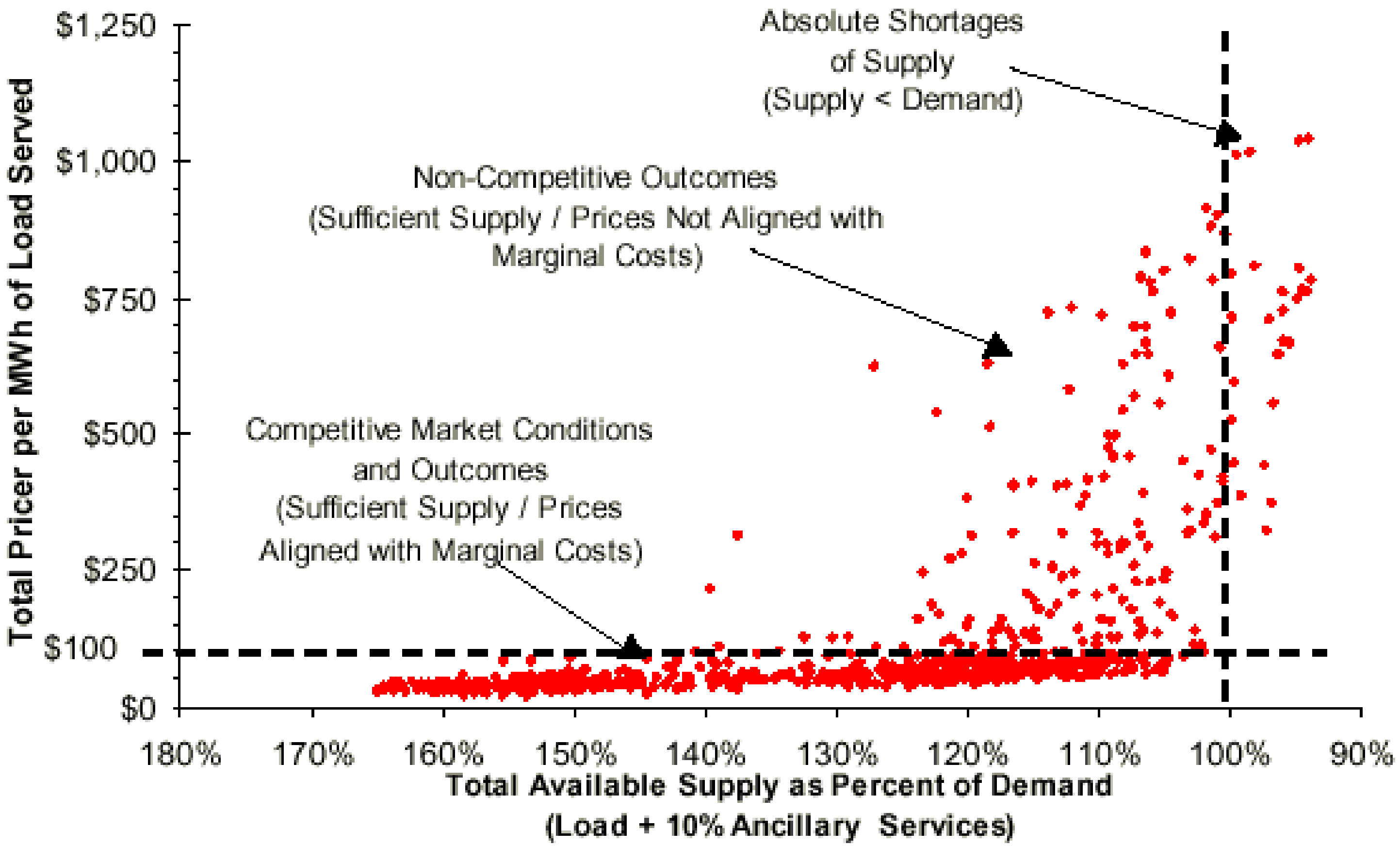


Calibrated for England 1990

CALIFORNIA ISO

California Independent System Operator

Scarcity or Market Power?



* Source: *Report on California Energy Market Issues and Performance: May-June, 2000*, Prepared by the Department of Market Analysis, August 10, 2000

Objections to continuous SFE

- Power exchanges require stepped offers and bids ("price ladders")

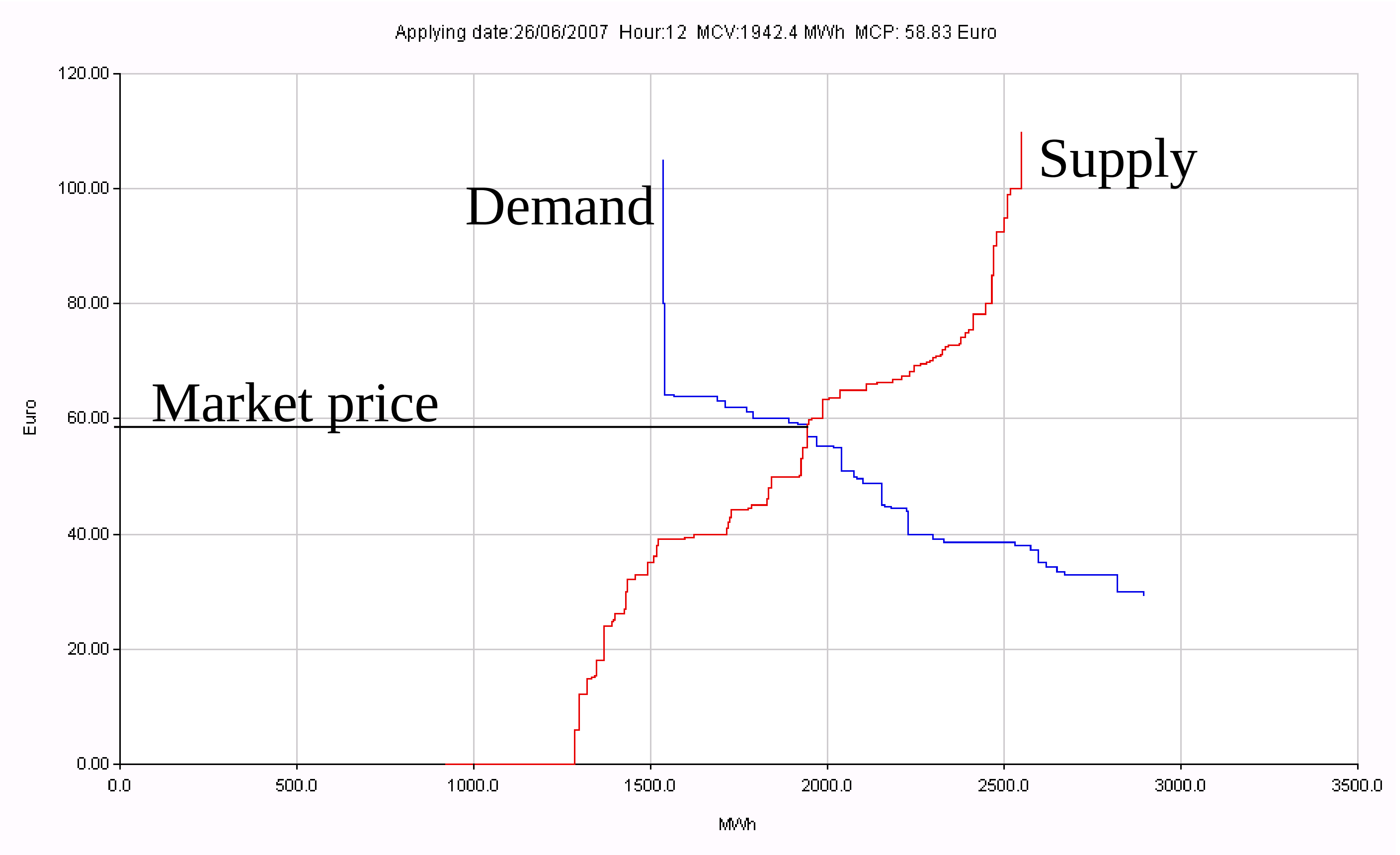
=> Residual demand stepped

=> poorly defined marginal revenue

=> multi-unit auctions

=> mixed strategies, unstable prices

Example from the Amsterdam Power Exchange



Related literature

- Dasgupta and Maskin (1986): Nash Equilibria (NE) of discrete approx of continuous game need not converge to NE of continuous game *if payoff functions are discontinuous*
- Empirical studies of Texas balancing market

  => large producers bid to satisfy f.o.c.s of continuous SFE
- Wolak (04), Anderson-Xu (04) derive best step function responses given prior choice of prices
  - do not analyse convergence to continuous SFE

Hortacsu-Puller model of ERCOT

- bids of all gencos available to regulator
- cost functions common knowledge => MC_i
- demand *less* other firms' bids = $RD_i(p)$
- can compute slope $RD_i'(p)$
- can compute p - $MC_i(S_i(p))$
- can compare this with actual bids
- can estimate θ (degree of market power) in

$$p - MC_i(S_i(p)) = \theta\{[S_i(p) - QC_i]/ RD_i'(p)\}$$

$\theta = 0$: competitive; $\theta = 1$: non-collusive optimum

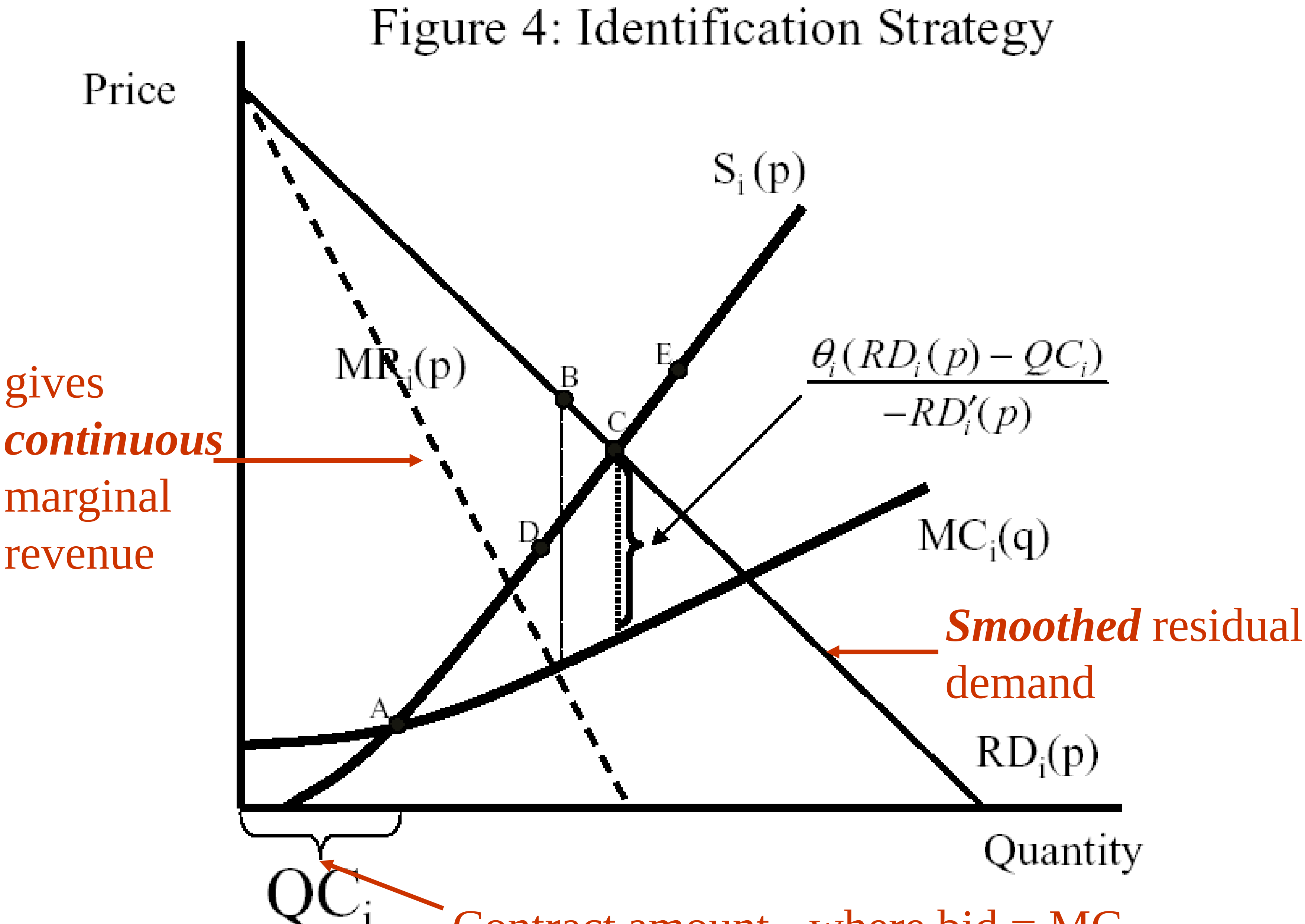
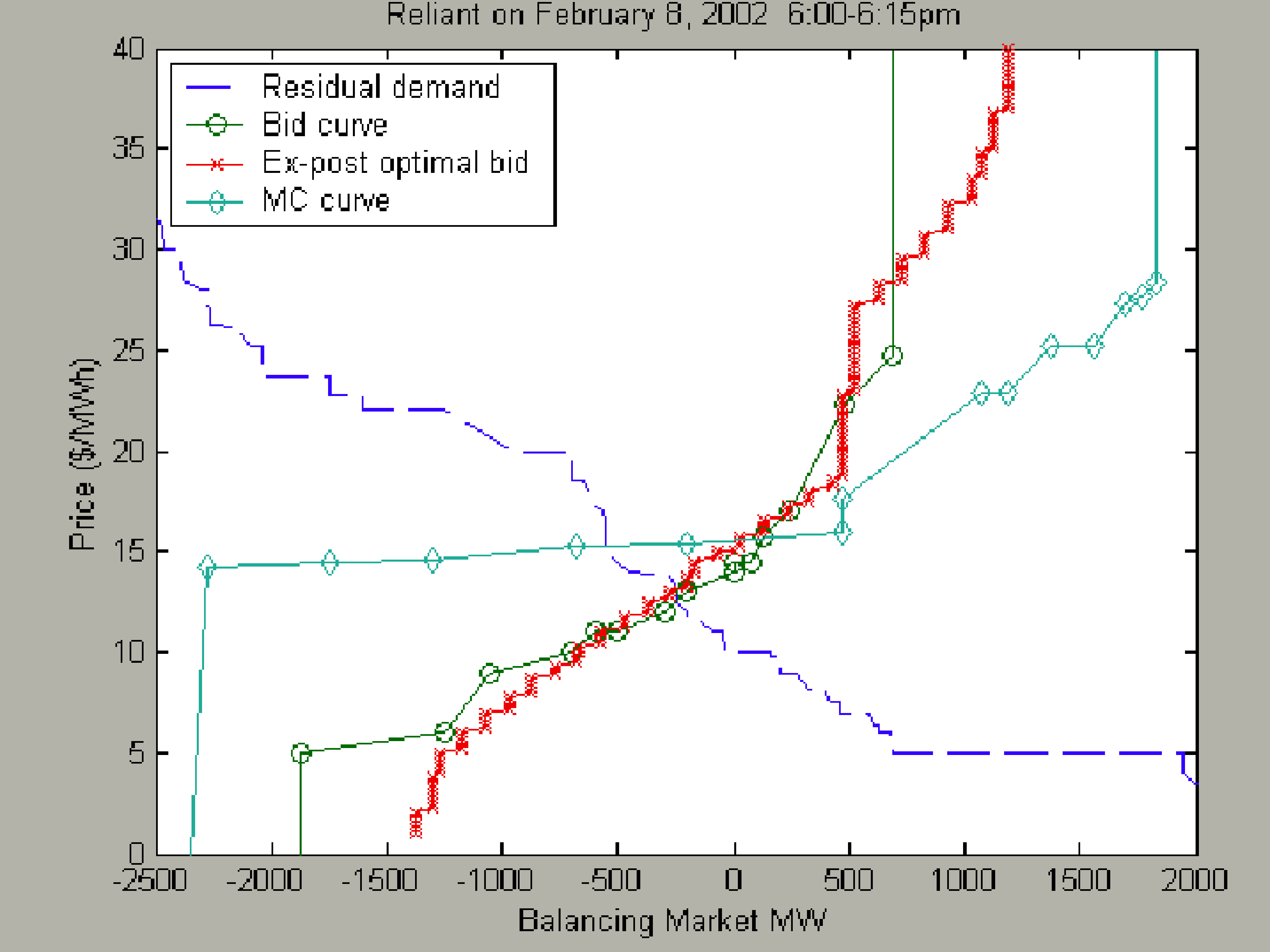
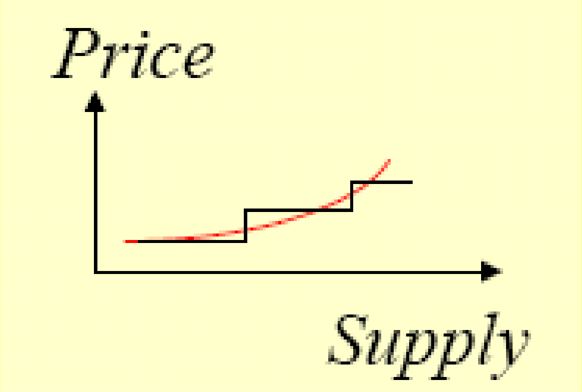


Figure 4: Identification Strategy
Price
Quantity
$S_i(p)$
$MR_i(p)$
$MC_i(q)$
$RD_i(p)$
$\frac{\theta_i(RD_i(p) - QC_i)}{-RD_i'(p)}$
A
B
C
D
E
QC_i
gives ***continuous*** marginal revenue
Smoothed residual demand
Contract amount - where bid = MC



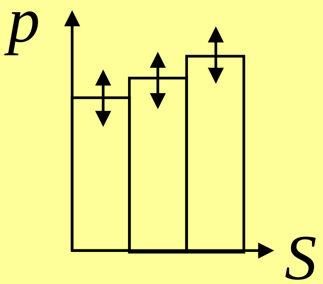
Reliant on February 8, 2002 6:00-6:15pm
Residual demand
Bid curve
Ex-post optimal bid
MC curve
Price ($/MWh)
Balancing Market MW
0
5
10
15
20
25
30
35
40
-2500
-2000
-1500
-1000
-500
0
500
1000
1500
2000

Summary

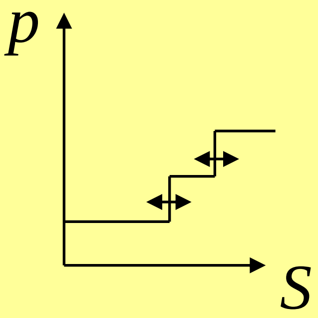


Continuous supply functions are convenient => pure-strategy SFE

von der Fehr and Harbord (1993) argue for step offers that are **discrete in quantity**
=> unstable prices
=> do not converge to continuous SFE



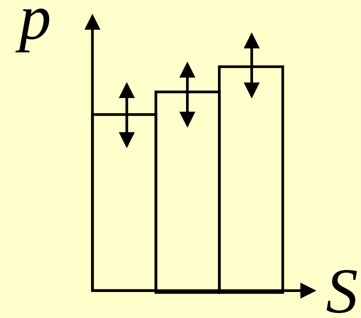
We derive pure-strategy NE of game with step offers **discrete in prices**
=> stable prices
=> converge to continuous SFE.



Offer constraints in wholesale electricity markets

Market	*Installed capacity*	*Max steps*	*Price range*	*Price tick size*	*Quantity multiple*	*No. quantities/ No. prices*
Nord Pool spot	90,000 *MW*	64 *per bidder*	0-5,000 *NOK/MWh*	0.1 *NOK/MWh*	0.1 *MWh*	18
ERCOT balancing	70,000 *MW*	40 *per bidder*	-$1,000/*MWh*-$1,000/*MWh*	*$0.01/MWh*	0.01 *MWh*	35
PJM	160,000 *MW*	10 *per plant*	*0-$1,000/MWh*	$0.01/*MWh*	0.01 *MWh*	160
UK (NETA)	80,000 *MW*	5 *per plant*	-£9,999/*MWh*-£9,999/*MWh*	£0.01/*MWh*	0.001 *MWh*	4
Spain Intra-day market	46,000 *MW*	5 *per plant*	*Yearly cap on revenues*	€0.01/*MWh*	0.1 *MWh*	—

Multi-unit auctions (discrete quantitities)



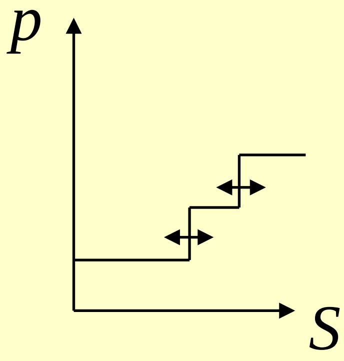
- von der Fehr and Harbord (1993):
  - multi-unit auction; *continuum* of prices: $p \in [\underline{p}, \overline{p}]$,
  - goods are *indivisible*: $s_i(p) \in \{q_1, q_2, \ldots, \overline{q}\}$.

=> pure-strategy equilibria may not exist

- infinitesimal undercutting profitable
- even if units are arbitrarily small

=> mixed NE => unstable prices.

Stepped supply function discrete prices

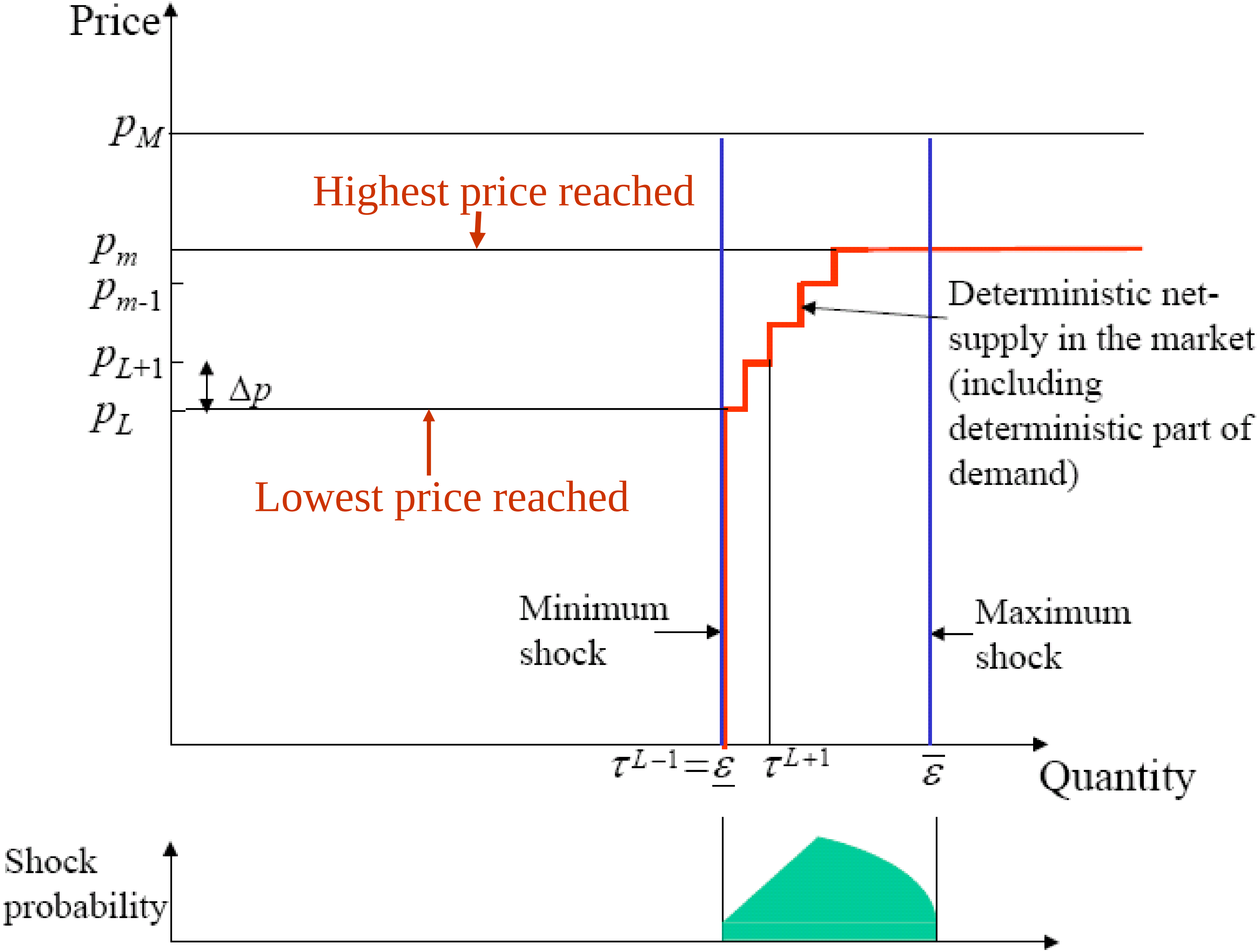


- Our model
  - *finite* set of prices: $p \in \{p_1, p_2, \ldots, p_M\}$
  - goods are *divisible*: $s_i(p) \in [0, \bar{q}]$
- offers below MCP accepted, at the MCP in proportion to offers at the MCP

=> pure-strategy equilibrium exists

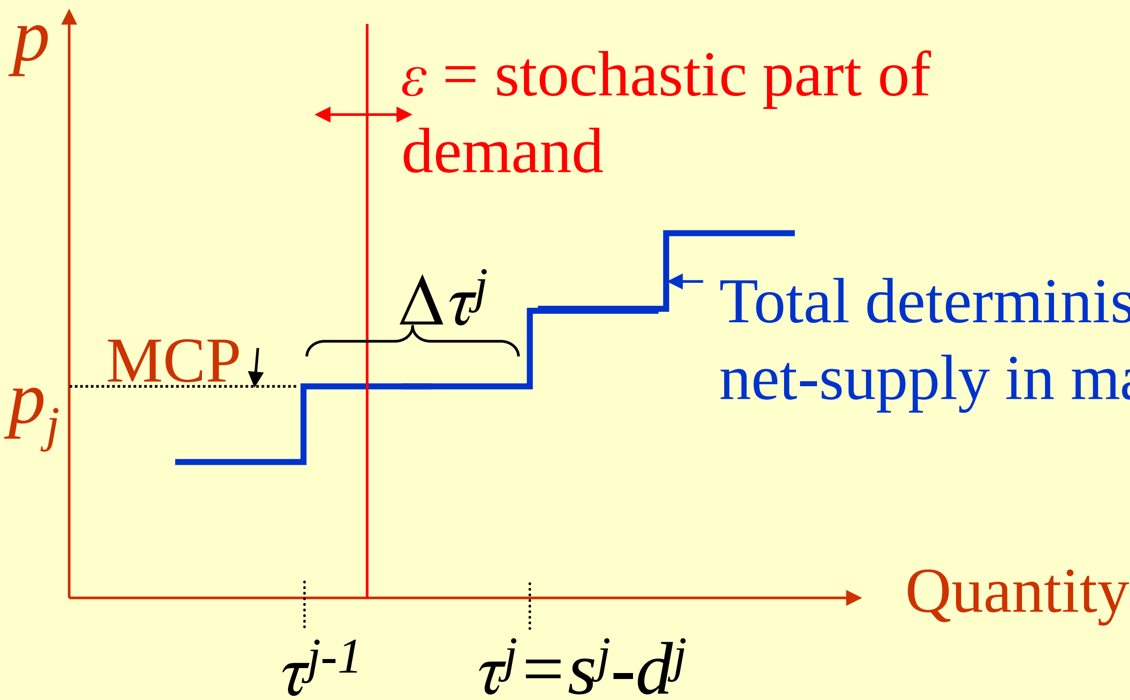
- converges to continuous SFE as $M \rightarrow \infty$

cannot marginally undercut rival



Price
p_M
Highest price reached
p_m
p_{m-1}
p_{L+1}
Δp
p_L
Deterministic net-supply in the market (including deterministic part of demand)
Lowest price reached
Minimum shock
Maximum shock
$\tau^{L-1} = \underline{\varepsilon}$
τ^{L+1}
$\overline{\varepsilon}$
Quantity
Shock probability

The expected profit of firm *i*



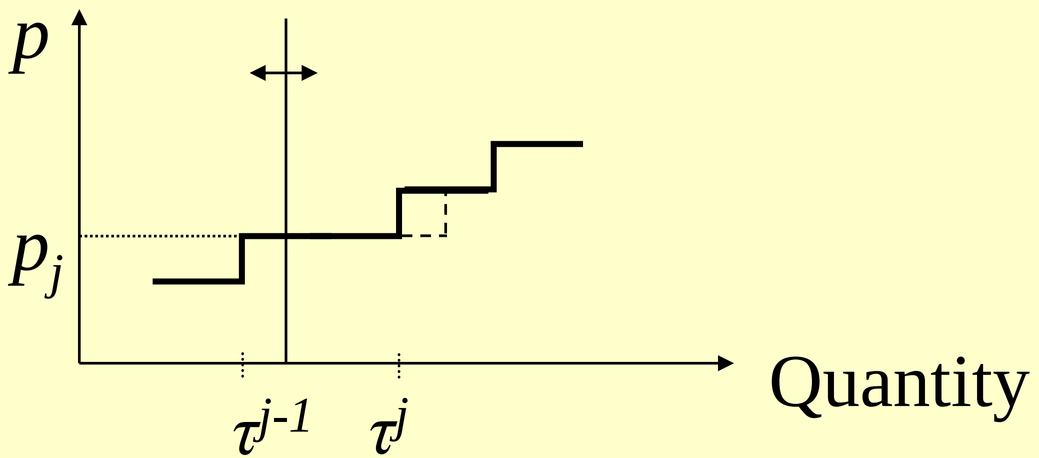
Market clearing => price is p_j if $\varepsilon \in (\tau^{j-1}, \tau^j]$.

Accepted supply of producer *i* if $\varepsilon \in (\tau^{j-1}, \tau^j]$: $s_i = s_i^{j-1} + \frac{\Delta s_i^j (\varepsilon - \tau^{j-1})}{\Delta \tau^j}$.

Expected profit of producer *i*:

$$E(\pi_i) = \sum_{i=1}^{m} \int_{\tau^{j-1}}^{\tau^j} \left[p_j \left(s_i^{j-1} + \frac{\Delta s_i^j (\varepsilon - \tau^{j-1})}{\Delta \tau^j} \right) - C_i \left(s_i^{j-1} + \frac{\Delta s_i^j (\varepsilon - \tau^{j-1})}{\Delta \tau^j} \right) \right] g(\varepsilon)\, d\varepsilon$$

First-order condition: discrete prices



Neg. contribution from $\varepsilon=\tau^j$

Pos. contribution from $\tau^{j-1}\leq\varepsilon<\tau^j$

$$\frac{\partial \mathrm{E}\left(\pi_i(s)\right)}{\partial s_i^j} = -\Delta p s_i^j g\left(\tau^j\right) + \int_{\tau^{j-1}}^{\tau^j} \left[p_j - C_i'\left(s_i(\varepsilon)\right)\right]\left(\frac{\Delta\tau_{-i}^j\left(\varepsilon-\tau^{j-1}\right)}{\left(\Delta\tau^j\right)^2}\right) g(\varepsilon)\, d\varepsilon \; +$$

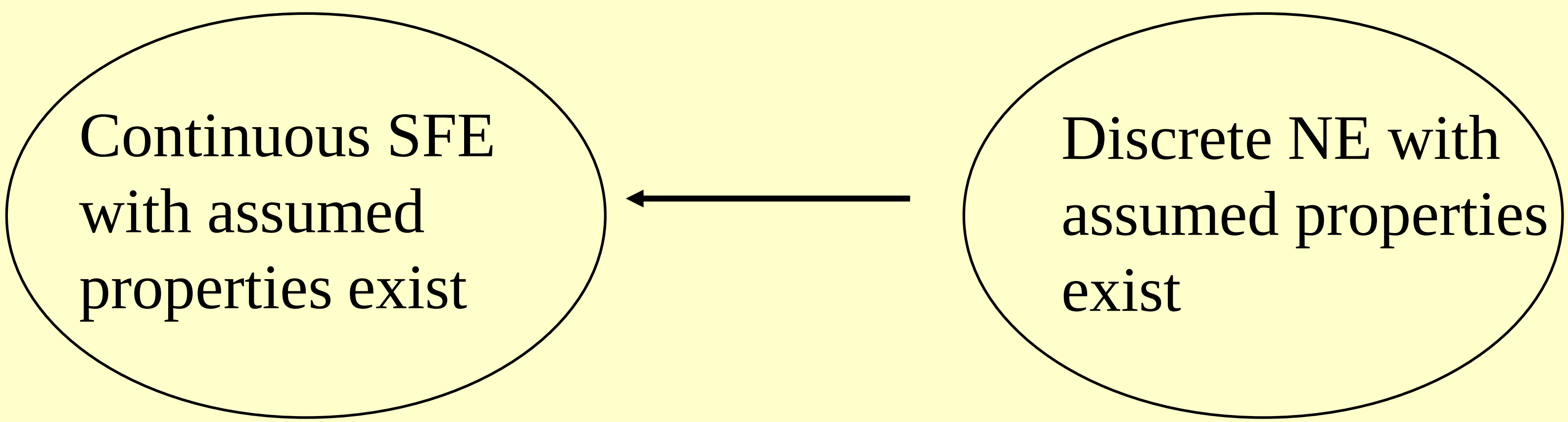
$$+ \int_{\tau^j}^{\tau^{j+1}} \left[p_{j+1} - C_i'\left(s_i(\varepsilon)\right)\right]\Delta\tau_{-i}^{j+1}\left(\frac{\Delta\tau^{j+1} - \left(\varepsilon-\tau^j\right)}{\left(\Delta\tau^{j+1}\right)^2}\right) g(\varepsilon)\, d\varepsilon = 0.$$

Pos. contribution from $\tau^j<\varepsilon\leq\tau^{j+1}$

Convergence of discrete NE to conts. SFE

Assumptions: Concave demand, fine enough price grid

• Consider equilibria, such that supply functions are bounded, increasing and have positive mark-ups for all realized prices.

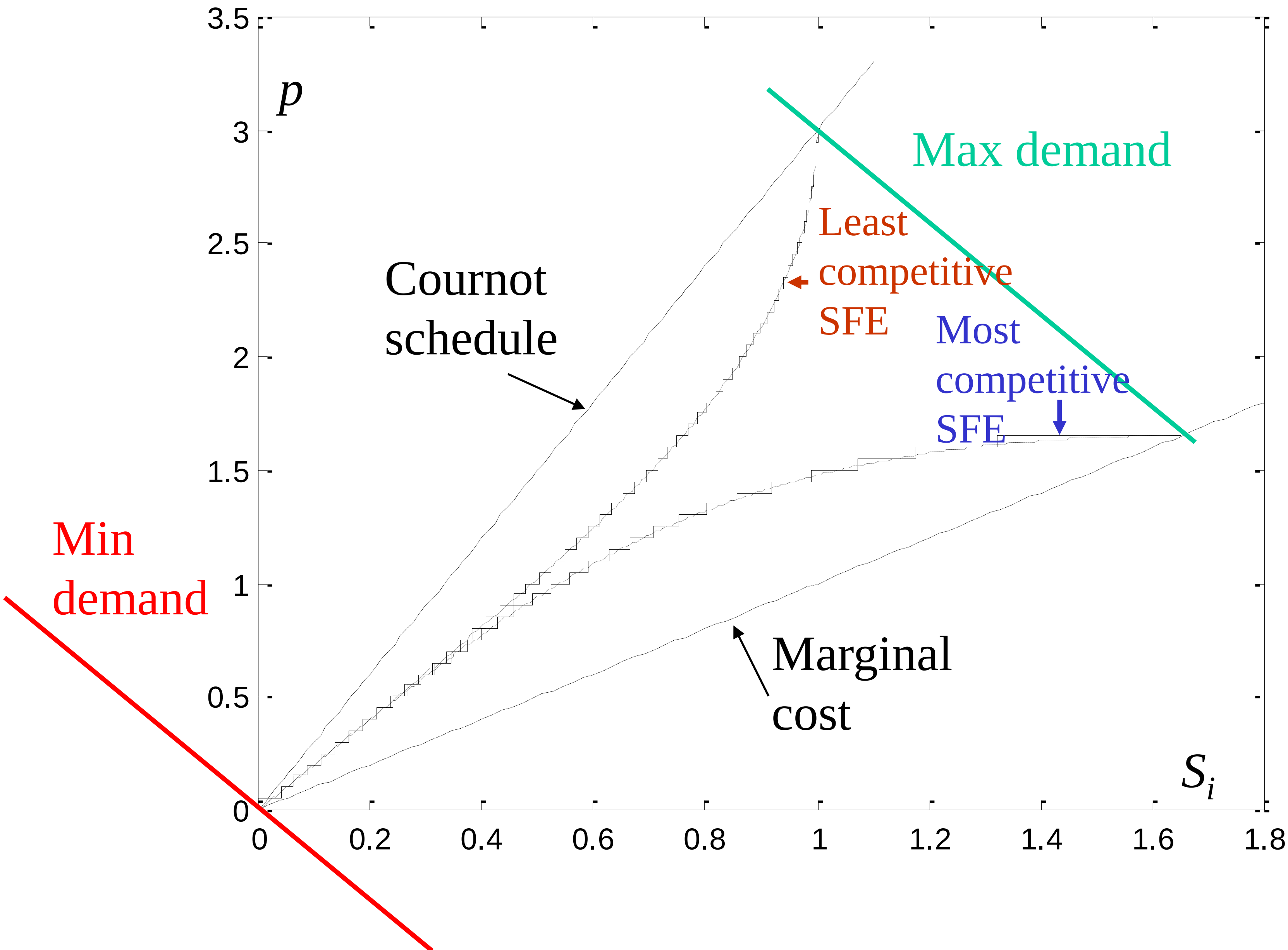


In the limit, as the price grid gets finer, discrete NE converges to continuous SFE

Outline of convergence proof

1. Solutions of difference eqns (ΔE) are consistent with f.o.c's of continuous SF's (CSF's)
   - if bounded and non-decreasing
2. Discrete solution exists and is stable
   - based on LeVeque
3. As number of price steps $M \to \infty$ the solutions to the ΔE's converge to the CSFE
4. Non-decreasing solutions to ΔE are NE
5. Increasing solutions to DE's are NE

Example



3.5
3
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
p
Max demand
Least competitive SFE
Cournot schedule
Most competitive SFE
Min demand
Marginal cost
S_i

Conclusions

- Convergence of stepped SFs to CSFE depends on nature of discreteness
- Price stability depends on market design: <= continuous payoff functions
  - piecewise linear offers (Nord Pool)
  - require large Δp, allow small Δq
- **Conjecture**: mixed strategy equilibria converge to CSFE as number of price steps increases, Δp falls
- Discrete solutions (which depend on pdfs) avoids need to smooth residual demand, and may improve empirical work (and solving CDEs)

UNIVERSITY OF CAMBRIDGE | Electricity Policy Research Group

E·S·R·C ECONOMIC & SOCIAL RESEARCH COUNCIL

Supply Function Equilibria: Step Functions and Continuous Representations

Pär Holmberg, **David Newbery** and Daniel Ralph

OR 50

9-11 September 2008, York

http://www.electricitypolicy.org.uk