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Abstract How can the electricity system operator determine the optimal quantity and quality of electricity ancillary services (such as frequency response) to procure in a market increasingly characterized by intermittent renewable electricity generation? The paper presents a system operator's utility function to calculate the exchange rates in monetary values between different frequency response products in the electricity system. We then use the utility function in a two-sided Vickrey-Clarke-Groves (VCG) mechanism combined of two frequency response products – enhanced and primary – in the context of the system in Great Britain. This mechanism would allow the market to reveal to the system operator the welfare optimal mix of speed of frequency response and quantity to procure. We show that this mechanism is the efficient way to support new faster sources of frequency response, such as could be provided by grid scale batteries.

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A system operator's utility function for the frequency response market

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ABSTRACT

How can the electricity system operator determine the optimal quantity and quality of electricity ancillary services (such as frequency response) to procure in a market increasingly characterized by intermittent renewable electricity generation? The paper presents a system operator's utility function to calculate the exchange rates in monetary values between different frequency response products in the electricity system. We then use the utility function in a two-sided Vickrey-Clarke-Groves (VCG) mechanism combined of two frequency response products – enhanced and primary – in the context of the system in Great Britain. This mechanism would allow the market to reveal to the system operator the welfare optimal mix of speed of frequency response and quantity to procure. We show that this mechanism is the efficient way to support new faster sources of frequency response, such as could be provided by grid scale batteries.

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1. Introduction

The electricity system is facing a significant challenge of renewables integration. Current EU28 targets for decarbonisation by 2030 would seem to imply renewable electricity shares of 55%+ by 2030, with massive reductions in the share of fossil fuels on the electricity system (to below 20%).[2] Ambitious reductions in carbon emissions will result in a large scale deployment of wind and solar generation in particular. The integration of a large share of

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[2] See Newbery et al. (2017).

renewable generation will increase the system requirements for various types of ancillary services to maintain the stability of the electricity system.

The rise of renewables, combined with the decline of conventional fossil fuel generation, on the electricity system increases the requirement for new sources of ancillary services, such as frequency response, to maintain the reliability and security of the grid (by matching of supply and demand in real time at appropriate levels of power quality). Such ancillary services provide 'flexibility' to the system to respond to the intermittent weather conditions affecting the amount of renewable energy that is instanteously available (see Strbac et al., 2016). Electrical energy storage facilities, such as lithium ion batteries or compressed air storage (hereafter denoted as suppliers), can provide the necessary ancillary services.

National Grid (NG) is the system operator (SO) in Great Britain (GB) and therefore, it is responsible for the reliability and security of the grid. It procures ancillary services for use in the everyday operation of the grid, such as reserve and frequency response, to avoid blackouts, interruptions and to manage peak demand. For instance, to deal with sudden generation loss, NG uses services like primary frequency response (PFR), which requires a supplier to deliver response in 10 seconds (10s), and the newly designed enhanced frequency response (EFR), which is a service to deliver in 1 second (1s).

To ensure the reliability and efficiency of the system, the SO needs the right services to be delivered. However, what are the right services? A SO might have chosen to work with a 1s service, 10s service and/or 1 hour service. Why not a 0.5s service or a guaranteed delivery in 13s? Current services may have been of interest to the majority of suppliers, maybe the bigger ones, maybe they were the right services some years ago. As we are in a world of asymmetric information between the SO and the market, the selling mechanism needs to be able to test the market for the right services at each allocation procedure.

Just as the suppliers want to maximise their own outcomes, a SO wants to balance the system. The current market design does not allow the SO to express complex and consistent preferences to balance the system. What the SO needs is an opportunity to express preference/willingness to pay (WTP). One way is to allow it to submit a utility function for the market to react on.

This paper presents a utility function of a SO. The function was created with GB in mind, but the idea can be applied to all countries. It connects two frequency response services (PFR and EFR) into one function and places monetary values on each of the services. It is built up in different scenarios, including different levels of demand and inertia. To simplify the analysis, we show that the exchange rate between PFR and EFR is 1.3 if demand is 60

GW, inertia is 4s and we expect to have 500 MW of EFR available in the system. In other words, if the value of 1 MW/h of PFR is £8.2, then the price of 1 MW/h of EFR is 1.3 times greater than £8.2, hence, £10.7. In another case if demand is 40 GW, inertia is 3s and we expect to have 500 MW of EFR available, the exchange rate is 3.5.

We then show how the utility function can be applied in a two-sided Vickrey-Clarke-Groves (VCG) mechanism to sell ancillary services. The VCG has two important properties needed when discussing the market for ancillary services. First, the most convincing part of the literature defines the security and reliability of the electricity grid as a public good (e.g. see Kiesling and Giberson, 1997; Schulze et al., 2008). The selling mechanisms for ancillary services used today are not based on social welfare. Second, a step forward to ensuring the welfare optimum is allocatively efficient (hereafter efficiency), i.e. licencses to deliver electricity end in the hands of those who value them the most. Current designs do not by themselves deliver efficiency. Without these two properties, the desired welfare optimum may not be achieved. The design in this paper also determines the optimal speed of frequency response by allowing the SO to express complex and consistent preferences over the different frequency response services it is simultaneously procuring (following Greve and Pollitt, 2016). This is the truly novel part of our design and involves the SO defining its utility function with respect to ancillary services products.

There is a large literature on energy storage and how to integrate storage into the system, e.g. see Sioshansi (2010), Newbery (2016) and Ruz and Pollitt (2016). Even classic mechanism design has recently been discussed for energy storage. For example, Keles et al. (2016) study energy and incentives to invest in renewables. Zou et al. (2015) examine a mechanism design in the energy market. In contrast to Zou et al. (2015), our design is a modified VCG mechanism built up to test the market for different responses, where a SO is part of the bidding process. We test the market for the optimal responses.

The papers closest to ours are Teng and Strbac (2016) and Greve and Pollitt (2016).[3] Teng and Strbac (2016) present a methodology to connect the different types of frequency responses. The function they present is developed for the GB power system. However, a similarly calibrated function can work in other systems. We extend the methodology to create a function that delivers both the exchange rates in monetary values on PFR and EFR and the implied quantities. The methodology can, in theory, be extended to include all frequency response services.[4] The function gives a SO's values of at least two services, which we apply using the VCG mechanism presented by Greve and Pollitt (2016). The mechanism

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[3] See also Greve (2017).

[4] In GB this would also include Secondary Frequency Response.

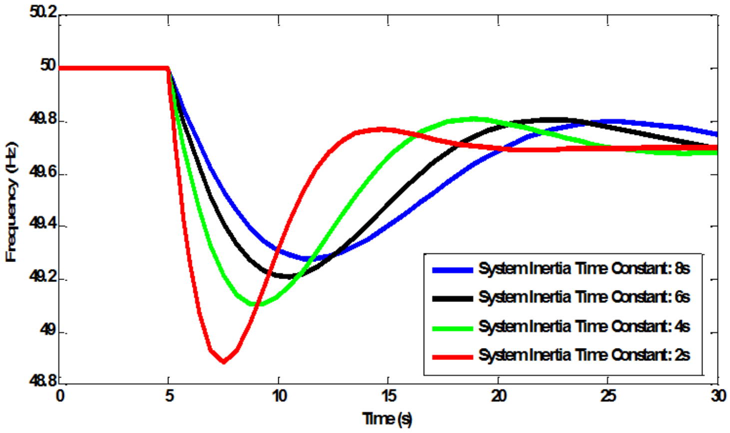
of Greve and Pollitt (2016) is extended from the area of energy to create a market for ancillary services products that can be supplied by electrical energy storage.

The rest of the paper is organised as follows: Section 2 discusses the need for further flexibility with the increased use of renewables. Section 3 presents a SO's utility function. Section 4 applies the utility function to be used in our two-sided VCG mechanism. Section 5 discusses the practical implications of our mechanism for frequency services procurement. Section 6 concludes.

2. **Renewables and the system**

The increased use of renewables will impose challenges on system balancing (or the instantenous matching of supply and demand within narrow frequency bounds). If the SO cannot maintain the system frequency within prescribed limits, this may even compromise the system's ability to integrate renewables. Increasing renewable generation share on the system makes maintaining system frequency more difficult because conventional turbines within fossil fuel power plants provide rotating mass which can easily dampen instanteous fluctuations in system frequency. However, as wind and solar generation displaces conventional plants, the system inertia provided by the rotating mass reduces, which leads to faster decline of system frequency in the events of generation failures.

Figure 1: Impact of reduced inertia on the system frequency response to a large generation loss in GB (Teng et al., 2017)



As shown in Figure 1, the rate of change of frequency (RoCoF) in response to a major system fault event (such as the sudden loss of a large conventional power plant) will increase as system inertia falls, potentially causing disconnections of distributed generators by actuating RoCoF-sensitive protection schemes. RoCoF relay protection actually has been found to be a main limitation to achieve high penetration of non-synchronous generation in Ireland. Moreover, if frequency drops rapidly, conventional generators may not be fast enough to provide frequency response; the resulting frequency nadir could activate the costly under frequency load shedding. Higher shares of renewables on the system combined with lower system demand reduces the system inertia and requires the SO to have contracts in place for combinations of faster and higher quanties of frequency response services. In response to the challenge of lower system inertia, NG has introduced a new faster frequency response product in April 2016 – EFR – in addition to its existing products (PFR and other response products) to cope with need for flexibility to support the integration of renewable energy in order to maintain grid security and reliability.

3. A SO's utility function in the GB case

This section develops constraints to ensure a minimum level of the post-fault frequency above the pre-defined limits considering inertia, EFR and PFR.

In a power system with multiple generators, by assuming that all generators swing synchronously at a common frequency, an approximation to the system post-fault frequency evolution can be obtained as an equivalent single-machine swing equation (Kundur et al., 1994):

$$2\mathrm{H} * \frac{\partial \Delta \mathrm{f}(\mathrm{t})}{\partial \mathrm{t}} = \Delta \mathrm{R}(\mathrm{t}) - \Delta \mathrm{P}_{\mathrm{L}} \tag{1}$$

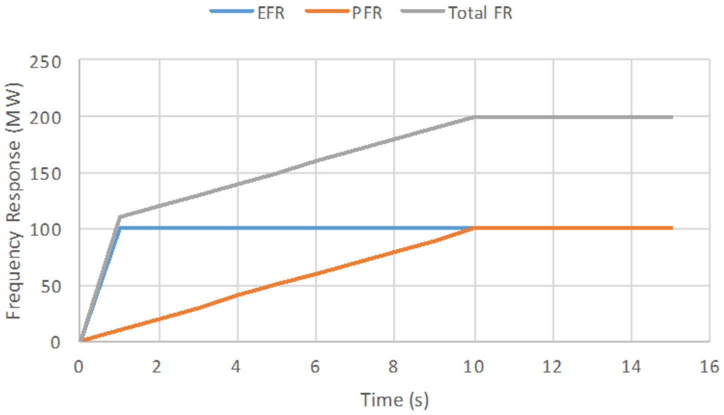
where $\Delta \mathrm{f}$ is the frequency deviation, t is time after fault happens, H [MWs/Hz] is the inertia from thermal plants and load, and ΔR [MW] describes the additional power delivered as frequency response after the generation loss $\Delta \mathrm{P}_{\mathrm{L}}$ [MW].

The system is assumed to be at nominal frequency (f_0) in the normal operation state. After a sudden generation loss, the inherent physical characteristic of the rotating machines is to draw on the stored kinetic energy to restore the balance between generation and load, referred as inertia response. The level of system inertia can be calculated as:

$$\mathrm{H} = \frac{\sum_{\mathrm{g} \in \mathcal{G}} \mathrm{H}_{\mathrm{g}} * \mathrm{P}_{\mathrm{g}}^{\mathrm{max}} * \mathrm{N}_{\mathrm{g}}^{\mathrm{up}} + \mathrm{H}_{\mathrm{d}} * \mathrm{P}^{\mathrm{D}}}{\mathrm{f}_0} \tag{2}$$

where $\mathrm{H_g}$ / $\mathrm{H_d}$ is inertia time constant for generator group g or load and $\mathrm{P^D}$ is the load level.

Figure 2: Delivery of frequency response



At the same time, the scheduled EFR and PFR is activated through governor response. As shown in (3), this paper applies a conservative assumption that frequency response is delivered by linearly increasing the active power with a fixed slope $(\frac{EFR}{\mathrm{T_{EFR}}}, \frac{PFR}{\mathrm{T_{PFR}}})$ until the required delivery time ($\mathrm{T_{EFR}}$, $\mathrm{T_{PFR}}$) and then keeps constant. Different delivery speeds of frequency response are demonstrated in Figure 2.

$$\Delta\mathrm{R(t)} = \begin{cases} (\frac{EF\mathrm{R}}{\mathrm{T_{EFR}}} + \frac{PF\mathrm{R}}{\mathrm{T_{PFR}}}) * \mathrm{t} & \text{if } \mathrm{T_{EFR}} \geq \mathrm{t} \geq 0 \\ EFR + \frac{PFR}{T_{PFR}} * \mathrm{t} & \text{if } \mathrm{T_{PFR}} \geq \mathrm{t} \geq \mathrm{T_{EFR}} \\ EFR + PFR & \mathrm{t} \geq \mathrm{T_{PFR}} \end{cases} \tag{3}$$

The combined provision of inertia response, EFR and PFR needs to limit the frequency within a pre-specified range ($|\Delta f_{min}|$). Otherwise, under frequency load shedding will be triggered to prevent the system from a wider blackout. By solving (1), (2) and (3) through similar procedures as in Teng and Strbac (2016), the following constraint can be developed to maintain frequency response adequacy.

$$(H - \frac{EFR * T_{EFR}}{4 * |\Delta f_{min}|}) * R \geq \frac{(\Delta P_L - EFR)^2 * T_{PFR}}{4 * |\Delta f_{min}|} \tag{4}$$

The key challenge there is how to procure the optimal portfolio of EFR and PFR, as both of the services contribute to the same goal. By using the constraint (4) developed above, Figure 3 and Figure 4 demonstrate how much more a SO should be willing to pay for EFR over PFR under different system conditions. It is clear that the willingness to pay (WTP) for EFR over PFR varies dynamically, depending on system demand, system inertia level and the amount of EFR that is expected to be procured. These exchange rates define the SO's utility function for frequency response products, by allowing any given pair of quantities of EFR and PFR (e.g. 1MW of EFR and 2MW of PFR) to be evaluated against any other pair of quantities of EFR and PFR.

Figure 3: WTP for EFR under 40GW system demand

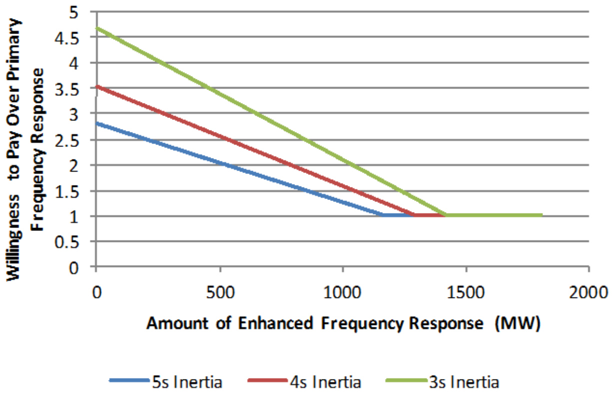
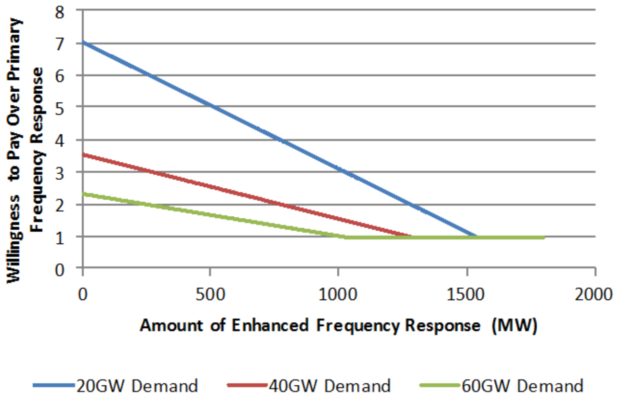


Figure 4: WTP for EFR under 4s system inertia



Using Figure 3, we can see that the exchange rate is 3.5 if demand is 40 GW, inertia is 3s and 500 MW of EFR is expected to be available. Using Figure 4, the exchange rate is 1.3 if demand is 60 GW, inertia is 4s and 500 MW of EFR is expected to be available. Assuming PFR is competitively available in sufficient quantities, the exchange rates place limits on the WTP for EFR. Assuming the system is at 60 GW, 4s and 500 MW of EFR is expected to be available and suppose the price of 1 MW of PFR is 8.2 £/hour, we have the following two set-points for the SO's revealed preferences: (1 MW of EFR, 1 MW of PFR) = (£10.7, £8.2).[5] This vector will be used in the next section.

4. A new mechanism for frequency response products

This section introduces a VCG mechanism for frequency response services. Compared to current design, the SO is part of the allocation process by submitting a 'complex bid', i.e. a utility function. Hence, it is now given a tool to better manage the network.

The presented VCG mechanism can overcome other problems of the current market. Today, there is no simultaneous clearing process across auctions/between different ancillary services. Further, today's procedures do not allow for package bidding - either a supplier (e.g. an electrical energy storage facility) gets its desired services or it does not. A supplier wants to supply electricity services to maximise profit or at least minimise cost. By allowing for a simultaneous clearing process and package bidding, a supplier can choose its optimal combination of services to submit a bid in one clearing process. There might be large gains from cost synergies and therefore lower consumer prices. Our design allows bidders to submit bids on all the services covered by the mechanism at the same time. It also allows for package bidding, i.e. combinations of different products bid at single price.

The presented design is a VCG mechanism, which has the desired welfare elements as part of the design, given by the efficiency. The design delivers the optimal length of response times via vector bidding. The following examples illustrate how the utility function can be applied in a selling mechanism. For illustration, see each 1 MW to be sold as one license. In the examples we specify only two potential response times 1s and 10s (roughly corresponding to EFR and PFR). However the mechanism could test multiple potential response times, as long as the SO can identify exchange rates over these (and they can be usefully technologically distinguished). As the SO's exchange rates vary according to demand

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[5] £8.2*1.3 = £10.7.

and inertia in real time, we might imagine our design as representing the mechanism to determine prices in a spot (half-hourly or hourly) market for frequency response.

Example 1: a licence to deliver 1 MW of frequency response

Consider a SO that wants to procure frequency response from a supplier. A licence of 1 MW of frequency response is for sale. In principle, the 1 MW could be delivered at any potential response time, but for simplicity suppose the SO offers the market the opportunity to deliver the 1 MW via the response times: 1s or 10s. We have two suppliers – Suppliers 1 and 2. Table 1 shows the submitted bids, where, for example, Supplier 1 has submitted a bid of £6.8 if the response is 1s and £6 if it is 10s, or written as (6.8, 6). As faster response is higher quality, we can say that the bidders' costs are decreasing with the response time, shown by Greve (2017).

Table 1: Submitted bids, 1 MW

Supplier	Response time (s)	Bids (£/hour)
1	1	6.8
	10	6
2	1	6.5
	10	6.1

The VCG chooses the winner and the response time to be delivered, where social value is maximised. We calculate the social value by taking the differences between the SO's values of the different responses and the submitted bids, hence, we have a social value per response time. The SO's values can be derived from an appropriate scaling of the exchange rate utility function discussed in section 3. In our example we anchor the WTP for slower response (10s) at 8.2 and apply the 1.3 exchange rate of EFR for PFR to get a WTP of 10.7 for the faster response (1s). In reality the anchor figure of 8.2 can be thought of as a bid cap (or maximum WTP) on the basic service.

Table 2: Welfare effect

Response time (s)	SO (£/hour)	Supplier 1 (£/hour)	Supplier 2 (£/hour)	Welfare effect (£/hour)
1	10.7	6.8	<u>6.5</u>	**+4.2**
10	8.2	<u>6</u>	6.1	+2.2

Table 2 shows that Supplier 2 has submitted lowest price if the response time is 1s. Supplier 1 has submitted lowest prices if it is 10s. Further, the table shows that social welfare is maximised if Supplier 2 is asked to deliver 1 MW with a response of 1s (+£4.2).

To determine the price that Supplier 2 can charge the consumers, the VCG evaluates again the welfare effect, but this time without the winning supplier, hence, without Supplier 2.

Table 3: Welfare effect, without the winner

Response time (s)	SO (£/hour)	Supplier 1 (£/hour)	Welfare effect (£/hour)
1	10.7	6.8	**+3.9**
10	8.2	6	+2.2

Table 3 shows that, in the absence of Supplier 2, social value is maximised if Supplier 1 is “asked to deliver” 1 MW with a response of 1s (+£3.9). Using the VCG, Supplier 2 therefore wins the 1 MW-licence with a response of 1s and should be paid £6.8[6] by customers for this 1 MW of frequency response.

Example 2A: Up to two licences of 1 MW each, package bidding is an opportunity

Now, suppose that the SO wants to procure up to two licences of 1 MW each. Compared to Example 1, we now also allow for package bidding. The bids for single units are as in Table 1. Table 4 shows the package bids. For simplicity, suppose that the suppliers are only interested in the packages 1s and 1s, 1s and 10s and 10s and 10s.

Table 4: Submitted bids for packages of licences

Supplier	Response time (s)	Bids (£/hour)
1	1 and 1	11.8
	1 and 10	9.9
	10 and 10	8.9
2	1 and 1	10.7
	1 and 10	9.6
	10 and 10	9.3

Besides the individual bids, Table 5 shows the welfare effect of the submitted package bids. It shows that social welfare is maximised if Supplier 2 is asked to deliver the package of

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[6] -1*(£10.7-£6.8)+£10.7=£6.8.

2 MW-licences of 1s (+£9.4), which is higher than the sum of all possible combinations of individual licences.

Table 5: Welfare effect

Response time (s)	SO (£/MW)	Supplier 1 (£/hour)	Supplier 2 (£/hour)	Welfare effect (£/hour)
1 and 1	20.1	11.8	10.7	**+9.4**
1 and 10	17.9	9.9	9.6	+8.3
10 and 10	15.9	8.9	9.3	+7

Again, to determine the price that Supplier 2 can charge the consumers, the VCG evaluates the welfare effect, but without Supplier 2.

Table 6: Welfare effect, without the winner

Response time (s)	SO (£/hour)	Supplier 1 (£/hour)	Welfare effect (£/hour)
1 and 1	20.1	11.8	**+8.3**
1 and 10	17.9	9.9	+8
10 and 10	15.9	8.9	+7

Table 6 shows that, in the absence of Supplier 2, social value is maximised if Supplier 1 is "asked to deliver" the package 1s and 1s (+£8.3). Using the VCG, Supplier 2 therefore wins the package of 2 MW-licences of 1s and is paid £11.8[7] for these 2 MWs of frequency response.

Example 2B: The package 10s and 10s is the winning licence to deliver

The outcome of the allocation procedure depends on the welfare effect. For example, suppose Supplier 1 submitted a bid of £6.4 on the package 10s and 10s instead of the £8.9 and Supplier 2 a bid of £7.5 instead of the £9.3. Other things being equal, we have the following tables.

Table 7: Welfare effect

Response time (s)	SO (£/hour)	Supplier 1 (£/hour)	Supplier 2 (£/hour)	Welfare effect (£/hour)
1 and 1	20.1	11.8	10.7	+9.4
1 and 10	17.9	9.9	9.6	+8.3
10 and 10	15.9	6.4	7.5	**+9.5**

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[7] -1*(£20.1-£11.8)+£20.1=£11.8.

Table 7 shows that the VCG now chooses Supplier 1 as the winner of the allocation procedure to deliver the package 10s and 10s. As before, the VCG evaluates the welfare effect, but without Supplier 1.

Table 8: Welfare effect, without the winner

Response time (s)	SO (£/hour)	Supplier 2 (£/hour)	Welfare effect (£/hour)
1 and 1	20.1	10.7	+8.3
1 and 10	17.9	9.6	+8
10 and 10	15.9	7.5	**+8.4**

Table 8 shows that Supplier 1 can charge a price of £7.5[8] for the winning package. This shows that even though the faster response is more valuable to the SO, the 10s response is sufficiently cheap as to maximise social welfare by being selected in the allocation process.

Example 3A: The value of procuring 1s instead of 10s

The SO can also indicate with its WTPs what packages it prefers. A real-life example could be the tradeoff between procuring a package made up of 1s services instead of a package made up of 10s services. A SO may be prepared to pay more for 1s because it allows multiples of savings in contracting 10s. Our Figures 3 and 4 show this. In other words, if it procures, for example, 2MW of 1s it might be able to save 3MW of 10s. The following example shows and emphasizes how the SO can signal these potential savings via its WTPs. Suppose we have the same two suppliers which can supply up to 3 MWs of response services. At the same time, the SO has published its utility function showing that it is ready to buy up to 3 MW. For simplicity, suppose that the bidders only put in bids on the packages: 3*1s (1s and 1s and 1s), 3*10s (10s and 10s and 10s), 2*1s (1s and 1s) and 2*10s (10s and 10s). Table 9 shows the submitted bids.

Table 9: Submitted bids for packages of licences

Supplier	Response time (s)	Bids (£/hour)
1	1 and 1 and 1	17.2

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[8] -1*(£15.9-£7.5)+£15.9=£7.5.

	10 and 10 and 10	12.3
	1 and 1	11.8
	10 and 10	8.9
2	1 and 1 and 1	17.0
	10 and 10 and 10	11.9
	1 and 1	10.7
	10 and 10	9.3

Table 10 shows how the SO signals its preferences. The table shows that SO is prepared to pay almost the same for 1s and 1s (£20.1) as for 10s and 10s and 10s (£20.5).

Table 10: Welfare effect

Response time (s)	SO (£/hour)	Supplier 1 (£/hour)	Supplier 2 (£/hour)	Welfare effect (£/hour)
1 and 1 and 1	25.2	17.2	17.0	+8.2
10 and 10 and 10	20.5	12.3	11.9	+8.6
1 and 1	20.1	11.8	10.7	**+9.4**
10 and 10	15.9	8.9	9.3	+7

Using the same technique as in previous examples, Supplier 2 is determined as the winner of the allocation procedure to deliver the package 1s and 1s (+9.4).

Table 11: Welfare effect, without the winner

Response time (s)	SO (£/hour)	Supplier 1 (£/hour)	Welfare effect (£/hour)
1 and 1 and 1	25.2	17.2	+8
10 and 10 and 10	20.5	12.3	+8.2
1 and 1	20.1	11.8	**+8.3**
10 and 10	15.9	8.9	+7

Table 11 shows that Supplier 2 can charge the consumer £11.8[9] for these 2 MWs of electricity.

Example 3B: The value of procuring 10s instead of 1s

Now take package 10s and 10s and 10s and lower the bids, so that Supplier 1 submits a bid of £11.7 and Supplier 2 a bid of £11. We have the following tables.

Table 11: Welfare effect

Response time	SO	Supplier 1	Supplier 2	Welfare effect

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[9] -1*(£20.1-£11.8)+£20.1=£11.8.

(s)	(£/hour)	(£/hour)	(£/hour)	(£/hour)
1 and 1 and 1	25.2	17.2	17.0	+8.2
10 and 10 and 10	20.5	11.7	11.0	**+9.5**
1 and 1	20.1	11.8	10.7	+9.4
10 and 10	15.9	8.9	9.3	+7

The VCG determines Supplier 2 as the winner of the allocation procedure, but to deliver the package 10s and 10s and 10s (+9.5).

Table 12: Welfare effect, without the winner

Response time (s)	SO (£/hour)	Supplier 1 (£/hour)	Welfare effect (£/hour)
1 and 1 and 1	25.2	17.2	+8
10 and 10 and 10	20.5	11.7	**+8.8**
1 and 1	20.1	11.8	+8.3
10 and 10	15.9	8.9	+7

The payment that Supplier 2 can charge the consumer is now £11.7[10], which is lower than the payment if it was to deliver the package 1s and 1s (given in Example 3A). This illustrates that as long as the bids for the slower frequency response time are sufficiently low the SO may prefer more slower response to less faster response. Or conversely, as long as the bid premium on faster frequency response is low enough the SO would prefer less faster response to more slower response. By specifying WTP for several different response times the SO can test the market for faster response and see whether faster response is cost effective. It would indeed be possible in theory to test whether combinations of 0.5s or 20s responses did maximise social welfare relative to 1s or 10s.

5. Contracting in practice

Our examples illustrate a process in which only one frequency response supplier is required to satisfy the SO’s demand for frequency response products. We did this in order to illustrate how a mechanism can consistently select the optimal frequency response time. In reality the market for frequency response products (in a market the size of Great Britain) will need to select multiple winning bidders in order to provide system security and reliability. In principle, this can be done by using a similar VCG mechanism in a multi-unit/object auction

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[10] -1*(£20.5-£11.7)+£20.5=£11.7.

setting[11] which allows for both package bidding on the part of individual bidders and multiple winning bids.

As of mid 2017, the different ancillary services are procured for different and longer time intervals, e.g. PFR is procured monthly, where EFR was tendered for the first in June 2016 (and has not being re-procured). Under the current procurement exercises each supplier indicates time interval over which it can deliver. For example, a supplier can indicate that it can deliver the submitted MWs from 6am to 11pm on working days or from 7am to 11pm on Saturdays and Sundays. The services are contracted in MW, but the payment given to a winning supplier depends on the payment structure of the different services. For example, the main fees for the PFR are an availability fee (for being available to the system operator) and a nomination fee (for being called upon to deliver) (NG, 2015). The tenders are assessed on the capability of response services (e.g. EFR and/or PFR) and the hours (quantity og MWs) suggested/submitted (NG, 2015).We have envisioned a spot market for frequency response products, though the market could be for the supply of frequency response services under longer term contracts. Our design differs from the current design as the different services (i.e. EFR, PFR) are procured at the same time allowing for package bidding and a response time that can minimise a supplier's cost. Hence, in term's of implementation of the ideas in the current paper, the SO only has to decide the period over which to contract for frequency response services (say, one month), the modelled baseline PFR quantity (3MW in Example 3) and exchange rates (implied by the relative WTP for EFR and PFR) and the maximum WTP on baseline PFR. For a given anticipated demand and inertia profile there is an average exchange rate over the period between EFR and PFR and a required baseline PFR quantity and its associated maximum WTP. These rates vary within any given period, and hence there may be increased market efficiency from shorter contract periods (i.e. moving towards a spot market), which allows better matching of real time supplier availability (reflected in supplier bids) with real time system condition (reflected in the SO utility function). However longer term markets offer better risk hedging opportunities and are likely to be more competitive.

6. Conclusion

This paper has presented a method to calculate the exchange rates between different ancillary services. In theory, it allows the specification of a utility function for an SO. We

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[11] See Krishna (2009) for a discussion of multi-unit/object auctions or the mechanism presented by Greve (2017).

discussed it in the context of just two services: enhanced and primary frequency response (EFR and PFR). In general, it can be specified with respect to multiple packaged services over a range of quantities. We have applied the utility function in a two-sided VCG mechanism to show how our mechanism can let the market determine the type and quantity of very fast response that might be needed for ancillary services. This would allow the market, rather than the SO, to determine the prices and quantities of very fast frequency response to procure. This would allow the SO to market test the ability to deliver of new frequency response providers, such as electrical energy storage facilities. For future research it would be of interest to extend the SO utility function which we define to include more than two services.

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