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Unintended consequences: The snowball effect of energy communities

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Abstract

Following the development of decentralized generation and smart appliances, energy communities have become a phenomenon of increased interest. While the benefits of such communities have been discussed, there is increasing concern that inadequate grid tariffs may lead to excess adoption of such business models. Furthermore, snowball effects may be observed following the effects these communities have on grid tariffs. We show that restraining the study to a simple cost-benefit analysis is far from satisfactory. Therefore, we use the framework of cooperative game theory to take account of the ability of communities to share gains between members. The interaction between energy communities and the DSO then results in a non-cooperative equilibrium. We provide mathematical formulations and intuitions of such effects, and carry out realistic numerical applications where communities can invest jointly in solar panels and batteries. We show that such a snowball effect may be observed, but its magnitude and its welfare effects will depend on the grid tariff structure that is implemented, leading to possible PV over-investments.

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*The opinions expressed in this paper are those of the authors alone and might not represent the views of ENGIE

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1 Introduction and literature review

The US Community and Shared Solar initiatives provide four main vehicles to broaden the participation of individuals in the energy transition by giving access to an investment option into solar PV. Shared solar falls under the community solar umbrella, allowing multiple participants to benefit from a jointly produced energy. This US initiative finds its (more limited) equivalent in the ongoing European discussion about a new legislative package for the electricity market design, called the winter package. The winter package pushes for more customer empowerment in general but provides several new concepts that fall into the category of community solar (local energy community, renewable energy community). Most notably, it introduces the concepts of collective auto-consumption (Renewable Energy Directive, Article 21, the full article can be found in appendix G): *1. Member States shall ensure that renewable self-consumers (...) are entitled to carry out self-consumption and sell (...) production of renewable electricity (...) and receive a remuneration for the self-generated renewable electricity they feed into the grid which reflects the market value of the electricity fed in (...). Member States shall ensure that renewable self-consumers living in the same multiapartment block (...) are allowed to jointly engage in self-consumption as if they were an individual renewable self-consumer.*

Some member states have already implemented such geographically limited auto-consumption. The German Mieterstromgesetz allows residents in collective houses to share the generation of the PV panels on the common roof (on-site shared solar). If consumers form such a community they can avoid network charges on the self-consumed energy and they receive a return on the renewable energy surcharge (effectively a tax reduction) like a classical prosumer in a stand-alone building. In addition, if network charges contain fixed or capacity-based components, the community pays this component only once (fixed parts) or for the whole capacity of the community. The community then receives an aggregation benefit due to the mitigation of the overall consumption capacity.

In the described German case, the collective owner of the house essentially becomes the electricity provider of the energy community. Since retail markets are open to competition, residents cannot be forced to participate (or they might form more than one community). The first challenge is, hence, in allocating the benefit of collective PV in a way that leads to a stable coalition among the residents. The second challenge is in the determination of the origin of the benefit itself, which might lead to some externalities that have to be accounted for: not paying for the grid component of the auto-consumed energy creates value for the community but can lead to a revenue shortfall for the grid operators, most importantly the distribution companies. As Lucas Davis puts it, "Rooftop solar isn't getting rid of the utility. It's just changing who pays for it"[1]. Low-contributing consumers will result in higher tariffs and hence to more consumers investing in PV. A more in-depth description of this effect, which we call a snowball, can be found in section 2. The discussion so far has mainly concentrated on individual prosumers and on the fact that the presence of the grid provides a service (in terms of reliability). Proposed solutions include demand charges or fixed charges (see discussions in Oklahoma, North Western...).

For the German case of the energy community, the avoidance of fixed charges and to some extent demand charges is explicitly foreseen. As long as the grid cost is not paid by some form of general taxation, the rise of collective auto-consuming communities will not be stopped by fixed or variable volumetric charges. However, the choice between fixed or variable charges will affect the investment choices of the community.

[1]see https://energyathaas.wordpress.com/2018/03/26/why-am-i-paying-65-year-for-your-solar-panels/

In particular, this paper proposes three main insights. First, community formation can lead to a snowball effect through the medium of grid tariffs. This comes as a result of a very simple mechanism. When communities form, they often save on grid tariffs. A grid operator's costs, however, are largely unchanged. In turn the operator may need to modify its tariffs, so as to recover costs. As a consequence, prospective communities may also form. We then analyze more closely the precise structure of grid tariffs. A second insight is that per-connection fees is the structure most favourable to community formation, while capacity-based or energy-based tariffs lead to more inertia. It is also the structure that best avoids welfare-destructive efforts to reduce payments to the grid operator. This second insight is to be contrasted with the third one. Namely, capacity- and energy-based tariffs are the ones most effective in promoting investments in PV and batteries. From this we derive a simple policy implication: a policy maker willing to promote communities per-se should favour per-connection tariffs, with the risk of increasing coordination costs. If the focus is rather on PV or battery installations, capacity- or energy-based tariffs should be preferred. We however note that such tariffs may induce excessive investment in these technologies, from a social-welfare point of view.

This paper is closely related to [1]. In that paper, we developed a game theoretical approach so as to analyze the stability of energy communities sharing a PV panel with respect to grid tariffs. In particular, we showed that reducing the analysis of the emergence of communities to a cost-benefit analysis was unsatisfactory. In the very spirit of energy communities, our main contribution was to exclude the possibility that a social planner would impose a given structure and rules on the communities, with households instead creating coalitions on their own. We showed that adding these cooperative-game theoretical considerations complicated substantially the formation of communities and that a wise choice of a sharing rule was crucial for communities to materialize. However, the paper focused on gains sharing within a given neighborhood and ignored potential system effects of community formation. In the present paper we observe in particular that the cost savings of communities in terms of grid tariffs may translate into a loss for a DSO. As a consequence, we now impose a grid cost-recovery constraint that allows for potential spillovers among communities through the medium of grid tariffs. The primitives of the model and data sources are kept the same as in [1].

More broadly, the present paper is related to at least three strands of the literature. First, it is an application of the seminal literature on cooperative games ([53],[63]) to the energy sector. [39] analyzes the surplus sharing among liquefied natural gas (LNG) exporters and casts doubts on the credibility of a logistic cooperation that would be exempt from market power. Cooperative games have also been successfully applied to the allocation of CO2 emission rights ([32],[47]), the allocation of network costs among customers ([59], [16], [31], [30]) or optimal system planning [60]. However, the concept of energy communities has gained significant interest only recently, and to the best of our knowledge they have not yet been analyzed through the lens of cooperative game theory.

Second, this paper pertains to the literature on decentralized energy systems. Operational research on energy communities and micro-grids has been very active recently, showing an increased interest in these business models ([44], [5], [57], [40]), of which the benefits have been widely stressed, both theoretically ([36], [13], [12]) and empirically ([35] , [15]). However, the literature has so far restricted the analysis to the technically achievable benefits yielded by such communities, while very little research has been made to date on the actual viability of the community seen as a coalition. [37], [61], [62], [41] and [17] discuss how energy communities or micro-grids may be integrated in the existing system but, likewise, do not address whether these coalitions hold in practice. Close in spirit to our paper, [38] exposes how cooperative game theory may shed light on the desirability of micro-grids. Echoing our results, they find that the misalignment between private and social objectives can lead to inefficient deployment. Similarly, [34] discusses how the gains of decentralized trading can be shared between end-users and suppliers. However, [38] and [34] mainly focus on the allocation of gains between energy communities and other players of the energy system. Instead, the present paper focuses on the interaction between agents acting *within* the energy community and how this may impact other energy communities with whom it shares the grid.

Finally, we contribute to an already rich literature on grid tariffs. As electricity networks are characterized by high fixed costs, competitive pricing may not generate enough revenue to ensure grid cost recovery. In case government subsidies are unavailable, several options exist, including Ramsey pricing ([49], implemented in [6]). However, concerns over equity have led to the development of two-parts tariffs ([26]). The benefits of cost-reflective tariffs are well documented in the literature and are summarized in [28] or [11]. Within these discussions, most relevant to our analysis is the recent debate on the interaction between decentralized generation and grid tariffs. There is increasing agreement that volumetric tariffs are unsatisfying, and that a capacity component should be reinforced ([24]; [48], [45]). In particular, net metering is accused of unduly favoring decentralized generation, at the expense of consumers who cannot invest in renewable technologies ([18], [23], [46]). Our research suggests, however, that not only the volumetric but also the capacity components of the grid tariff may induce excessive efforts to reduce grid payments. This results in a prisoner's dilemma, while per-connection tariffs are more resilient. This finding echoes those of [9] who suggests that a combination of fixed charges and volumetric prices may result in a reasonable balance between efficiency and equity. Recently, some literature has underlined the possibility of a snowball effect that is similar to ours. [51] envisage several tariff structures and examine how individual decisions affect them and in turn the decisions of other individuals. They also show that capacity-based charges may lead to over-investment in new technologies. However, players still act as individuals and no cooperative game-theoretical considerations were made. In the case of energy communities, we believe these considerations are necessary.

To the best of our knowledge, the present paper is the first attempt to study the interaction between stable energy communities as mediated through grid tariffs, using an equilibrium formulation in a non-cooperative game theory framework. The inner stability of an energy community is properly addressed using cooperative game theory. The main contribution of this paper is then to model and solve the equilibrium between the DSO and energy communities while capturing stability considerations using cooperative game theory.

The rest of the paper is organized as follows. Section 2 shows in a simple setting how a snowball effect may arise and how game theoretical considerations may modify conditions for such phenomena to occur. Section 3 provides the formal setting of our analysis, which we apply numerically in section 4 on stylized communities. Section 5 concludes the paper. In particular, we were able to draw some interesting policy recommendations regarding the grid tariff composition, that allow us to bring the overall structure of energy communities close to the optimal investment setting.

2 Stability considerations shed new light on the snowball effect

In the present section we propose a simple illustration aimed at providing intuitions about the emergence of a "snowball" effect which will in turn motivate the paper. We also show that adding stability considerations to the traditional cost-benefit analysis modifies the conditions for the emergence of such a snowball effect.

Definition 1. *We use "snowball effect" to define a situation such that:*

1. *Given existing grid tariffs, some communities form but not all players join a community.*
2. *To ensure grid cost recovery following community formation, grid tariffs are modified.*
3. *Following this modification, new communities form or existing communities increase in size.*

In order to highlight the effect of gains sharing on stability, we investigate under which conditions two potential communities of a given size may form. These communities are independent but are connected to the same power grid, whose costs need to be recovered through a tariff. For simplicity, we restrict ourselves in this section to questioning whether communities will fully form (i.e. all potential members join the same community) or not. The remaining sections allow smaller communities to form. We will first analyze community formation when each community is managed by a benevolent planner. The latter will therefore maximize the overall gains of his community, without a need to consider gains sharing. To that aim, a simple cost-benefit analysis is needed.

Cost-benefit analysis: Assume that we have two buildings, 1 and 2, composed of n_1 and n_2 households respectively. They are very similar, except building 2 has more households than building 1: $n_2 > n_1 > 1$. Within each building, households may decide to collaborate and form a community. In this basic illustration, this will simply consist of the whole community sharing one meter instead of having one meter per household. Thus, community members may save on grid costs. We set the cost of each meter at $\delta > 0$. The value creation when the n-th household is added is δ, since there is one less meter to install. However, joining a community entails some coordination costs $c(n)$, that increase and, for the sake of illustration, are assumed to be convex in the number of community members. A community of $n-1$ households accepting an n-th player sees its coordination costs increase by $g(n) \equiv c(n) - c(n-1)$. Coordination costs being convex, it is easy to show that if the n-th household is accepted, so, too, will the $(n-i)$-th with $i \in [1 : n-1]$. The planner of a community composed of $n-1$ households will include an additional household if and only if the benefits from aggregation exceed the additional costs of coordination. Thus, the condition for a community of n_i households in building i to form is:

$$g(n_i) \leq \delta \tag{1}$$

Function $g(.)$ increases in n meaning the condition is less likely to be met as n grows: small communities are likely to be easier to form than bigger ones. Because households are symmetric, we can state that communities may either gather all households of a given building, or may not form at all (that is, each household keeps its own meter). This means, because $n_1 < n_2$, that building 1 is likely to form a community before building 2. To illustrate our ideas, let us assume that the grid tariff δ allows the formation of a community in building 1 and not in building 2: $g(n_1) < \delta < g(n_2)$.

Now that we have a condition for community formation, let us see how this condition evolves as communities form and the grid cost-recovery constraint induces changes in δ.

This constraint imposes that whatever the community formation, the DSO needs to recover her costs. To that aim, she may define tariff parameter δ' such that her revenues are maintained after building 1 has formed:

$$(n_1 + n_2)\delta = (1 + n_2)\delta' \Longrightarrow \delta' > \delta \tag{2}$$

Then we will have a snowball effect as shown in Definition 1 if and only if:

$$g(n_1) < \delta < g(n_2) < \delta' \tag{3}$$

The first inequality means that with initial grid tariffs, community 1 forms (relation (1) and condition 1 in Definition 1). The second inequality means that community 2 would not form under initial grid tariffs (condition 1 in Definition 1). The third inequality means that the increase in grid tariffs finally prompts the formation of community 2 (condition 3 in Definition 1).

Interestingly enough, the snowball effect is twofold. It is not only that some communities may form as a consequence of another community having formed, also, the first communities to form are the smallest ones, prompting bigger ones to form at later stages.

Adding gain-sharing considerations The situation becomes slightly more complicated when communities manage themselves without a central authority – in the spirit of current regulation. In [1], we provided necessary and sufficient conditions that ensure the non-emptiness of the core of the game and the stability of the community. It is reported here for convenience.

Theorem 3 in [1]. *When the coordination cost is taken into account and players are symmetric, the following two propositions are equivalent:*

1. *The core of the game is not empty and the community forms*
2. $$g(n) + h(n) \leq \delta \tag{4}$$

with $h(n) = (n-2)c(n) - (n-1)c(n-1)$. $h(n)$ essentially represents the additional cost related to community formation once cooperative considerations are considered – on top of the traditional cost-benefit analysis. Because coordination costs are convex, one can easily show that both $g(.)$ and $h(.)$ are increasing functions of n. Unfortunately the precise formulation of $h(n)$ doesn't have a clear intuitive interpretation. For the purpose of this analysis it suffices however to note that $h(.)$ adds to the standard cost/benefit approach to account for community stability, is positive, increasing and convex.

The condition for a snowball effect to arise is:

$$g(n_1) + h(n_1) < \delta < g(n_2) + h(n_2) < \delta' \tag{5}$$

Comparing relations (3) and (5), one may observe that when gain-sharing considerations (to ensure the stability of the community) are added to the traditional cost-benefit analysis:

1. Condition for the formation of any community is more stringent.
2. Conditions for a snowball effect are modified.

The proof of point (1) is straightforward: to form a community, one needs $g(n)+h(n) < \delta$ with $h(n)$ positive and increasing in n (condition 4), instead of the less restrictive $g(n) < \delta$ (condition 1). To illustrate point (2), we show that if the coordination cost of a coalition is proportional to the number of handshakes in that coalition, $c(n) = \psi(n-1)n$, the set of grid tariffs δ that lead to a snowball effect is of larger measure when communities' stability is considered, than when only a cost benefit analysis is performed. The proof is given in appendix H. This means that gain sharing issues make a sole community less likely to form, but if it does form, a snowballing effect impacting other communities is more likely to arise in the sense that more buildings may then see the formation of energy communities.

3 Theoretical developments and mathematical formulation of the problem

3.1 Setting of the problem: Energy communities and the distribution system operator

In this section, we give an overall mathematical description of the energy communities we are interested in, as well as of the distribution system operator.

We assume that there are B buildings in a neighborhood where energy communities might be created (we will sometimes call such a community a coalition). Households of each building $b \in \{1, 2, ..., B\}$ can join in an energy community in order to share the cost of investment in a photo-voltaic panel with a battery and aggregate their energy consumption. All households of building b are gathered into one set I_b. If households $S_b \subset I_b$ form a community, they will subsequently share all the benefits of producing solar energy and aggregating their consumption profiles, as well as all related costs of investment. The stability of a community in a building will depend on the existence of an allocation rule of this benefit among all its households so that the formation of any smaller coalition (or sub-community) of the community is deterred. In [1], we have shown how this problem can be tackled using cooperative game theory and we refer to this work for an in-depth description of the value sharing in the community. In particular, to analyze the stability of a community I, we will have to consider all its smaller coalitions $S \subset I$, and estimate their best payoff if they decide not to join the overall community I.

For the sake of simplicity, we will assume that energy communities among households living in different buildings cannot be formed. This can be justified by the fact that the roof of a building is usually owned by residents only and any access to it in order to build a PV panel is regulated by them. However, this assumption can easily be relaxed in theory and we believe that it does not alter the main findings of this paper. Following [1], we will consider that a community will be stable if it can share its overall benefit so that no smaller coalition finds it profitable to leave the grand community. In game theoretical vocabulary, this means that a community is stable if the game that has a non-empty core. Besides, if all households in a building fail to create a whole stable community, we will assume that they can form smaller stable sub-communities in a way that optimizes the overall value (in a sense that has to be defined) over the entire building. This notion was introduced and explained in [1] via the concept of optimal partitioning. In particular, it has been shown that such a partitioning always exists in any building. We will further elaborate on these notions in section 3.2.

Time t is discretized into T periods of one hour representing a characteristic consumption year: $t \in \{1, 2...T\}$. The energy consumed by household $i_b \in I_b$ living in building b is denoted $f_{i_b}(t)$ and is expressed in kilowatt-hour (kWh). The installation cost of a PV panel is assumed to be a function of its capacity K. Once installed, we assume that the panel will deliver, on average, a yearly profile $Kg(t)$ (in kWh) with $g(t) \in [0, 1]$ representing the shape of PV production over a year and $\text{Max}_t g(t) = 1$. Peak production occurs around noon (under clear sky conditions). The investment cost (expressed in €) of a PV panel with capacity K is denoted $c(K)$. Each individual household has access to an area in the premises to install a PV panel that is proportional to her living area. We assume her energy consumption is also proportional to her living area. Hence, if a group of households $S_b \subset I_b$ want to go green, we will make the assumption that they can install a PV capacity K. This capacity will be upper bounded by the size of the roof they can have access to, which is proportional to their annual expected consumption $\sum_t \sum_{i_b \in S_b} f_{i_b}(t)$. To make communities comparable, we perform a change of variable and consider, in an equivalent way, an investment decision $\mu(S_b) \equiv K \frac{\sum_t g(t)}{\sum_t \sum_{i_b \in S_b} f_{i_b}(t)}$ instead of capacity K. In other words, a set of households $S_b \subset I_b$ that install a PV panel will invest in a capacity that will allow them to cover a percentage $\mu(S_b) \in [0,1]$ of their yearly expected consumption $\sum_{i_b \in S_b} \sum_{t=1}^{T} f_{i_b}(t)$. In our setting, $\mu(S_b)$ is effectively the decision variable of communities.[2] This corresponds to a cost of PV installation $c\left(\sum_{i_b \in S_b} k_{i_b}(\mu(S_b))\right)$ (€), where $k_{i_b}(\mu(S_b)) = \mu(S_b) \frac{\sum_{t=1}^{T} f_{i_b}(t)}{\sum_{t=1}^{T} g(t)}$. Parameter $\mu(S_b)$ is calculated such that the benefits (in terms of local consumption and injection in the grid) for the set of households is optimized (and therefore, $\mu(S_b)$ represents the PV investment decision for coalition S_b). We bound parameter $\mu(S_b)$ by an upper limit $\bar{\mu}_b$ that reflects the fact that coalition S_b has access to a limited area on the roof and we set the same upper limit $\bar{\mu}_b$ to all coalitions of the same building, to model the fact that any coalition can have access on the roof to a surface that is proportional to its expected annual energy consumption (with the same proportionality factor for all coalitions). It is worth mentioning that the upper limit $\bar{\mu}_b$ depends on the considered building, since two buildings might have different roof sizes. At time t, the amount of solar energy injected into the grid is denoted $inj(S_b)_t$ (expressed in kWh) and the amount that is locally consumed (or auto-consumption) $aut(S_b)_t$ (expressed in kWh).

Depending on the incentives one has to create an energy community (economic, environmental, etc.), it could be possible that the effect of joining an energy community might modify the consumption habits of its members: people might want to reduce their consumption during peak hours or shift their consumption to hours where solar production is at the highest. This demand sensitivity can be modeled in different ways: introducing a demand elasticity to the price, modeling demand shedding and shifting, etc. In this work, we have depicted this effect by introducing the possibility that a community will invest in a battery to store energy. Similarly to the way we model the PV investment, we will assume that the community will also mutualize the investment of the battery. A coalition S_b in building b will also have to estimate the optimal battery capacity $Bat(S_b)$ (expressed in kWh) to install with the PV panel. Naturally, a battery comes with a technical charge and discharge constraints, that we denote ch and dch (expressed in kW), limiting the amount of energy that can be stored in or extracted from a given battery during one hour. The stored (respectively withdrawn) energy from the battery at time t will be denoted $st(S_b)_t$ (respectively $with(S_b)_t$) and the storage level of the battery $l(S_b)_t$. The investment cost (expressed in €) of a battery with capacity Bat is denoted $c_{Bat}(Bat)$.

If an energy community is formed in a building, it will have to organize in order to mutualize the PV and battery installation and to share any subsequent benefit (grid cost savings, PV injection, etc.) between its members. It is then reasonable to assert that members of the community will have to meet on a regular basis to manage its functioning. These meetings will incur some additional sunk costs for the community that have to be accounted for. To do so, we introduce a coordination cost function $c_{coo}(S)$ that depends on

[2] This means that once community S_b forms, it chooses the share of its total yearly expected consumption it wants to cover with PV (which directly translates in a given level of PV installation). The PV capacity to install is then $\mu(S_b) \frac{\sum_{i_b \in S_b} \sum_{t=1}^{T} f_{i_b}(t)}{\sum_{t=1}^{T} g(t)} \equiv \sum_{i \in S} k_{i_b}(\mu(S_b))$ (kW) with $k_{i_b}(\mu(S_b)) = \mu(S_b) \frac{\sum_{t=1}^{T} f_{i_b}(t)}{\sum_{t=1}^{T} g(t)}$.

each possible coalition S to represent all the coordination costs of the coalition. In these theoretical developments we keep $c_{coo}(.)$ quite general, the only basic requirement being that coalitions of size one (consisting of individuals) should have a zero coordination cost.

The local power distribution operator, denoted DSO throughout this paper, has the mission to deliver electricity to all households of the considered neighborhood. The DSO is not a profit maximizer: it imposes non-discriminatory electricity tariffs to the households only to recover the grid costs, which we denote Cost_{DSO} (expressed in €). We assume that the electricity tariff has three components: one fixed and related to the installation of the meter, one related to energy consumed, and one to peak demand. Typically, a community with a profile $f(t)$ will pay a fixed subscription fee δ per connection (expressed in €), $\alpha \text{Max}_t f(t)$ for her capacity (α is expressed in euro per kilowatt €/kW) and $\phi \sum_{t=1}^{T} f(t)$ for the energy it has consumed (ϕ is expressed in €/kWh). This particular linear form of the electricity tariff is not too restrictive, as our setting can easily be generalized to other more elaborate tariff formulas. Power grid tariffs α, ϕ and δ are calculated and fixed by the DSO to recover the grid costs.

The solar energy produced by an energy community can either be injected into the distribution network or locally consumed. When locally consumed, PV production reduces the energy bill of the community with marginal value equal to the electricity retail tariff β (expressed in €/kWh). When injected into the network, PV production is remunerated by the distribution system operator (DSO) at a marginal price of γ (expressed in €/kWh), that can represent a feed-in tariff or a market price with a premium attributed to the willingness to consume renewable energy. We assume in this paper that priority is given to local consumption, as we believe that this is the main objective of fostering energy communities from the point of view of policymakers (see [25]). This means $\beta > \gamma$.

As pointed out in [1], there are some externalities between communities of different buildings through grid tariffs: if a community forms in one building, the DSO will have to adjust her fees in order to recover her costs, which in turn might incentivize other communities to form or break. As a consequence, there is a clear link between grid tariffs (decided by the DSO) and the structure and stability of energy communities in different buildings. In this paper, we propose generalizing the setting of [1] by capturing these externalities via an equilibrium formulation between energy communities and the DSO. This equilibrium will encompass any possible snowball effect, as presented in section 2.

For the sake of clarity and simplicity, in this paper we only concentrate on the economic benefits of joining a community. Non-economic motivations of the energy community, like willingness to go green or to become energetically independent, are therefore neglected but can be included by considering different utility functions of the households. We assume all households have access to the same PV and battery technologies and that, if they decide to form an energy community, they cannot anticipate the reaction of the DSO or the one of the other households to this action.

A summary of our notation is given in appendix A.

3.2 Stability of a community and optimal partitioning

This part of the paper defines the stability of an energy community and elaborates on the optimal formation of communities in a given building.

3.2.1 Calculation of the value of a coalition

This section calculates the value of a given set of households S_b of size s_b living in the generic building b. Given grid tariffs α, ϕ and δ, the households will have to decide not only their optimal investment in the PV and battery but also their optimal functioning (injection in the grid, auto-consumption, stored and extracted energy from the battery) in order to maximize their payoff (we will refer to this payoff as the "value" throughout this paper). In order to simplify the exposition of the paper, we have included the exact calculations of the value of the community in appendix B.

The value of coalition S_b is the maximum possible benefit made by S_b when optimally deciding the PV and battery investment variables $\mu(S_b)$ and $Bat(S_b)$ and their operations $(aut(S_b)_t, inj(S_b)_t, st(S_b)_t, with(S_b)_t, l(S_b)_t)$. This value will be denoted $v(S_b, \alpha, \phi, \delta)$ to emphasize the fact that it depends on the DSO's decisions (the grid tariff components):

$$
\begin{array}{lll}
v(S_b,\alpha,\phi,\delta) = & \text{Max} & \alpha\left(\sum_{i_b\in S_b}\text{Max}_t\left(f_{i_b}(t)\right) - \text{Max}_t\left(\sum_{i_b\in S_b} f_{i_b}(t) - aut(S_b)_t\right)\right) \\
& & +\phi\sum_{t=1}^{T} aut(S_b)_t + \delta(s_b-1) \\
& & +\beta\sum_{t=1}^{T} aut(S_b)_t \\
& & +\gamma\sum_{t=1}^{T} inj(S_b)_t \\
& & -c_{PV}\left(\mu(S_b)\dfrac{\sum_{i_b\in S_b}\sum_{t=1}^{T} f_{i_b}(t)}{\sum_{t=1}^{T} g(t)}\right) - c_{Bat}\left(Bat(S_b)\right) \\
& & -c_{coo}(S_b) \\
& \text{s.t.} & 0\le\mu(S_b)\le\bar{\mu}_b \\
& & 0\le aut(S_b)_t,\ 0\le inj(S_b)_t \\
& & 0\le Bat(S_b),\ 0\le st(S_b)_t,\ 0\le with(S_b)_t,\ 0\le l(S_b)_t \\
& \forall t=1,...,T & \mu(S_b)\dfrac{\sum_{i_b\in S_b,t} f_{i_b}(t)}{\sum_{t=1}^{T} g(t)} g(t) + with(S_b)_t - st(S_b)_t = aut(S_b)_t + inj(S_b)_t \\
& \forall t=2,...,T & l(S_b)_t = l(S_b)_{t-1} + st(S_b)_t - with(S_b)_t \\
& \forall t=1,...,T & l(S_b)_t \le Bat(S_b) \\
& \forall t=1,...,T & st(S_b)_t \le ch\ Bat(S_b) \\
& \forall t=1,...,T & with(S_b)_t \le dch\ Bat(S_b)
\end{array}
\tag{6}
$$

The value of coalition S_b results from an optimization program with constraints. The objective function is simply the sum of the payoffs due to the grid tariff savings, PV auto-consumption and injection in the grid, minus the investment and coordination costs. All constraints are straightforward except the last five which we now explain:

- Constraint

$$\forall t=1,...,T \quad \mu(S_b)\frac{\sum_{i_b\in S_b,t} f_{i_b}(t)}{\sum_{t=1}^{T} g(t)} g(t) + with(S_b)_t - st(S_b)_t = aut(S_b)_t + inj(S_b)_t$$

  states that at each time period, the PV production (which is equal to the product of the PV invested capacity with the solar profile $g(t)$) should be equal to the amount of energy that is auto-consumed plus the one that is injected into the grid, plus (or minus) the amount of energy that is charged in (or discharged from) the battery.

- Constraint

$$\forall t=2,...,T \quad l(S_b)_t = l(S_b)_{t-1} + st(S_b)_t - with(S_b)_t$$

  is the state equation of the battery at each time step, linking the level at time t, to the one at time $t-1$ plus or minus the amount of energy that is charged in (or discharged from) from the battery.

- Constraint

$$\forall t = 1, ..., T \quad l(S_b)_t \leq Bat(S_b)$$

bounds the level of the battery by the capacity that is invested.

- Constraint

$$\forall t = 1, ..., T \quad st(S_b)_t \leq ch\ Bat(S_b)$$

bounds the storage variable by the technical constraint of the battery.

- Constraint

$$\forall t = 1, ..., T \quad with(S_b)_t \leq dch\ Bat(S_b)$$

bounds the withdrawal variable by the technical constraint of the battery.

To simplify the calculation of the value of coalitions, we have assumed that all investment variables are continuous. A more realistic description will consider them as integers since PV panels and batteries are commercialized as standardized products. Nevertheless, we believe that this assumption is not too restrictive as it does not alter the main findings of this paper.

From the empirical data we have (current prices of commercial PV panels and batteries), it appears that investment costs for the PV and batteries are largely linear in the range of capacity we are considering for a standard building or village. In that case, it can be shown that for any grid tariff structure (α, ϕ, δ), optimization program (6) always has a unique solution because it is linear and nondegenerate.

Once the value of a coalition is calculated, one then wants to examine whether it can remain stable or not, this can be done by analyzing allocations of this value among members of the coalition that are in the core of the game (see [1] for more details about the notion of the core applied to energy communities).

3.2.2 Stability of a coalition

Following [1], we will consider that coalition S_b will remain stable if it can split its value $v(S_b, \alpha, \phi, \delta)$ among its members so that no subset of S_b has an interest in leaving the coalition S_b and creating one on its own. To simplify notation, we will consider that S_b is the set $\{1, 2, ..., s_b\}$. Formally, the stability of the coalition can be rewritten as follows:

Definition 2. *Coalition $S_b = \{1, 2, ..., s_b\}$ is stable if there exists an allocation $x = (x_1, x_2, ...x_{s_b}) \in \mathbb{R}^{s_b}$ of its value $v(S_b, \alpha, \phi, \delta)$ among its members satisfying:*

$$\forall T_b \subset S_b, \quad \textstyle\sum_{i \in T_b} x_i \geq v(T_b, \alpha, \phi, \delta) \tag{7}$$

$$\textstyle\sum_{i=1}^{s_b} x_i = v(S_b, \alpha, \phi, \delta) \tag{8}$$

Relation (7) states that if coalition S_b is stable the sharing of the total benefit $v(S_b, \alpha, \phi, \delta)$ should be done in such a way that satisfies all smaller coalitions: members of any sub-coalition T_b receive more than what they would get if the sub-coalition stood alone: $v(T_b, \alpha, \phi, \delta)$. Relation (8) states that we only consider allocations that split the whole value $v(S_b, \alpha, \phi, \delta)$: there is no gain or loss of the value of the whole coalition when splitting it between its members.

It has been shown in [1] that the whole community I_b composed of all households living in building b is not always stable. The stability of an entire building is a property that is actually quite hard to obtain if coordination costs are convex and if households have quite different consumption profiles. In that case, we will look for an optimal partition of the building into smaller stable coalitions that maximizes a pre-specified criterion across the entire building. The choice of the criterion depends on the main motivation to form the energy community in a considered building. For instance, if the driver of the community is its concern about the environment then it should be reasonable to look for the stable partition that maximizes the overall PV installed capacity. Since we made the assumption that agents are driven by economic incentives, for the sake of coherence, we will assume that households will look for the stable partition that maximizes the overall value of the building. We elaborate more on this in the next section.

3.2.3 Optimal partitioning of a building

A partition is simply a subdivision P_b of the whole set I_b into smaller (non-empty) coalitions or subsets that never intersect (see appendix C for a formal definition). We will consider that the value of partition P_b is the sum of the values of all its coalitions:

$$VP(P_b, \alpha, \phi, \delta) := \sum_{S \in P_b} v(S, \alpha, \phi, \delta) \tag{9}$$

, and in our notation we made clear that such a value depends on the grid tariffs α, ϕ and δ. The main rationale behind considering only energy communities that partition a building is that it is usually not economically interesting for a group of people to join two different communities as this artificially increases coordination costs for them. Besides, a partitioning never forces any group of households to join a community, as it is always possible to invest in a zero kW of PV for any coalition: a situation with no investment in a PV and battery is always a feasible point of the optimization program (6).

The optimal partition of a building is the one that maximizes the value among all partitions containing only stable coalitions (a stable coalition is a one the yields a non-empty core. See appendix C for a formal definition of an optimal partition).
Since there is a finite number of partitions, one can deduce that there always exists an optimal partition for any grid tariff structure (α, ϕ, δ). Uniqueness is not always guaranteed and depends on the consumption profiles of the households. However, all our numerical simulations indicate that if there does not exist in the building any pair of households that have exactly the same consumption profiles, the optimal partition is always unique.

Given a partition $P_b = \{S_b^1, S_b^2, ..., S_b^p\}$ of building b, we already know that each coalition $S_b^k,\ k = 1, ..., p$ will operate following the solution of optimization program (6), that we concatenate into the vector (the dependence on the tariffs (α, ϕ, δ) is omitted here to ease the notation) $\big(\mu(S_b^k), aut(S_b^k)_t, inj(S_b^k)_t, Bat(S_b^k), st(S_b^k)_t, with(S_b^k)_t, l(S_b^k)_t\big) \in \mathbb{R}^{5T+2}$. We then gather all these investment and operations variables of all coalitions into one vector, depending on the whole partition P_b, as follows:

$$\begin{gathered}(\boldsymbol{\mu}, \mathbf{aut}, \mathbf{inj}, \mathbf{Bat}, \mathbf{st}, \mathbf{with}, \mathbf{l})\,(P_b, \alpha, \phi, \delta) \\ = \\ \left[\big(\mu(S_b^k), aut(S_b^k)_t, inj(S_b^k)_t, Bat(S_b^k), st(S_b^k)_t, with(S_b^k)_t, l(S_b^k)_t\big)\,,\quad k = 1, ..., p\right] \\ \in \mathbb{R}^{p(5T+2)}\end{gathered} \tag{10}$$

3.3 The DSO and the grid cost recovery

In this section, we intend to present how the DSO is modeled in our setting. With respect to consumers, the main role of any European DSO is to guaranty equal access to the grid at the same tariff and at any time. The DSO is also responsible for quality of service, as well as long-term investments in the distribution grid to ensure adequacy and security. As a consequence, the DSO incurs some grid costs that it has to recover from the grid tariffs (α, ϕ, δ).

Tariffs are non-discriminatory which involves the DSO charging the same amount for capacity α, for energy ϕ and the same fixed tariff δ to all consumers or communities of any building b. Given partitions of the buildings $(P_1, P_2, ..., P_B)$ with $P_b \subset \mathcal{P}(I_b),\ b = 1, ..., B$, recall that the investment and operational variables of the functioning of the energy communities are $(\boldsymbol{\mu}, \mathbf{aut}, \mathbf{inj}, \mathbf{Bat}, \mathbf{st}, \mathbf{with}, \mathbf{l})(P_b, \alpha, \phi, \delta),\ b = 1, ..., B$. Therefore, payments given to the DSO by all energy communities can be calculated as follows (the detailed calculation is given in appendix D) where we assume that partition P_b splits building b into p_b coalitions, or in other words, that partition P_b is of size p_b:

$$\begin{aligned}
\mathrm{DSO}_{\mathrm{Payoff}}(P_1, P_2, ..., P_B) &= \sum_{b=1}^{B}\sum_{k=1}^{p_b}\left(CP\left(S_b^k\right) + EP\left(S_b^k\right) + FP\left(S_b^k\right)\right) \\
&= \alpha\sum_{b=1}^{B}\sum_{k=1}^{p_b}\mathrm{Max}_t\left(\sum_{i_b\in S_b^k} f_{i_b}(t) - aut(S_b^k)_t\right) \\
&\quad +\phi\sum_{b=1}^{B}\sum_{k=1}^{p_b}\sum_{t=1}^{T}\left(\sum_{i_b\in S_b^k} f_{i_b}(t) - aut(S_b^k)_t\right) \\
&\quad +\delta\sum_{b=1}^{B} p_b
\end{aligned} \tag{11}$$

We can state that the DSO will recover her costs if her payments balance out the grid costs:

Definition 3. *Given partitions $(P_1, P_2, ..., P_B)$ of the buildings, we will consider that the grid tariffs (α, ϕ, δ) allow the DSO to recover her cost if and only if:*

$$DSO_{Payoff}(P_1, P_2, ..., P_B) = COST_{DSO} \tag{12}$$

Note that condition (12) allows the DSO to only recover her costs and not make any extra profit, which reflects the public service mission of any DSO in Europe.

3.4 An equilibrium formulation between the DSO and energy communities

As explained in section 3.2 and highlighted in Definition 6, any optimal partition of a building into different energy communities clearly depends on grid tariffs set by the DSO. Furthermore, as developed in section 3.3 and highlighted in Definition 3, the grid tariffs that the DSO should impose to recover her costs depend on how energy communities are structured in each building. It is then relevant to look for the Nash equilibrium between energy communities and the DSO that we define as follows:

Definition 4. *The energy system composed of coalitions $(P_1^*, P_2^*, ..., P_B^*)$ and grid tariffs $(\alpha^*, \phi^*, \delta^*)$ is in equilibrium if and only if:*

- *At each building $b \in \{1, ..., B\}$, P_b^* is an optimal partition of b according to Definition 6, given grid tariffs $(\alpha^*, \phi^*, \delta^*)$.*
- *The DSO recovers the grid costs according to Definition 3, given partitions $(P_1^*, P_2^*, ..., P_B^*)$.*

We would like to emphasize the fact that our definition of the Nash equilibrium does not only imply that all energy communities are in equilibrium with respect to the grid tariffs. It also requires that the partition into coalitions is stable from a game-theoretical point of view. Also, note that the definition of an optimal partition in a building depends only on grid tariffs and does not explicitly involve how other communities in the other buildings are structured: the dependence between optimal partitions is actually implicit and is exerted through grid tariffs. Furthermore, we only focus on the Nash equilibrium because we have assumed that neither the energy communities nor the DSO can anticipate the reaction function of the other players to their decisions.

It is reasonable to think of the Nash equilibrium as the natural outcome of the interaction between energy communities and the DSO. Indeed, in equilibrium the DSO meets its obligations as it sets tariffs that allow it to recover all the grid costs, and each building is split into communities so as to maximize the total value. The snowballing effect, that we have described to motivate this paper, occurs because it often happens that one building where energy communities form is not in equilibrium with the DSO and other buildings without energy communities: to react to the formation of energy communities, the DSO will have to increase the grid tariffs, which might motivate the creation of energy communities in the other buildings, which in turn might further change the tariffs of the DSO, and so forth. The equilibrium is the final outcome of this snowballing effect.

The equilibrium between energy communities and the DSO is in general neither unique nor isolated, for the simple reason that the DSO has three levers to recover the grid costs: α, ϕ and δ and many combinations of these three variables can shape the communities in the same way. To seek a uniqueness result, one will have to allow the DSO to set only one of the three grid tariff components while fixing the other two. This is concretely what has been done in our numerical applications (see section 4). Unfortunately, even this assumption is not enough to guarantee the uniqueness of the equilibrium in the general case, without additional assumptions on the composition of buildings (considering, for instance, that consumers are never identical). Regarding existence, all our numerical experiments lead to an equilibrium, which gives some hope to proving existence in the general case. However, standard fixed point theorems (Brower's or Kakutani's) generally invoked to prove the existence of equilibrium solutions,and degree theory arguments (such as the ones developed in [2]), cannot be used in our setting because the optimal partitioning of a building involves integer decision variables and therefore the first order conditions (or KKT conditions) of optimization program (20) do not characterize its solution. We propose leaving the analysis of existence and uniqueness of the equilibrium for future research.

Appendix E gives a simple algorithm to calculate the equilibrium. The algorithm is iterative and mimics the unravelling of the snowball effect.

4 Numerical developments: A case study of the snowball effect

4.1 Two synthetic buildings

For the purpose of this analysis, we constructed a neighborhood of two synthetic buildings $B = 2$, each composed of six households. The PV and battery installation costs are the same in both buildings. However, these buildings differ in their household composition: each building will have a different consumption pattern, not only overall, but also at the individual household level. This will affect the value of each coalition, through the channel of PV revenues and installation costs, and in turn the optimal partitions.

We focus on typical buildings in north-west Germany. The timestamp granularity is one hour and we consider consumption and solar profiles over the course of a year (we thus capture the seasonality inherent to power systems). Each household demand has been generated by a load profile generator that allows us to simulate detailed demand curves for various types of households, including parameters such as their size, employment status, age, and family status, etc. The PV production has been calibrated on regional data for solar generation, in year 2014. PV installation costs are calibrated on standard commercial PV panel prices. We assume a panel lifetime of 30 years and a discount factor of 5%. PV production is valued at the German retail price when it is consumed within the community. Excess production is injected into the grid at German wholesale prices. It is worth mentioning that PV installation costs are concave in theory. However, in practice, the concavity occurs above an installed capacity of the range of the megawatt and for the rather small capacities we are interested in in this paper, the cost can be considered as linear. Battery installation costs and technical constraints are those of the Tesla Powerwall battery ([58]).

As previously explained, we simulate two buildings which we believe are reasonable representations of neighborhoods one can find in developed countries, and yet provide rather contrasted occupation patterns. The first one is completely mixed and accommodates students, retired people, families with various occupations and, most importantly a storekeeper. The latter has a particular consumption profile: it is broadly flat in the range between 9am and 6pm for all days of the year except weekends. Outside these periods, consumption is very low. The second building is composed of retired people (i.e., rather symmetric households). To summarize, the first building is composed of rather mixed households, while the second is more homogeneous, in terms of activity and size of the households. As previously discussed, the mathematical complexity of the problem constrains us to make use of numerical applications. While this allows our illustration to be as realistic as possible, such applications inherently lack the generality of a formal proof. To gain confidence of the generality of our insights, many other simulations on buildings of various compositions were carried out. While numerical results obviously differed, the main insights of the paper generally carried over. These results should nevertheless be considered as illustrative, rather than normative.

The DSO seeks to recover its cost COST_{DSO} that we estimate for our two buildings as follows: the German grid tariff[3] is composed only by the fixed component δ that is equal to 143.7€ (per year and per connection). Applied to our two buildings each composed of six households, this corresponds to 1724.4€ (per year) and we will thus make the simplifying assumption that $\text{COST}_{\text{DSO}} = 1724.4$. In each case we consider, we shall calculate the equilibrium in three different situations, depending on whether the DSO recovers the grid cost by tuning the capacity component α (in that case, we will consider that $\phi = \delta = 0$), the individual connection cost δ ($\alpha = \phi = 0$) or energy component ϕ ($\alpha = \delta = 0$). The initial values of the numerical algorithm α^0, β^0 and δ^0 are calculated so that the DSO recovers its cost when there is no energy community.

The coordination cost of a coalition of size s is assumed to be proportional to the number of handshakes between its members, $\frac{s(s-1)}{2}$, which is strictly convex with respect to the size of the coalition. We then treat two different sensitivities of this coordination cost: either 10€ per handshake or 50€ per handshake.

The convergence criterion of the algorithm that calculates the equilibrium is $\epsilon = 10^{-5}$. More detailed information on the data sources can be found in appendix F. All values are reported in 2018 €/annum. Table 1 reports on the composition of households.

[3]As prevails in the city of Cologne, Germany. See `https://www.rheinenergie.com/media/portale/downloads_4/rheinenergie_1/preisblatt_1/preisbestandteile/Preis-Kostenuebersicht_Strom_RE_Haushalt_final.pdf`

	Building 1 (Mixed household)			Building 2 (Homogeneous household)		
Household number	Type	Annual demand (kWh)	Peak demand (kW)	Type	Annual demand (kWh)	Peak demand (kW)
1	Couple, working	2623	10.1	Retired man	1101	5.4
2	Family, working 1 child	2613	6.7	Retired woman	1016	5.1
3	Man, work from home	1601	2.1	Retired couple	2680	8.2
4	Student	1563	5.4	Retired couple	2088	7.3
5	Storekeeper	4003	1.4	Retired couple	1747	7.0
6	Retired couple	1747	7.0	Retired couple	1747	7.0
	Whole building	14150	32.7	Whole building	10379	40.0

Table 1: Composition of buildings 1 and 2

4.2 The snowball effect at play: Results at each step of the dynamic process

In a first numerical application, we give a step-by-step description of the unravelling of a snowball effect. To that aim, we only treat the case when the DSO tariff is purely capacity-based and we set other tariff components to zero. This means each consumer (that is, each community) pays a grid cost α per kW of peak demand, and $\phi = \delta = 0$. The process toward equilibrium is outlined in section E. We summarize it here for convenience. It consists of a succession of a (possibly infinite) number of periods. Each period is divided into two stages: first, households are assumed to be myopic and make their decision to form coalitions based on the assumption that α is an exogenous parameter. Second, the DSO observes community formation and adjusts α such that it recovers the grid costs. In the following period, households observe the new α and reconsider how to best form coalitions, etc. The game ends when a convergence (as defined in equation (22)) is reached. A summary of each period is shown in table 2. We report on how each building is split into energy communities, along with the total value of the communities (that is, the sum of the values of the equilibrium partitions over all coalitions) and PV and battery installations in each period. The table assumes a handshake cost of $\psi = 50$€.

Several observations can be made. First, we notice that the system stabilizes after five periods. This is in sharp contrast with a model without grid-cost recovery, or where households would ignore grid tariffs when they make investment decisions: in these systems an equilibrium would be reached at the end of the first period. Also, there is evidence that community decisions in different buildings are intertwined. Indeed, any modification in community formation in a given building may affect α which in turn affects both buildings. This explains in particular why some participants may change partners from one period to the other: while stable under certain tariff conditions, a given coalition may no longer be stable once tariffs are adjusted so as to meet a cost-recovery constraint. This change, unanticipated by individuals, is a direct consequence of the very formation of communities in either building in the first place.

Second, the value of community formation dramatically increases over periods. It is important to stress here that this value is the sum of the values, as it is perceived by each individual community – and not the welfare as calculated by a hypothetical social planner based on true system costs. At the end of the game, each community sees a very high α, which makes it prohibitively costly to consider separation. Hence, their perception of the value of community formation is artificially inflated by extremely high grid tariffs making these communities particularly precious in their eyes. However, no building forms into a

	Iteration	0 (no community)	1	2	3	4	5 (final)
Grid cost	α (Eur/kW)	52.5	196.7	265.41	268.9	269.1	269.1
Building 1	Optimal partition	{1}{2} {3}{4} {5}{6}	{1,4} {2,3} {5,6}	{1,5,6} {2,3,4}	{1,5,6} {2,3,4}	{1,5,6} {2,3,4}	{1,5,6} {2,3,4}
	Value (Eur)	629.3	805.2	2509.5	3352.8	3396.4	3397.8
	PV (kW)	7.1	7.6	7.9	8.1	8.2	8.2
	Battery (kWh)	11.9	11.6	12.0	12.5	12.6	12.6
Building 2	Optimal partition	{1}{2} {3}{4} {5}{6}	{1,4} {2,6} {3,5}	{2} {3,6} {1,4,5}	{2} {3,5} {1,4,6}	{2} {3,5} {1,4,6}	{2} {3,5} {1,4,6}
	Value (Eur)	573.6	770.1	2792.1	3764.9	3815.2	3816.9
	PV (kW)	6.0	5.9	6.1	6.1	6.1	6.1
	Battery(kWh)	13.8	11.3	11.9	12.3	12.3	12.3
Total	Number of coalitions	12	6	5	5	5	5
	Value	1202.9	1575.3	5301.6	7117.7	7211.6	7214.7
	PV(kW)	13.1	13.5	14.0	14.2	14.3	14.3
	Battery(kWh)	25.7	22.9	23.9	24.8	24.9	24.9

Table 2: Community building over iterations with capacity-based grid cost recovery

single community: instead households prefer to form sub-coalitions (two coalitions in building 1, three coalitions in building 2). This is due to the asymmetry of households and the existence of coordination costs that can only be avoided by forming smaller coalitions.

Finally, community formation induces changes in PV and battery installations. Gathering into a community may modify PV installation decisions in two opposite directions. On the one hand, community formation allows PV production to be jointly managed. In particular, an excess of production may be profitably shared with another member of the community instead of being fed into the grid. This tends to make PV panels more profitable and, hence, encourages their installations. On the other hand, communities allow an increase in the value that can be extracted from a single panel. This effect is strongly linked to the aggregation benefit of forming a community. Indeed, if demand curves are sufficiently different amongst community members, a single panel may serve more than one household who may in turn install fewer panels. In our simulations, the first effect dominates the second one, with inflated PV installations following the formation of communities.

Similarly, battery installations could go in either direction following the formation of communities. Keeping PV fixed, batteries are less needed in a community since some of the excess supply of an individual at any given time may be absorbed by other community members in real time. More PV in a given coalition nevertheless means more need for a battery, since PV production is unlikely to be perfectly correlated with demand. The first building exhibits large investments in PV, meaning the second effect dominates. In the second building PV is broadly constant, resulting in a decrease in battery installations prompted mainly by community formation. Overall, PV increases while batteries decrease.

4.3 Resilience of various DSO tariff structures

So far, we have assumed for simplicity that the DSO recovers its costs through a capacity-based tariff. However, many other tariff structures could be chosen. As an illustration and similar to our theoretical developments, we describe here the effect of three tariff structures: Capacity-based as before ($\alpha > 0$ and $\delta = \phi = 0$), per-capita ($\delta > 0$ and $\alpha = \phi = 0$), or volumetric ($\phi > 0$ and $\alpha = \delta = 0$). While these tariff specifications may be rather extreme, we believe they provide a fair representation of the effect of each component (capacity, capita, volume) on community formation. Table 3 shows the results of such assumptions in buildings 1 and 2. Columns 2 to 4 show the case of a handshake cost $\psi = 10$€. Columns 5 to 8 show a handshake cost of $\psi = 50$€ as previously reported. These costs affect only community formation but not investments once communities are formed. Hence, we will focus our interpretation of the results on the case with a handshake cost of $\psi = 50$€ and our conclusions carry over to the case with lower handshake costs. Table 3 also shows the value of the grid tariffs parameter (either α, δ or ϕ) before community formation ("initial" value) and after ("final" value, at the equilibrium), how many periods are needed for the system to reach equilibrium, the final formation of coalitions, and the total level of PV and battery installations. Furthermore, we show the optimal level of such installations if a benevolent planner were to manage investments and gains sharing without coordination costs (instead of the households themselves) in order to optimize the social welfare (we then add the prefix "best" to the results). We report on this optimal welfare as well as the one ("Final welfare") obtained at the equilibrium when communities are self-managed. This final welfare does not account for the coordination costs of community formation and can therefore be compared to the optimal best welfare. Finally, to compare between all our outcomes, we will report on the value of the welfare obtained at the equilibrium when communities are self-managed but when one also accounts for the coordination cost ("Final welfare coor").

	Handshake cost = 10€			Handshake cost = 50€		
	α (€/kW)	δ (€/connection)	ϕ (€/kWh)	α (€/kW)	δ (€/connection)	ϕ (€/kWh)
Initial value	52.5	143.7	0.07	52.5	143.7	0.07
Final value	309.4	862.5	0.20	269.1	431.2	0.19
Number of iterations	5	2	4	5	2	4
Final partition build.1	{1,2,3,4,5,6}	{1,2,3,4,5,6}	{1,4,5}{2,3,6}	{1,5,6}{2,3,4}	{1,4,6}{2,3,5}	{3}{4}{1,5}{2,6}
Final partition build.2	{3,5}{1,2,4,6}	{1,2,3,4,5,6}	{1,3,6}{2,4,5}	{2}{3,5}{1,4,6}	{1,3,6}{2,4,5}	{1}{2}{3}{4}{5}{6}
# of communities	3	2	4	5	4	10
Final PV (kW)	14.1	9.6	17.0	14.3	9.8	17.1
Final battery (kWh)	22.3	6.6	28.0	24.9	7.8	29.3
Final value (€)	8956	9206	3667	7215	3677	3248.9
Final welfare (€)	735.4	881.2	516.8	598.4	826.4	341.5
Final welfare coor (€)	515.4	581.2	396.8	98.4	226.4	241.5
Best PV (kW)	9.2	9.2	9.2	9.2	9.2	9.2
Best battery (kWh)	5.4	5.4	5.4	5.4	5.4	5.4
Best welfare (€)	941.4	941.4	941.4	941.4	941.4	941.4

Table 3: Community formation in buildings 1 and 2 depending on coordination costs and tariff structure

A first conclusion is that community formation is not neutral to the DSO tariff structure: while we observe a strong tariff increase with all three structures, the resulting coalitions, PV and battery installations differ substantially. Unsurprisingly the per-connection fee is the most favorable to community formation with four communities in equilibrium, while a capacity-based or energy-based tariff results in five and ten (smaller) communities respectively. It is also the structure where convergence to equilibrium is the fastest, in only two periods. This is due to the strong direct incentive to reduce the number of meters caused by a per-connection tariff. We also observe that this tariff yields the outcome that is the closest to the optimal system ("Best PV," "Best battery," "Best welfare"). Indeed, with capacity- or energy-based tariffs, households have a strong incentive to exert high efforts to decrease their bill once they have formed into a community: they engage in a race to build more PV and batteries so as to decrease their peak or energy demand from the grid, and as a consequence, a massive over-investment in PV is observed, as a comparison between "Final PV" and "Best PV" values reveals. However, grid costs remain the same and ultimately still have to be shared among consumers. It results in a prisoner's dilemma equilibrium whereby all coalitions try to ensure that other coalitions bear the grid costs. No such effect happens with a per-connection fee, that do not affect consumers' usage of the grid - set aside from the initial incentive to form into a community.

Our results indicate that imposing a fixed tariff to recover the cost partially brings the buildings closer to the optimal system: PV and battery over-investments are avoided and the social welfare is quite high as compared with the other cases. This result is similar to what has been empirically found in part of the literature related to efficient tariffs. In markets where fixed costs are important, [14] was a pioneer in understanding what would be an efficient pricing and his proposal was to set a two-part tariff: one fixed and one proportional to consumption, the variable tariff component being correlated with the marginal supply cost. In [7] and [9], the author analyzed the pricing policy of natural gas and electricity and demonstrated that a strong increase of the fixed-part tariff restores efficiency without harming equity. Similarly, we find that a pure per-connection tariff ($\alpha = \phi = 0$) generates a welfare close to the optimal one. This derives from a virtuous property of such a tariff, namely that communities' contribution to grid cost is independent from how they manage their net load. They will therefore not engage in possibly welfare-destructive investments and load shifting. Mathematically, this means that a given community's optimization program does not depend on their contribution to grid costs (see equation (6)). Because communities optimize investments to reduce energy costs, without a possibility to escape a contribution to grid costs, fixed grid tariff leads to an equilibrium that is closer to the socially optimal one. The optimal social welfare is not reached, though, because both buildings cannot gather in a whole community. On the contrary, grid tariff components α and ϕ strongly influence the PV and battery investment variables, which diverts the equilibrium choices from the optimal ones.

The previous considerations ignored coordination costs. Indeed, setting grid tariffs that prompt the formation of the biggest communities might prevent over-investments, but induce more coordination costs. To go further in the comparison between all our outcomes, one should then analyze the *final welfare coor* figure that also includes coordination costs and is therefore always smaller than *final welfare*. It then appears that when the coordination cost is moderate (10€ per handshake), setting a fixed tariff δ is still the best option for the DSO: not only over-investments are limited, but the *final welfare coor* is also the highest. On the contrary, when coordination costs are too high (50€ per handshake), the aggregation benefits of the formation of big communities, that would also limit over-investments, are not sufficient to compensate for the coordination cost: in that case, setting a grid tariff that is proportional to consumption (ϕ) is the option that maximizes the welfare of communities.

All these considerations lead us to several policy implications that will depend on policy-makers' dominant motivation for communities. If energy communities are seen as a vector to promote investment in PV and battery solutions then capacity-based or energy-based tariffs are suitable: a strong incentive to reduce peak demand or withdrawals from the grid leads households to invest in these new technologies. Furthermore, this paper shows that this effect is magnified by a potential snowball effect that motivates further investments and further community formation. However, the prisoner's dilemma effect we have observed potentially leads to some over-investments in PV, that incur strong costs to the system which may outweigh the benefits of green energy. A policymaker who is willing to moderate over-investments may then want to increase the per-connection tariff component. A policymaker that is more keen on increasing the welfare of communities while reducing their cost of coordination should design the grid tariff as a function this cost. When this cost is small, setting a fixed tariff is still the best option. When this cost is too high, setting an energy-based tariff becomes the best solution. However, one should keep in mind that our welfare calculations are based on purely financial primitives that correspond to a simple cost-benefit analysis from the joint point of view of buildings 1 and 2. A comprehensive picture should take account of the benefits in terms of environmental conservation which would then increase the appeal of a higher α or ϕ. Similarly, if a policymaker is primarily motivated by the sense of community that may be created by such coalitions, per-connection fees are most suitable as it forms the largest communities. We wish to stress here that these results are derived from numerical analysis rather than formal proofs. While we are confident that our insights carry over to many cases, a practical implementation and in particular the precise understanding of the magnitudes at play would require the computation of similar equilibria on a much bigger system. We believe this goes beyond the scope of this paper. More modestly, we aim at drawing attention to the existence of a potential "snowball" effect of communities through grid tariffs, and suggests tools to make sure this effect may serve rather than contrast the purpose of policy makers.

5 Conclusion

Our research shows that grid tariffs are a strong determinant of community formation, and of the subsequent installation of new technologies once communities are formed. In addition, a grid cost recovery constraint implies that there may be substantial spillovers between communities and a snowball effect will likely be observed.

To capture this effect, we have modeled an interaction between energy communities and the DSO. The formation of energy communities is treated using cooperative game theory, that we embed in a non-cooperative game framework to define the equilibrium between themselves and a local DSO, and come up with a simple way to calculate it. We have applied our model to the schematic interaction between the energy communities of two different buildings of the same neighborhood. Our results are multiple: depending on policymakers' motivations, the snowball effect may be deemed beneficial especially if the policy goal is to increase PV or battery installations. However, if communities form too large a share of the consumer-base, investments in new technologies may be excessive and may prompt policymakers to favour tariff structures that induce weaker spillovers.

Future research can focus on considering a bigger size of the neighborhood (more buildings) that could mitigate the impact of the grid recovery constraint and its inherent snowball effect. Other drivers to form communities, such as environmental concerns, could also be taken into consideration. Finally, a theoretical study of the existence and uniqueness of equilibria can also be developed.

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APPENDIX

A Notation

The following array summarizes the notation used in the paper. We call "variables" all the decisions that will be taken by the coalitions (PV and battery investments, operations of the battery, auto-consumption and PV injection in the grid) or the DSO (grid tariffs):

Sets	
$\{1, 2, ..., B\}$	set of buildings. Indexed by b
I_b	set of households of building b. Indexed by i_b
T	time. Indexed by t
$\mathcal{P}(I_b)$	set of all coalitions of I_b, that we denote $S_b \subset I_b$
Parameters	
$f_i(t)$	consumption profile of household i (kWh)
$g(t)$	PV production profile (kWh per kW)
$\bar{\mu}_b$	upper bound of all $\mu(S_b)$, $\mu(S_b) \in [0, 1]$
$c_{PV}(.)$	PV investment cost as a function of capacity (€)
$c_{Bat}(.)$	battery investment cost as a function of capacity (€)
$c_{coo}(.)$	coordination costs of coalitions (€)
ch	battery charging limit (in kW)
dch	battery discharging limit (in kW)
β	electricity retail price (€/kWh)
γ	electricity wholesale price or feed-in tariff (€/kWh)
COST_{DSO}	the grid cost incurred by the DSO that she has to recover (€)
Variables decided by the communities	
$\mu(S_b)$	factor of proportionality of the invested PV capacity with respect to consumption (no unit).
$inj(S_b)_t$	energy produced and injected into the grid, time t (kWh)
$aut(S_b)_t$	energy produced and locally consumed, time t (kWh)
$Bat(S_b)$	battery capacity (kW)
$st(S_b)_t$	energy stored in battery, time t (kWh)
$with(S_b)_t$	withdrawn energy from battery, time t (kWh)
$l(S_b)_t$	level of the battery, time t (kWh)
Variables decided by the DSO	
α	grid tariff component related to capacity (€/kW)
ϕ	grid tariff component related to energy (€/kWh)
δ	fixed component of the grid tariff (€)

B Detailed calculation of the value of a coalition

This appendix calculates the value of a given set of households S_b of size s_b living in the generic building b. The payoff and cost of the coalition can be split into different terms:

- **Savings from the grid tariff.** Before joining the coalition S_b, each household $i_b \in S_b$ had to pay the following individual grid tariff (IGT stands for Individual Grid Tariff):

$$\text{IGT}_{i_b} = \alpha \text{Max}_t \left(f_{i_b}(t) \right) + \phi \sum_{t=1}^{T} f_{i_b}(t) + \delta \tag{13}$$

The coalition as a whole has only one meter to install and pays a different grid tariff that depends on its auto-consumption $aut(S_b)_t$ as follows (CGT stands for Coalition Grid Tariff):

$$\text{CGT}(S_b) = \alpha \text{Max}_t \left(\sum_{i_b \in S_b} f_{i_b}(t) - aut(S_b)_t \right) + \phi \sum_{t=1}^{T} \left(\sum_{i_b \in S_b} f_{i_b}(t) - aut(S_b)_t \right) + \delta \tag{14}$$

Therefore, the savings from the grid tariff earned by the coalition are:

$$\begin{aligned} \text{CGS}(S_b) &= \sum_{i_b \in S_b} \text{IGT}_{i_b} - \text{CGT}(S_b) \qquad (15) \\ &= \alpha \left(\sum_{i_b \in S_b} \text{Max}_t \left(f_{i_b}(t) \right) - \text{Max}_t \left(\sum_{i_b \in S_b} f_{i_b}(t) - aut(S_b)_t \right) \right) + \phi \sum_{t=1}^{T} aut(S_b)_t + \delta(s_b - 1) \end{aligned}$$

- **Payoffs of auto-consumption.** Auto-consumption is remunerated at the retail price β. The payoff of the coalition due to local consumption is then expressed as follows (CAC stands for Coalition Auto-Consumption payoff):

$$\text{CAC}(S_b) = \beta \sum_{t=1}^{T} aut(S_b)_t \tag{16}$$

- **Payoffs of energy injection in the grid.** Part of the solar energy that is produced by the coalition can be injected into the grid at a tariff γ. The corresponding payoff is then expressed as follows (CGT stands for Coalition Grid Injection payoff):

$$\text{CGI}(S_b) = \gamma \sum_{t=1}^{T} inj(S_b)_t \tag{17}$$

- **PV and battery investments costs.** Recall that coalition S_b chooses the PV investment variable $\mu(S_b)$ (that corresponds to a PV capacity of $\mu(S_b) \frac{\sum_{i_b \in S_b} \sum_{t=1}^{T} f_{i_b}(t)}{\sum_{t=1}^{T} g(t)}$) with the battery investment variable $Bat(S_b)$. The total cost of investment is therefore given as follows (CIC stands for Coalition Investment Cost payoff):

$$\text{CIC}(S_b) = c_{PV} \left(\mu(S_b) \frac{\sum_{i_b \in S_b} \sum_{t=1}^{T} f_{i_b}(t)}{\sum_{t=1}^{T} g(t)} \right) + c_{Bat} \left(Bat(S_b) \right) \tag{18}$$

- **Coordination costs.** Members of coalition S_b will have to organize in order to manage the functioning of the coalition. This comes at the coordination cost $c_{coo}(S_b)$:

The value of coalition S_b is then the maximum possible benefit made by S_b, $\text{CGS}(S_b) + \text{CAC}(S_b) + \text{CGI}(S_b) - \text{CIC}(S_b) - c_{coo}(S_b)$, when optimally deciding the PV and battery investment variables $\mu(S_b)$ and $Bat(S_b)$ and their operations $(aut(S_b)_t, inj(S_b)_t, st(S_b)_t, with(S_b)_t, l(S_b)_t)$. This value will be denoted $v(S_b, \alpha, \phi, \delta)$ to emphasize the fact that it depends on the DSO's decisions (the grid tariff components):

$$
\begin{array}{lll}
v(S_b,\alpha,\phi,\delta) = & \text{Max} & \alpha\left(\sum_{i_b\in S_b}\text{Max}_t\,(f_{i_b}(t)) - \text{Max}_t\left(\sum_{i_b\in S_b} f_{i_b}(t) - aut(S_b)_t\right)\right) \\
& & +\phi\sum_{t=1}^{T} aut(S_b)_t + \delta(s_b-1) \\
& & +\beta\sum_{t=1}^{T} aut(S_b)_t \\
& & +\gamma\sum_{t=1}^{T} inj(S_b)_t \\
& & -c_{PV}\left(\mu(S_b)\dfrac{\sum_{i_b\in S_b}\sum_{t=1}^{T} f_{i_b}(t)}{\sum_{t=1}^{T} g(t)}\right) - c_{Bat}\left(Bat(S_b)\right) \\
& & -c_{coo}(S_b) \\
& & \\
& \text{s.t.} & 0 \le \mu(S_b) \le \bar{\mu}_b \\
& & 0 \le aut(S_b)_t,\ 0 \le inj(S_b)_t \\
& & 0 \le Bat(S_b),\ 0 \le st(S_b)_t,\ 0 \le with(S_b)_t,\ 0 \le l(S_b)_t \\
& \forall t = 1,...,T & \mu(S_b)\dfrac{\sum_{i_b\in S_b,t} f_{i_b}(t)}{\sum_{t=1}^{T} g(t)} g(t) + with(S_b)_t - st(S_b)_t = aut(S_b)_t + inj(S_b)_t \\
& \forall t = 2,...,T & l(S_b)_t = l(S_b)_{t-1} + st(S_b)_t - with(S_b)_t \\
& \forall t = 1,...,T & l(S_b)_t \le Bat(S_b) \\
& \forall t = 1,...,T & st(S_b)_t \le ch\ Bat(S_b) \\
& \forall t = 1,...,T & with(S_b)_t \le dch\ Bat(S_b)
\end{array}
\tag{19}
$$

C Partitioning of a building

A partition of building b is a collection of different separate subsets of I_b (recall that I_b is the set of all households living in building b) that add up to the whole set I_b:

Definition 5. *A partition $P_b = \{S_b^1, S_b^2, ..., S_b^p\}$ of size p of building b is a collection of subsets of I_b satisfying:*

- $\forall k \in \{1,2,...,p\},\ S_b^k \subset I_b$
- $\forall k \in \{1,2,...,p\},\ S_b^k$ *is not empty:* $S_b^k \neq \Phi$
- $\forall k,l \in \{1,2,...,p\},\ k \neq l \Longrightarrow S_b^k \cap S_b^l = \Phi$
- $\cup_{k=1}^{p} S_b^k = I_b$

We now define the notion of optimal partition:

Definition 6. *Given grid tariffs (α,ϕ,δ), partition $P_b = \{S_b^1, S_b^2, ..., S_b^p\}$ of size p of building b is optimal if:*

- *P_b contains only stable coalitions according to Definition 2: $\forall S_b \in P_b$, S_b is a stable coalition. We thus say that P_b is then a stable partition of building b.*
- *Among all stable partitions of building b, P_b has the highest value:*

$$\forall Q_b \text{ stable partition of building } b,\ VP(P_b,\alpha,\phi,\delta) \ge VP(Q_b,\alpha,\phi,\delta) \tag{20}$$

D Calculation of the payment received by the DSO

Payments given to the DSO by all energy communities can be calculated as follows (we assume that partition P_b splits building b into p_b coalitions, or in other words, that partition P_b is of size p_b):

- **Capacity payment.** Each coalition S_b^k of partition P_b of building b has a peak residual consumption, that is, the sum of the demand minus the amount of auto-consumption: $\text{Max}_t \left(\sum_{i_b \in S_b^k} f_{i_b}(t) - aut(S_b^k)_t \right)$. The capacity payment charged by the DSO is then:

$$CP\left(S_b^k\right) = \alpha \text{Max}_t \left(\sum_{i_b \in S_b^k} f_{i_b}(t) - aut(S_b^k)_t \right)$$

- **Energy payment.** Each coalition S_b^k of partition P_b of building b pays to the DSO an amount that is proportional to its residual consumption: $\sum_{t=1}^{T} \left(\sum_{i_b \in S_b^k} f_{i_b}(t) - aut(S_b^k)_t \right)$. The energy payment charged by the DSO is then:

$$EP\left(S_b^k\right) = \phi \sum_{t=1}^{T} \left(\sum_{i_b \in S_b^k} f_{i_b}(t) - aut(S_b^k)_t \right)$$

- **Fixed payment.** Each coalition S_b^k of partition P_b of building b pays δ to the DSO corresponding to the installation of their meter in the building:

$$FP\left(S_b^k\right) = \delta$$

Therefore, the total payment received by the DSO depends on the partitions of the buildings $(P_1, P_2, ..., P_B)$ as follows:

$$\begin{aligned} \text{DSO}_{\text{Payoff}}\left(P_1, P_2, ..., P_B\right) &= \sum_{b=1}^{B} \sum_{k=1}^{p_b} \left(CP\left(S_b^k\right) + EP\left(S_b^k\right) + FP\left(S_b^k\right) \right) \\ &= \alpha \sum_{b=1}^{B} \sum_{k=1}^{p_b} \text{Max}_t \left(\sum_{i_b \in S_b^k} f_{i_b}(t) - aut(S_b^k)_t \right) \\ &\quad + \phi \sum_{b=1}^{B} \sum_{k=1}^{p_b} \sum_{t=1}^{T} \left(\sum_{i_b \in S_b^k} f_{i_b}(t) - aut(S_b^k)_t \right) \\ &\quad + \delta \sum_{b=1}^{B} p_b \end{aligned} \tag{21}$$

E Calculation of the equilibrium

This section gives a simple algorithm to calculate the equilibrium. The algorithm mimics the snowballing effect:

- **Initialization:** Starting from the initial value of grid tariff parameters $(\alpha^0, \phi^0, \delta^0)$, we calculate optimal partitions of all buildings, as shown in Definition 6, that we denote $\left(P_1^0, P_2^0, ...P_B^0\right)$. This calculation is done by listing all possible partitions of each building and selecting one that is stable and provides the highest value.
- **Iteration:** Given the optimal partition at step $n-1$, $\left(P_1^{n-1}, P_2^{n-1}, ...P_B^{n-1}\right)$, the DSO adjusts the grid tariff to $(\alpha^n, \phi^n, \delta^n)$ to recover its cost as explained in Definition 3. As a reaction to this change of tariff, communities might find another way to optimally reorganize: given $\alpha^n, \phi^n, \delta^n$, we calculate optimal partitions of all buildings, as shown in Definition 6, that we now denote $(P_1^n, P_2^n, ...P_B^n)$.

- **Convergence criterion:** The algorithm iterates until convergence of the tariff α^n (which then ensures that the structures of the energy communities in all buildings also converge). Formally, given a convergence criterion ϵ, the algorithm will stop when:

$$|\alpha^n - \alpha^{n-1}| + |\phi^n - \phi^{n-1}| + |\delta^n - \delta^{n-1}| \leq \epsilon \tag{22}$$

As explained above, it is difficult to obtain a unique equilibrium if we let the DSO set all three grid tariff components α, ϕ and δ at the same time. Therefore, in the following numerical applications we will assume that she can set only one of these parameters to react to the creation of energy communities, whereas the other two are kept constant and equal to their historical values (α^0, ϕ^0 or δ^0). Section 4.2 describes in detail the snowball effect when the DSO changes only the energy component α of her tariff. Section 4.3 examines the effect of a change in the other two components of the grid.

The algorithm we have just presented has two advantages. First, it can be shown that when it converges, the algorithm converges toward an equilibrium. Second, by iterating between the organization of energy communities and the DSO grid recovery constraint, the algorithm mimics a possibly realistic succession of reactions of energy communities to the DSO tariffs and the one of the DSO to the formation of energy communities. The snowballing effect is the consequence of these successive reactions to the formation of communities. This succession of interactions could also justify how the equilibrium among communities and the DSO could be reached in reality.

F Data sources

The energy sector being particularly complex, there exists a wide diversity of candidate parameters on which to base our analysis, reflecting the diversity of existing grid tariffs, PV costs, and profiles of demand and production, etc. In addition, PV installations are characterized by a relatively long duration, and many economic or regulatory parameters are likely to evolve throughout the lifetime of a community. Even though we are aware of these difficulties we believe the present sources and estimates give a reasonable account of the information available to households when they take their decision to form a community, as of today. All the sources of our numerical applications are public.

On the demand side, we simulated composite household load curves using a publicly-available load generator LoadProfileGenerator (www.loadprofilegenerator.de). The timestamp is per minute, reduced to hourly averages for ease of computation. On the supply side, the shape of the solar generation curve is calibrated on 2014 solar production, as reported by ELIA (in Belgium). The data is per 15-minutes granularity, converted into hourly granularity by averaging. We assumed each household or community could cover a maximum of 80% of their annual demand with PV production ($\mu_1 = \mu_2 = 0.8$). This value has been calibrated so that it corresponds to up to 35 square meters of available surface to install PV panels, for a typical building composed of representative households.

Costs of PV have been calibrated on the observed current prices of commercial PV panels. We assume a 30-year panel lifetime with a 5% discount rate. These values are in line with most of the recent estimates for PV panels – see e.g., [29], [19] or [56]. Battery installation costs and technical constraints are those of the Tesla Powerwall battery ([58]). We assume a 15-year battery lifetime with a 5% discount rate.

Two sources of gains are taken into consideration in the numerical applications. First, the PV production can be injected into the grid. In that case we assume communities receive the average 2014 spot price in Germany.[4] Second, PV can be locally consumed.

[4]These prices can be found on the EPEX website.

This marginally decreases the community bill by the retail price of electricity in Germany (280 €/MWh).

Regarding coordination costs, we assume costs of 10 or 50€ per handshake, which results in a total coordination cost of 150€ or 750€ per year for an entire building (our buildings are composed of six households).

Results are in 2018 euro.

G Article 21 of the Renewable Energy Directive (European Commission)

1. Member States shall ensure that renewable self-consumers, individually or through aggregators: (a) are entitled to carry out self-consumption and sell, including through power purchase agreements, their excess production of renewable electricity without being subject to disproportionate procedures and charges that are not cost-reflective; (b) maintain their rights as consumers; (c) are not considered as energy suppliers according to Union or national legislation in relation to the renewable electricity they feed into the grid not exceeding 10 MWh for households and 500 MWh for legal persons on an annual basis; and (d) receive a remuneration for the self-generated renewable electricity they feed into the grid which reflects the market value of the electricity fed in. Member States may set a higher threshold than the one set out in point (c). 2. Member States shall ensure that renewable self-consumers living in the same multiapartment block, or located in the same commercial, or shared services, site or closed distribution system, are allowed to jointly engage in self-consumption as if they were an individual renewable self-consumer. In this case, the threshold set out in paragraph 1(c) shall apply to each renewable self-consumer concerned. 3. The renewable self-consumer's installation may be managed by a third party for installation, operation, including metering, and maintenance.

H Proofs

This appendix proves claim 2 of section 2:

1. Conditions for a snowball effect are modified when game sharing aspects are considered. In particular, if the coordination cost of a coalition is proportional to the number of handshakes in the coalition, $\psi(n-1)n$, the set of grid tariffs δ that lead to a snowball effect is of larger measure when the stability of communities is considered, than when only a cost benefit analysis is taken into account.

The coordination cost function is denoted $c(n) = \psi n(n-1)$. To avoid analyzing simple particular results, we will assume that $4 \leq n_1 < n_2$. Functions g, h and $j = g + h$ can then be calculated as follows:

$$\forall x \in \mathbb{R}, \quad \begin{aligned} g(x) &= \; 2\psi(x-1) \\ h(x) &= \; \psi(x-1)(x-2) \\ j(x) &= \; \psi x(x-1) \end{aligned} \tag{23}$$

- With a cost benefit analysis, a snowball effect occurs when:

$$g(n_1) \leq \delta \leq g(n_2) \leq \delta\prime = \delta\frac{n_1 + n_2}{n_2 + 1} \tag{24}$$

The set of δs that fulfill equation (24), which we denote CBA, is therefore:

$$CBA = \{\delta \geq 0, \text{ such that } \delta \geq g(n_1),\ \delta \leq g(n_2), \text{ and } \delta \geq g(n_2)\frac{n_2+1}{n_1+n_2}\} \quad (25)$$

or

$$CBA = \left[\text{Max}\left(g(n_1), g(n_2)\frac{1+n_2}{n_1+n_2}\right), g(n_2)\right] \quad (26)$$

and its size will be denoted $\Delta_{CBA} = g(n_2) - \text{Max}\left(g(n_1), g(n_2)\frac{1+n_2}{n_1+n_2}\right)$.

- Similarly, when the stability of a community and profit sharing are considered, the snowball effect occurs when δ is the set CGT:

$$CGT = \left[\text{Max}\left(j(n_1), j(n_2)\frac{1+n_2}{n_1+n_2}\right), j(n_2)\right] \quad (27)$$

and its size will be denoted $\Delta_{CGT} = j(n_2) - \text{Max}\left(j(n_1), j(n_2)\frac{1+n_2}{n_1+n_2}\right)$.

The goal now is to prove that $\Delta_{CGT} \geq \Delta_{CBA}$, whenever $4 \leq n_1 < n_2$.

Four cases have to be analyzed as follows:

1. **Case 1**: $g(n_1) \leq g(n_2)\frac{1+n_2}{n_1+n_2}$ and $j(n_1) \leq j(n_2)\frac{1+n_2}{n_1+n_2}$.
In that case, one can deduce that:

$$\begin{aligned}\Delta_{CBA} &= g(n_2) - g(n_2)\frac{1+n_2}{n_1+n_2} \\ \Delta_{CGT} &= j(n_2) - j(n_2)\frac{1+n_2}{n_1+n_2}\end{aligned}$$

Hence,

$$\Delta_{CGT} - \Delta_{CBA} = (j(n_2) - g(n_2))\frac{n_1-1}{n_1+n_2} \quad (28)$$

Because $j - g = h \geq 0$ over $[2, +\infty[$, one will have $\Delta_{CGT} \geq \Delta_{CBA}$.

2. **Case 2**: $g(n_1) > g(n_2)\frac{1+n_2}{n_1+n_2}$ and $j(n_1) > j(n_2)\frac{1+n_2}{n_1+n_2}$.
In this case, we will have:

$$\Delta_{CGT} - \Delta_{CBA} = (j(n_2) - j(n_1)) - (g(n_2) - g(n_1)) = (j-g)(n_2) - (j-g)(n_1) \quad (29)$$

$$\Delta_{CGT} - \Delta_{CBA} = h(n_2) - h(n_1) \quad (30)$$

Function h is a convex and increasing function on $[2, +\infty[$, which means that $\Delta_{CGT} \geq \Delta_{CBA}$.

3. **Case 3**: $g(n_1) > g(n_2)\frac{1+n_2}{n_1+n_2}$ and $j(n_1) \leq j(n_2)\frac{1+n_2}{n_1+n_2}$.
Here, we will have:

$$\begin{aligned}\Delta_{CGT} - \Delta_{CBA} &= \left(j(n_2) - j(n_2)\frac{1+n_2}{n_1+n_2}\right) - (g(n_2) - g(n_1)) \\ &= \frac{\psi}{n_1+n_2}\left((n_1-3)n_2^2 - (n_1-1)n_2 + 2n_1^2\right) \quad (31)\end{aligned}$$

Given n_1, we consider the term $\left((n_1-3)n_2^2 - (n_1-1)n_2 + 2n_1^2\right)$ as a function $k(n_2)$ of n_2 and we will demonstrate that $\forall n_2 > n_1 \geq 4$, $k(n_2) \geq 0$. Function k is polynomial of degree 2 whose discriminant is $disc(n_1) = -8n_1^3 + 25n_1^2 - 2n_1 + 1$. Function $disc(x)$ is polynomial and its derivative, $disc'(x) = -24x^2 + 50x - 2$, is negative if $x \geq 3$. This indicates that function $disc$ is decreasing over $[3, +\infty[$, or that $\forall n_1 \geq 4$, $disc(n_1) < disc(4) < 0$ and then, function k (as a function of n_2) has no roots and keeps a constant sign that is positive. k being positive, using equation (31), one will deduce that $\forall 4 \leq n_1 < n_2$, $\Delta_{CGT} - \Delta_{CBA} \geq 0$.

4. **Case 4**: $g(n_1) \leq g(n_2)\frac{1+n_2}{n_1+n_2}$ and $j(n_1) > j(n_2)\frac{1+n_2}{n_1+n_2}$.
   In this case, one will have:

$$\begin{aligned} \Delta_{CGT} - \Delta_{CBA} &= (j(n_2) - j(n_1)) - \left(g(n_2) - g(n_2)\frac{1+n_2}{n_1+n_2}\right) \\ &= \frac{\psi}{n_1+n_2}\left(n_2^3 + (n_1-1)n_2^2 + (-n_1^2 - 2n_1 + 2)n_2 - n_1^3 + n_1^2 + 2n_1 - 2\right) \end{aligned} \tag{32}$$

   Given n_1, we consider the term $\left(n_2^3 + (n_1-1)n_2^2 + (-n_1^2 - 2n_1 + 2)n_2 - n_1^3 + n_1^2 + 2n_1 - 2\right)$ as a function $l(n_2)$ of n_2 and we will demonstrate that $\forall n_2 > n_1$, $l(n_2) \geq 0$. The derivative l' of l is defined as follows: $\forall x \in \mathbb{R},\ l'(x) = 3x^2 + 2(n_1-1)x - n_1^2 - 2n_1 + 2$. The discriminant of l', $16n_1^2 + 16n_1 - 20$, is positive if $n_1 \geq 1$. Hence l' has two roots and it can be shown that if $x \geq n_1$, $l'(x) \geq 0$. Therefore l, as a function of n_2, is increasing over $[n_1, +\infty[$ and thus: $\forall n_2 > n_1,\ l(n_2) \geq l(n_1+1) = 2n_1(n_1+2) \geq 0$. Therefore, $\forall 4 \leq n_1 < n_2,\ \Delta_{CGT} - \Delta_{CBA} \geq 0$.

Unfortunately, we could not prove this result for any convex coordination cost function. Empirically, we have observed that it holds for a wide range of possible convex costs functions (exponential, which includes the situation when the coordination cost of a coalition of size s is proportional to the total number of its sub-coalitions 2^s, polynomial of degree 3 or 4, etc.), which gives us some confidence in the robustness of the result.