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Keywords Storage, electricity, market structure, investment, vertical relations

JEL Classification L22, L94.

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Storing Power:
Market Structure Matters*

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1 Introduction

The transition to a low carbon economy will require massive investments in renewable energy. Renewables provide substantial environmental and economic benefits (Borenstein (2012)), but their deployment is not free of obstacles. In particular, the intermittency of renewables poses a challenge for power systems, in which demand and supply have to be equal at all times. For this reason, the pathways to decarbonizing the power sector increasingly rely on energy storage as a means to counteract the volatility of renewable output.[1,2] Whether this objective is actually achieved will crucially depend on firms' incentives to operate and invest in storage facilities. The goal of this paper is to characterize how such incentives shape market outcomes, and to understand how they depend on the market structure.

By storing electricity when renewables' availability is high and releasing it when it is low, storage facilitates the integration of renewables in electricity markets. Furthermore, because storage improves security of supply, it reduces the need to invest in oil-fired or natural gas back-up generators (European Commission (2020)). And last, but not least, by smoothing production over time, storage reduces generation costs and flattens the price curve, which translates into improved production efficiency and lower prices for consumers. The downside is that the costs of investing in energy storage remain high, despite substantial cost reductions over the past decade (BloombergNEF (2020)).

Do markets send adequate signals for firms to invest in storage, or is it necessary to put in place other regulatory arrangements to align social and private incentives? As it is well known, markets fail in internalizing positive externalities, such as the ones listed above, and this naturally leads to under-investment. But, are such externalities the only market failures we should be concerned about? Leaving aside externalities, perfectly competitive markets (both in storage as well as in generation) induce the socially optimal storage decisions (Ambec and Crampes (2019) and Schmalensee (2019)). However, in this paper we show that market power (in storage and/or generation) distorts storage decisions (operation and investment) in ways that increase costs and consumer payments.

[1] Demand response is also an important source of flexibility. Some of the economic issues it raises are similar to the ones raised by storage, with two important differences. First, consumers are usually considered as price-takers. And second, storage requires heavier investments as compared to demand response. However, behavioral, informational and political considerations often introduce obstacles to demand response (Fabra et al. (2020)).

[2] For instance, in the big five markets in Europe (Great Britain, France, Germany, Spain and Italy), energy storage could grow from 3 GW today, to 26 GW in 2030, and 89 GW by 2040, representing one fifth of the total capacity additions that are needed to decarbonize the power sector- the rest being wind, solar, interconnectors and gas peakers (McCarthy and Eager (2020)).

For this reason, market structure is a key determinant of the ability of markets to send efficient signals for storage operators.[3]

We build a stylized theory model that captures the key drivers of storage investment and pricing incentives in wholesale electricity markets. In particular, we assume that the market is served by a fringe of non-strategic producers, one strategic producer, and a set of storage owners. In order to endogenize investment decisions, we assume that storage capacity is chosen once and for all, followed by competition in the wholesale market. Demand moves deterministically over time, from low to high levels over a compact interval,[4] while production entails increasing marginal costs.

Under the welfare maximizing solutions,[5] the planner uses storage to shift production from high to low demand periods in order to minimize generation costs. Moreover, it invests in storage capacity so as to equate the additional marginal cost savings brought about by storage with its per unit investment cost.[6] At the optimal capacity, production is not fully flattened across time as the marginal cost savings of adding storage would fall down to zero, i.e., below the investment cost. Under the competitive market solution, storage owners make profits by arbitraging price differences across demand levels. Since in the absence of market power prices reflect marginal costs, the arbitrage gains capture the cost savings that storage brings about. Hence, the social and private incentives are aligned, absent other market imperfections.

Market power in generation and storage distorts this outcome in opposite directions. Consider first the case in which there is market power in generation, but not in storage. Since the strategic firm's incentives to withhold output are stronger in high demand periods, the price curve becomes steeper the higher the degree of market power. This

[3]The nature of the different storage technologies may give rise to important differences in market structure. For instance, plug-in electric vehicles and in-home batteries are probably better thought as being price-takers, thus giving rise to competitive market structures. In contrast, pumped storage, large batteries and future compressed air facilities are more likely to be in the hands of large firms, possibly vertically integrated with the generators.

[4]Our model focuses on seasonal demand variation while omitting demand uncertainty. By ranking demand rather than adopting a temporal sequence we can make further progress, as compared to existing papers, in comparing equilibrium market outcomes under different market structures. The reason is that we can circumvent the complexity of dynamic decision problems. Although our problem is analytically simpler, in the absence of uncertainty, the properties of the comparison across market structures would remain unchanged in a fully dynamic problem.

[5]We characterize the first-best (the planner can decide both upon generation and storage) and the second-best (she can only decide upon storage, as generation decisions are market-based.

[6]Under the first-best, these marginal cost savings are computed along the industry marginal cost curve. Instead, under the second-best, they are computed along the market supply curve, which is steeper. This implies that the second-best capacity exceeds the first-best capacity.

makes arbitrage more profitable, inducing storage firms to over-invest.

Consider now the case in which a storage monopolist serves a perfectly competitive energy market. The storage firm is no longer a price taker, i.e., it internalizes the impact of its storage decisions on the prices at which it either buys or sells the stored amounts. This leads the storage owner to smooth its storage decisions over time in order to avoid a strong price reduction when it sells and a strong price increase when it buys (i.e., acting as a monopolist or as a monopsonist, respectively). In turn, this smoothing reduces the profitability of storage, and thus leads to under-investment.

These distortions are enhanced in the case in which a vertically integrated firm has market power in both storage and generation. The reason is that the vertically integrated firm not only internalizes the price impacts on its stored output but also on its own generation. This leads to a greater distortion in the allocation of output across firms. For this reason, under reasonable assumptions, this market structure yields the least efficient market outcome, the lowest level of investment in storage capacity, and the lowest level of consumer surplus.

In sum, we find that total welfare and consumers surplus decline as we introduce more layers of market power. Market power in production creates static productive inefficiencies as it distorts the optimal market shares across producers; while market power in storage creates dynamic productive inefficiencies as storage fails to flatten production across demand levels. In both cases, market power gives rise to additional inefficiencies as it distorts the incentives to invest in storage. These impacts ultimately translate into higher prices for consumers.

We illustrate the predictions of our model by simulating the Spanish electricity market under the 2030 energy and environmental targets (MITECO (2020)). Using detailed data on electricity demand, generation units and generation costs, we quantify the improvement in productive efficiency and the reduction in carbon emissions brought about by storage. We also compute the arbitrage profits made by competitive storage firms, and show that they decrease as installed storage capacity goes up. Interestingly, our results point at the complementarity between investments in renewables and storage. On the one hand, storage boosts the profitability of renewables by reducing curtailment at times of excessive renewables availability. On the other, renewables enlarge arbitrage profits, as a result of increased price volatility and more frequent zero-price episodes.

Importantly, even in scenarios with large renewables penetration, arbitrage profits are several orders of magnitude lower than the costs of investments. Accordingly, if regulators want to boost investments in storage (as shown in their decarbonization pathways), they will have to complement the market revenues of storage owners with public support. For this purpose, they could resort to storage capacity auctions to select those firms that are willing to carry out the investments at least cost. By bundling support to price caps or reliability options (Cramton and Stoft (2008)), they could at least partially correct the distortions created by market power on the optimal use of storage. The auctions' eligibility criteria could also serve to avoid that dominant generators increase their market power by investing in storage, as that would also result in an inefficient use of the storage facilities.

Related Literature Our paper relates to a long-standing literature on the role of storage technologies in commodity markets. The canonical theory (Newbery and Stiglitz (1979); Wright and Williams (1984)) focuses on the role of storage in balancing stochastic production in a perfectly competitive environment. Subsequent papers in this literature consider alternative market structures and explore the impact of storage on price volatility and social welfare (McLaren (1999); Newbery (1990); Allaz (1991); Williams and Wright (2005); Thille (2006); Mitraille and Thille (2014)). Our contribution to this literature is two-fold. First, we abstract from issues related to stochastic demand to put the spotlight on the role of strategic interactions and ownership structure. Encompassing different market structures in a single tractable framework allows us to provide a welfare ranking across market structures. Second, in contrast to the previous literature, we characterize endogenous storage investment decisions and relate them to the degree of market power. Interestingly, our results imply that analyzing production and storage decisions in isolation underestimates the welfare distortions created by the exercise of market power.

Within the energy economics literature, there is a long strand of papers analyzing the role of hydro storage and its impact on market power.[7] In an early paper, Borenstein and Bushnell (1999) already note that the availability of hydroelectric production is one of the most important determinants of the severity of market power in wholesale electricity markets. In turn, Bushnell (2003) characterizes how strategic hydro producers exercise market power: by shifting hydro production from peak to off-peak periods in order to avoid depressing market prices when their infra-marginal production is larger (see also Garcia et al. (2001)). A similar result also arises in our paper, but not only when firms decide how to allocate their stored amounts, also when deciding when to schedule their charging decisions. Indeed, there is a key difference between the strategic use of hydro-

[7]See Rangel (2008) for a survey of the papers analyzing the competition issues that arise in hydro-dominated electricity markets. For empirical papers, see Kauppi and Liski (2008) on the Nordic electricity market and McRae and Wolak (2018) and Fioretti and Tamayo (2020) on the Colombian electricity market.

power and pure storage: whereas the former involves allocating an exogenously given amount of output across time (i.e., determined by rainfalls or river flows), the latter involves four types of intimately linked decisions, i.e., when and how much to charge and discharge.[8]

An emerging strand of the literature specifically analyzes the economics of energy storage. First, a set of engineering-oriented studies quantify the value of electricity storage for small storage operators that take prices as given (e.g. Shardin and Szölgyenyi (2016); Steffen and Weber (2016)). In contrast to these papers, our analysis reveals that abstracting from strategic interaction and storage-induced price effects overestimates the profitability of storage investments. Related papers analyze the level of storage capacity needed to deal with the intermittency of renewables (Pommeret and Schubert (2019)), the complementarity between thermal production and storage (Crampes and Moreaux (2010)), or the economic properties of different storage technologies (Crampes and Trochet (2019)). More closely related to us, Ambec and Crampes (2019) and Schmalensee (2019) analyze whether perfectly competitive energy markets provide optimal incentives for investing in storage facilities. We depart from these papers by introducing strategic behavior in both generation and storage under various ownership configurations, revealing a wedge between private and social incentives regarding storage decisions.[9] This incentive misalignment is also present in an empirical paper by Karaduman (2020), who builds a quantitative model of the South Australian Electricity Market to estimate the expected market outcomes under various levels of storage capacity. Our stylized framework complements this analysis in two respects. First, we provide analytical closed-form solutions that single out the differences across different market structures. Second, we expand the set of cases considered by analyzing the effects of vertical integration between generation and storage, which is common in most electricity markets in practice. Last, Schill and Kemfert (2019) perform Cournot simulations of the German electricity market and conclude that strategic firms have incentives to underutilize storage facilities, in line with our theoretical predictions.

[8] Another notable difference between hydro power and pure storage regards their storage cycles: hydro plants are usually designed for seasonal storage to supply water during dry seasons, whereas batteries or pumped storage can store much smaller amounts of energy, with their storage cycle typically spanning over a day.

[9] Sioshansi (2014) and Schill and Kemfert (2011) also compare market outcomes under different market and ownership structures, but do not analyze investment decisions. Nasrolahpour et al. (2016) explores storage investment incentives, but only under the assumption of perfect competition. We also depart from these papers in that, instead of their two period configuration, we allow for a continuum of demand levels. With only two demand levels, storage smoothing would not be possible.

More broadly, our paper is related to the trade literature that allows for strategic arbitrage across countries. The reason is that trade links markets across space, while storage links markets across time. One notable difference is that trade flows are rarely constrained by the infrastructure linking two markets, while storage is typically limited by binding capacity constraints. Hence, while (in the absence of market power) the law of one price (up to transportation costs) applies to the trade context, it does not apply to the storage case. Energy trade is an exception, as electricity and gas trade require cross-border interconnection capacity. It is thus not surprising to find some similarities between our analysis and papers on electricity trade (Joskow and Tirole (2000, 2005) and Yang (2020)), or gas trade (Ritz (2014); Massol and Banal-Estanol (2018)). The main difference however is that the storage capacity allows to 'stock' energy over time, in contrast to the transmission capacity which allows energy to 'flow' at an instant of time. Hence, while it is particularly relevant to understand how and when is a binding storage capacity operated, this question becomes simpler in the context of energy trade (i.e., always use the transmission line at full capacity).

Last, our paper connects with the literature on exhaustible natural resources. Indeed, oil, gas, and minerals, among other natural resources, have two common features with electricity: they are storable and often vulnerable to the exercise of market power. This literature has shown that the optimal extraction path of natural resources follows the "Hotelling rule" both for price-taking storage firms (Hotelling (1931)) as well as for strategic firms (Salant (1976)). Interestingly, our analysis departs from the Hotelling model in that, unlike the case of natural resources in which reserves are exogenously given, in our storage problem firms also have to decide when to store, as well as how much to invest in storage capacity.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the solution to the social planner's problem when she can take production and storage decisions (first-best) or when she can only decide on storage (second-best). These solutions serve as benchmarks to assess the equilibrium market outcomes characterized in Section 4. The analysis considers three alternative market structures for storage ownership: a fringe of competitive storage owners, an independent storage monopolist, and a vertically integrated storage monopolist. Section 5 compares the resulting equilibrium outcomes in terms of consumer surplus and total welfare. Section 6 conducts simulations of the Spanish electricity market for different levels of storage capacity. Section 7 concludes. All the proofs are postponed to the Appendix.

2 The Model

Consider a market for a storable good (e.g., electricity). Its demand can be met through production or through storage. The costs of producing q units are captured by the function $c(q)$, which is increasing and convex, i.e., $c'(q) > 0$ and $c''(q) > 0$. The costs of storing and releasing the good are normalized to zero up to the storage capacity K,[10] while storing above K is impossible. The costs of investing in storage capacity K are given by the function $C(K)$, which is also increasing and (weakly) convex, i.e., $C'(K) > 0$ and $C''(K) \geq 0$. To obtain closed-form solutions, we will assume linear marginal costs of production, i.e. $c'(q) = q$.

Demand, denoted θ, is assumed to be perfectly inelastic up to the consumers' maximum willingness to pay v.[11] Demand takes values in the interval $\left[\underline{\theta}, \overline{\theta}\right]$ during a certain period of time (i.e., the storage cycle), which we will refer to as a 'day'. However, in the interest of tractability, rather than describing demand by the values it takes during the 'day', we describe it by a load duration curve (Green and Newbery (1992)), i.e., an increasing cumulative distribution function $G(\theta)$ that gives the fraction of the day when demand is below a certain level. This characterization implies that demand is monotonically increasing during the 'day'.[12,13] We assume that $G(\theta)$ is everywhere differentiable in the support, with density $g(\theta)$. The density is assumed symmetric around its expected value, denoted $\mathbb{E}(\theta)$.

Investment, production and storage decisions take place in two stages. In the first stage, storage capacity K is chosen once and for all. In the second stage, production and storage decisions are simultaneously chosen by firms (in those cases in which there is more than one) for each of the possible demand levels of the 'day'(s). It is inconsequential

[10] In reality, storage entails costs (the so-called round-trip inefficiencies typically imply that a 25% of the stored amounts are lost). Adding these to the model is feasible but would complicate the analysis without altering its main results. In the simulations presented in Section 6 we introduce these costs.

[11] In the context of electricity markets, θ can be interpreted as total demand net of renewables. We assume that v is large enough so that it never binds. Results do not depend on this assumption, or on demand being price-inelastic, but they make the analysis simpler.

[12] It is possible to extend our analysis to more general demand characterizations, as long as demand during the storage cycle had at most one minimum and one maximum, e.g. if demand follows a sine or cosine function during the day. The notation would be more involved but results would remain unchanged.

[13] This formulation is particularly convenient because it allows to focus on the analysis of market power without having to model the dynamics of energy storage. Most storage models introduce motion conditions, with the stored amounts at each moment of time being non-negative and depending on how much was charged/discharged in the past, and must thus be solved through dynamic programming. These models are complex, and adding market power would make them even more complex to solve.

whether there is a single day or several 'days' as along as they are all identical. At the beginning of each 'day', the storage capacity K is empty, but it can be filled up during the 'day'. The stored amounts can be released any time and without constraints.[14] However, any stored amount becomes valueless.

Market structure in the product market There are two types of producers: a dominant firm (D) and a set of fringe firms (F). Inspired by Perry and Porter (1985), we assume that the existing production assets are split between them: for each cost level, the dominant firm owns a fraction $\alpha \in (0,1)$, while the remaining fraction $(1-\alpha)$ is owned by the fringe. This means that their marginal costs are $c_D'(q) = q/\alpha$ and $c_F(q) = q/(1-\alpha)$, respectively. The competitive industry supply curve remains fixed at $q = c'(q)$ irrespectively of the distribution of assets across firms. Firms' market shares at an efficient output allocation are α for the dominant firm and $(1-\alpha)$ for the fringe. Any departure from those efficient shares would lead to higher production costs. Last, note that α is a measure of the dominant firm's size, i.e., at any given price, the higher α the more it can produce without incurring in losses. Equivalently, α is a measure of the dominant firm's efficiency, i.e., the higher α, the lower the costs that the firm incurs when producing a given quantity.

Market structure in the storage market Regarding storage, we will consider various market structures. First, we will analyze the first-best and the second-best solutions. Under both of them, a social planner chooses how much to invest in storage capacity and when to use it. The difference between the two is that under the first-best, the social planner can also take production decisions, whereas under the second-best, production decisions are market-based. We will compare these benchmarks with three alternative cases in which there is either (*i*) a continuum of competitive storage firms; (*ii*) a single independent storage monopolist; or (*iii*) a vertically integrated firm that owns both production and storage facilities.

[14] In reality, there are constraints on the rate of charging and discharging. In our model, adding these would lead to further storage smoothing over time. Since we want to highlight that storage smoothing arises because of strategic considerations (and not because of binding constraints), we omit these from the analysis.

3 The Social Planner Solutions

3.1 The First-Best

Under the first-best, the social planner takes investment, storage and production decisions in order to maximize total welfare. Because total demand is inelastic, total welfare is simply the sum of gross consumers' surplus net of production costs, minus the costs of investing in storage capacity. Let $q_B(\theta)$ and $q_S(\theta)$ be the quantities that are bought (similarly, charged) and sold (similarly, discharged) through the storage facility, when demand is θ. Since the total amount that has to be produced in order to meet demand is $(\theta - q_S(\theta) + q_B(\theta))$, the first-best solves the following maximization problem:

$$\max_{K, q_B(\theta), q_S(\theta)} W = \int_{\underline{\theta}}^{\bar{\theta}} \left[v\theta - c\left(\theta - q_S(\theta) + q_B(\theta)\right)\right] g(\theta) d\theta - C\left(K\right),$$

subject to two intertemporal constraints. First, the storage facilities cannot store beyond their capacity. And second, they cannot release more than what they have stored. Given our assumptions on demand, these two constraints can be written as

$$\int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta) g(\theta) d\theta \leq K \tag{1}$$

$$\int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta) g(\theta) d\theta \geq \int_{\underline{\theta}}^{\bar{\theta}} q_S(\theta) g(\theta) d\theta. \tag{2}$$

Our first lemma characterizes the optimal use of the storage capacity at the first-best solution, denoted as FB. Figure 1 provides an illustration.

Lemma 1 *At the first-best, for given K, the optimal storage decisions are given by:*

$$q_B^{FB}(\theta) = \max\left\{\theta_1^{FB}(\mu) - \theta, 0\right\} \text{ and } q_S^{FB}(\theta) = \max\left\{\theta - \theta_2^{FB}(\mu), 0\right\}$$

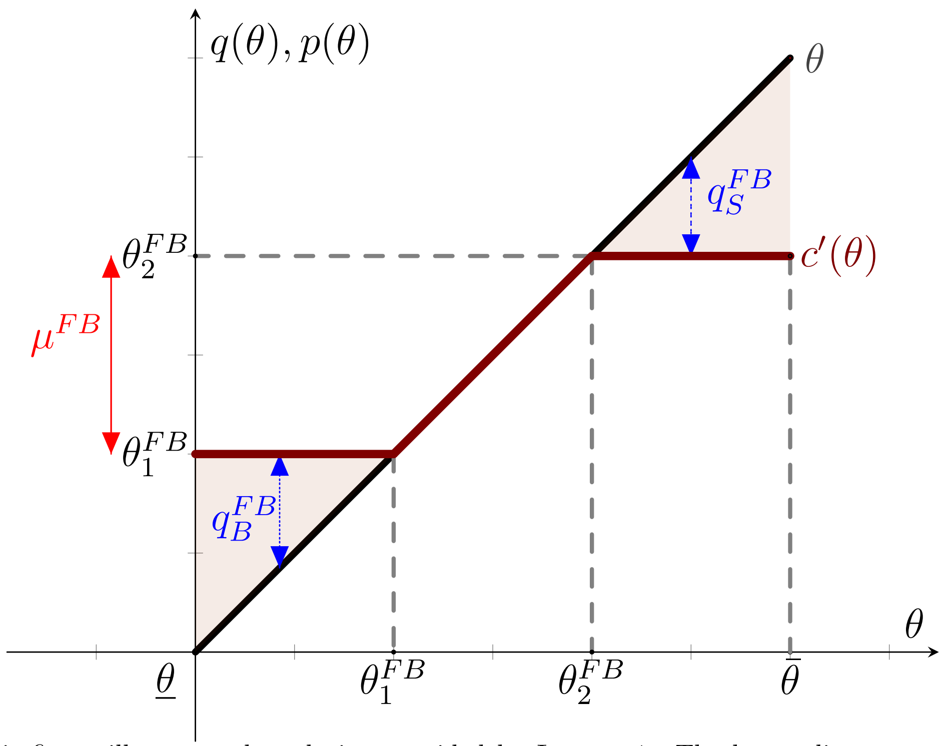
where

$$\theta_1^{FB}(\mu) = \mathbb{E}(\theta) - \frac{\mu}{2} \leq \theta_2^{FB}(\mu) = \mathbb{E}(\theta) + \frac{\mu}{2}, \tag{3}$$

and where μ solves the capacity constraint (1) with equality or it equals zero if the constraint is non-binding.

Proof. See the Appendix. ■

For given capacity K, storage reduces production costs by smoothing production across time. It is optimal to store so as to flatten production at θ_1^{FB} for $\theta < \theta_1^{FB}$, and to release the stored amounts so as to flatten production at θ_2^{FB} for $\theta > \theta_2^{FB}$. If the storage



Notes: This figure illustrates the solution provided by Lemma 1. The brown line represents market demand plus/minus storage decisions. The shaded area represents the amount of stored goods. As can be seen, demand and hence marginal costs are fully flattened whenever the storage facilities are active. The marginal value of storage is found along the industry's marginal cost curve, as depicted by the red arrow.

Figure 1: Optimal storage decisions under the first-best solution

capacity does not bind ($\mu = 0$), production and marginal costs are equalized at $\mathbb{E}(\theta)$ across all periods. Instead, a binding capacity constraint ($\mu > 0$) partially prevents this as, for demand levels between θ_1^{FB} and θ_2^{FB}, the storage capacity remains inactive.

The marginal value of storage capacity is given by $\theta_2^{FB} - \theta_1^{FB}$, i.e., the marginal cost savings from storing an extra unit of output that costs θ_1^{FB} in order to substitute production that would have cost θ_2^{FB} instead. The higher K, the lower the marginal value of storage as the cost savings from transferring output from θ_2^{FB} to θ_1^{FB} become smaller as θ_2^{FB} and θ_1^{FB} get closer to each other.

This leads to our first Proposition, which characterizes the optimal investment in storage.

Proposition 1 *At the first-best, the optimal investment in storage capacity, $K = K^{FB}$, is the unique solution to*

$$C'(K) = \theta_2^{FB}(K) - \theta_1^{FB}(K) > 0. \tag{4}$$

Proof. See the Appendix. ■

At the optimal investment, the marginal value of storage capacity is equal to its unit cost. This implies that the capacity constraint must be binding in equilibrium ($\mu^{FB} > 0$). Otherwise, the marginal value of storage capacity would fall below its unit cost. As a consequence, at the social optimum, storage allows to smooth production and marginal costs, but it does not lead to full price equalization across time.

3.2 The Second-Best

The first-best solution assumes that production is efficient, i.e., the market share allocation between the dominant and the fringe firms is efficient. However, in many instances, the social planner has no control over production decisions. Her role is limited to choosing how much to invest in storage capacity and how to operate it. We refer to the solution of the constrained planner's problem as the second-best.

The equilibrium in the product market is simultaneously determined by the storage decisions of the social planner, $\{q_S(\theta), q_B(\theta)\}$, and the output decisions of the dominant firm and the fringe, denoted $q_D(\theta)$ and $q_F(\theta)$ respectively (Cournot assumption). Since the fringe is willing to produce whenever prices are at or above its marginal costs, the fringe's supply is given by $q_F(\theta) = (1-\alpha)p(\theta)$. Last, because of market-clearing, the inverse residual demand faced by the dominant firm is given by

$$p\left(\theta; q_S, q_B, q_D\right) = \frac{\theta - q_S(\theta) + q_B(\theta) - q_D(\theta)}{1-\alpha}. \tag{5}$$

Taking $\{q_S(\theta), q_B(\theta)\}$ as given, the dominant producer chooses its output $q_D(\theta)$ in order to maximize profits over its residual demand for every demand level θ,

$$\max_{q_D(\theta)} \pi_D = \int_{\underline{\theta}}^{\bar{\theta}} \left[p\left(\theta; q_S, q_B, q_D\right) q_D(\theta) - c_D\left(q_D(\theta)\right)\right] g\left(\theta\right) d\theta. \tag{6}$$

Our next Lemma gives the resulting output allocation between firms, as well as the market price as a function of $\{q_S(\theta), q_B(\theta)\}$.

Lemma 2 *For given $q_B(\theta)$ and $q_S(\theta)$, the quantities produced by the dominant and fringe producers as a function of the storage decisions are given by*

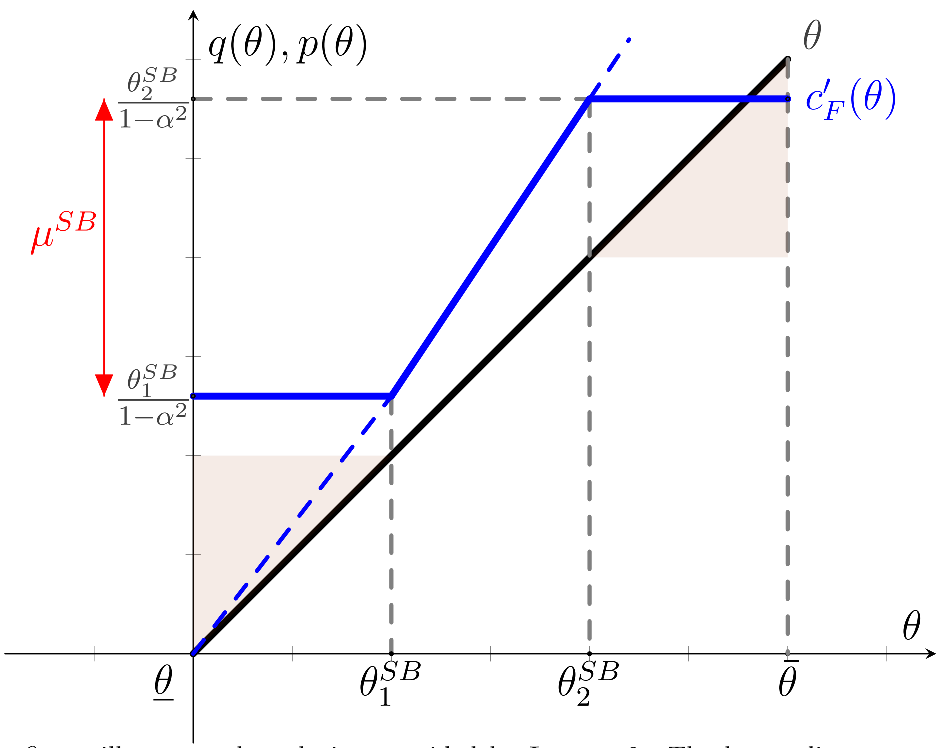
$$q_D(\theta) = \frac{\alpha}{1+\alpha}\big(\theta - q_S(\theta) + q_B(\theta)\big) < q_F(\theta) = \frac{q_D(\theta)}{\alpha},$$

resulting in a market price given by

$$p\left(\theta; q_S, q_B\right) = \frac{\theta - q_S(\theta) + q_B(\theta)}{1-\alpha^2}. \tag{7}$$

Proof. See the Appendix. ■

The dominant producer charges a constant price-cost markup equal to α, for all demand levels. Since the fringe operates at marginal costs, firms' market shares depart from the efficient allocation, giving rise to productive inefficiencies. The higher α, the stronger the dominant firm's market power, and the larger the degree of productive inefficiency.



Notes: This figure illustrates the solution provided by Lemma 3. The brown line represents market demand plus/minus storage decisions. The shaded area represents the amount of stored goods. The blue line gives prices at every demand level. As can be seen, demand is fully flattened, and the marginal value of storage is found along the price curve, as depicted by the red arrow.

Figure 2: Optimal storage decisions under the second-best solution

In turn, taking $q_D(\theta)$ as given, the social planner takes storage decisions $\{q_S(\theta), q_B(\theta)\}$ to maximize total welfare,

$$\max_{q_B(\theta), q_S(\theta)} W = \int_{\underline{\theta}}^{\bar{\theta}} v\theta g(\theta) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \left[c_D\left(q_D(\theta)\right) + c_F\left(\theta - q_S(\theta) + q_B(\theta) - q_D(\theta)\right)\right] g(\theta) d\theta,$$

subject to the intertemporal constraints (1) and (2).

Our next Lemma characterizes, for given K, the planner's storage decisions under the second-best. The solution is illustrated in Figure 2.

Lemma 3 *At the second-best, for given K, the optimal storage decisions are given by:*

$$q_B^{SB}(\theta) = \max\left\{\theta_1^{SB}(\mu) - \theta, 0\right\} \text{ and } q_S^{SB}(\theta) = \max\left\{\theta - \theta_2^{SB}(\mu), 0\right\}$$

where

$$\theta_1^{SB}(\mu) = \mathbb{E}(\theta) - (1-\alpha^2)\frac{\mu}{2} \leq \theta_2^{SB}(\mu) = \mathbb{E}(\theta) + (1-\alpha^2)\frac{\mu}{2}, \tag{8}$$

and where μ solves the capacity constraint (1) with equality or it equals zero if the constraint is non-binding.

Proof. See the Appendix. ■

The storage decisions under the second-best are the same as under the first-best. In particular, storage serves to flatten production at θ_1^{SB} for $\theta < \theta_1^{SB}$ and at θ_2^{SB} for $\theta > \theta_2^{SB}$. Since the storage capacity K is fully used, such demand thresholds are the same as under the first-best. There is however one key difference between Lemmas 1 and 3. Namely, μ^{SB} is now given by the fringe firms' marginal cost savings from moving production from θ_2^{SB} to θ_1^{SB}, and not by the marginal cost savings along the industry competitive supply. The reason is that the social planner takes the dominant firm's supply as given when deciding on the use of the storage facilities (Cournot assumption). Hence, the fringe's supply provides the production flexibility that accommodates the changes in the storage decisions. Since the fringe's supply is steeper than the industry competitive supply, $\mu^{SB} > \mu^{FB}$.

Turning into the optimal investment level, note that the impact of increasing storage capacity on total welfare can be decomposed into two terms:[15]

$$\frac{dW}{dK} = \frac{\partial W}{\partial K} + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial W}{\partial q_D(\theta)} \frac{\partial q_D(\theta)}{\partial K} g(\theta) d\theta.$$

The first term is a direct effect, which results from relaxing the storage capacity constraint, i.e., it is given by $\mu^{SB} = \left(\theta_2^{SB} - \theta_1^{SB}\right)/(1-\alpha^2)$. The second term is a strategic effect: an increase in storage capacity induces the dominant firm to withhold more output, which enlarges the productive inefficiencies and hence reduces total welfare. It follows that that marginal value of storage capacity is below μ^{SB}.

In particular, the marginal value of storage capacity is given by the marginal cost savings from storing an extra unit of output when demand is θ_1^{SB} in order to substitute production when demand is θ_2^{SB}. However, unlike the first-best, these cost savings are now evaluated at the equilibrium market shares, with the dominant firm (fringe) producing an

[15] Using the envelope theorem, the effect of the change in storage decisions vanishes out.

inefficiently low (high) market share (Lemma 2). In particular, for given θ, the average marginal costs (weighted by firms' market shares) are given by

$$\frac{\alpha}{1+\alpha}c_F'\left(\frac{\alpha}{1+\alpha}\theta\right)+\frac{1}{1+\alpha}c_D'\left(\frac{1}{1+\alpha}\theta\right)=\frac{1+\alpha-\alpha^2}{(1+\alpha)(1-\alpha^2)}\theta.$$

Therefore, the marginal cost savings brought about by an additional unit of storage are given by the difference of the above expression evaluated at θ_2^{SB} and θ_1^{SB}.

Our next Proposition characterizes the investment decision at the second-best.

Proposition 2 *At the second-best:*

(i) Equilibrium investment, $K=K^{SB}$, is the unique solution to

$$C'(K)=\frac{1+\alpha-\alpha^2}{(1+\alpha)(1-\alpha^2)}\left[\theta_2^{SB}(K)-\theta_1^{SB}(K)\right].$$

(ii) There is inefficient over-investment in storage,

$$K^{SB}=\frac{1+\alpha-\alpha^2}{(1+\alpha)(1-\alpha^2)}K^{FB}>K^{FB}, \tag{9}$$

which is increasing in α.

Proof. See appendix. ■

How does market power in the product market, α, affect the optimal capacity decision? The bigger the dominant firm, the more output it withholds. Hence, the marginal cost savings (weighted by firms' market shares) brought about by additional storage are greater the higher α. This implies that the optimal investment at the second-best is larger than at the first-best because it has the additional value of reducing the productive inefficiencies created by market power. This over-investment is nevertheless inefficient: if the product market were perfectly competitive, the investment costs of the extra storage capacity would exceed the production cost savings.

The first-best and the second-best serve to assess the market solutions under various market structures, an issue to which we turn next.

4 The Market Solutions

In this section we analyze the optimal storage and investment decisions under three alternative ownership structures: (i) there is a fringe of storage owners; (ii) there is an independent storage monopolist; or (iii) there is a vertically integrated storage monopolist.

4.1 Competitive Storage

We start by considering the case in which storage facilities are in the hands of a large set of small owners, with free entry in storage. Since the storage and the production facilities are independently owned, for given storage decisions $\{q_B(\theta), q_S(\theta)\}$, the equilibrium in the product market remains as in Lemma 2.

Storage operators earn a return from buying the good when prices are low and selling the good when prices are high. Therefore, for given K, at every demand level θ, their problem is simply to choose how much to buy, $q_B(\theta)$, and how much to sell, $q_S(\theta)$, so as to maximize their arbitrage profits, taking market prices as given. Formally, their problem can be written as

$$\max_{q_B(\theta), q_S(\theta)} \pi_S = \int_{\underline{\theta}}^{\bar{\theta}} p\left(\theta\right)\left[q_S(\theta) - q_B(\theta)\right] g\left(\theta\right) d\theta, \tag{10}$$

subject to the intertemporal constraints (1) and (2). The free entry condition implies that there is investment in storage capacity until the returns from storage just cover the investment costs.

Not surprisingly, the operation of storage facilities by competitive firms results in the same pattern of storage use as under the social planner solutions.[16] The planner flattens production, which is equivalent to flattening prices, just like the competitive owners do.

Lemma 4 *Under competitive storage, for given K, the equilibrium storage decisions are the same as under the second-best.*

Proof. See the Appendix. ∎

For the competitive storage owners, the marginal value of capacity is given by the extra arbitrage profits, i.e., the price difference between storing an extra unit at a price $\theta_1^C/(1-\alpha^2)$ in order to sell it at a price $\theta_2^C/(1-\alpha^2)$. Note that the market price is equal to the marginal cost of the fringe, which is steeper than both the industry marginal cost curve and the average marginal cost of the two firms at the market equilibrium. Hence, the marginal value of capacity for the storage owners is greater than under the first-best and the second-best.

This alone would imply that equilibrium investment is inefficiently high, a result that is further strengthened by the combination of free-entry and cost convexity. In particular, because of the free-entry condition, firms invest in storage capacity up to

[16] Indeed, with no risk and hence no missing markets and convexity, this result just derives from the standard welfare theorem. We state it here for completeness.

the level at which the marginal value of storage equals average investment costs. Due to cost convexity,[17] average costs are below marginal costs, giving rise to even greater over-investment, a result which is reminiscent of standard models of market power with fringe entry. In turn, investment is increasing in the degree of market power in the product market, α, as it enhances the marginal value of capacity by making the price curve steeper.

Proposition 3 *When storage is owned by a competitive fringe:*

(i) Equilibrium investment, $K = K^C$, is the unique solution to

$$\frac{C(K)}{K} = \frac{\theta_2^C(K) - \theta_1^C(K)}{1-\alpha^2}. \tag{11}$$

(ii) There is inefficient over-investment in storage, $K^C > K^{SB} > K^{FB}$, which is increasing in α.

4.2 Independent Storage Monopolist

Consider now the case in which the storage facilities are owned by an independent storage monopolist. The main difference with respect to the previous case is that the storage owner now internalizes the effects of its decisions on market prices, and thus on arbitrage profits. Hence, the problem of the storage monopolist can be re-written as in (10), now replacing $p(\theta)$ by the inverse demand (5),

$$\max_{q_B(\theta), q_S(\theta)} \pi_S = \int_{\underline{\theta}}^{\bar{\theta}} \frac{\theta - q_S(\theta) + q_B(\theta) - q_D(\theta)}{1-\alpha} \left[q_S(\theta) - q_B(\theta)\right] g(\theta)\, d\theta, \tag{12}$$

subject to the intertemporal constraints (1) and (2). The problem of the dominant producer is still given by (6).

Our next Lemma characterizes, for given K, the use of the storage facilities by the storage monopolist. Figure 3 illustrates the solution.

Lemma 5 *When storage is owned by an independent monopolist, for given K, the equilibrium storage decisions are given by:*

$$q_B^M(\theta) = \max\left\{\frac{\theta_1^M(\mu) - \theta}{2+\alpha}, 0\right\} \text{ and } q_S^M(\theta) = \max\left\{\frac{\theta - \theta_2^M(\mu)}{2+\alpha}, 0\right\},$$

where

$$\theta_1^M(\mu) = \mathbb{E}(\theta) - \frac{\mu}{2}(1-\alpha^2) \le \theta_2^M(\mu) = \mathbb{E}(\theta) + \frac{\mu}{2}(1-\alpha^2), \tag{13}$$

[17] In the investment cost function $C(K)$ were concave, then the comparison with the first-best and second-best would depend on the relationship between α and the degree of cost concavity.

and where μ solves the capacity constraint (1) with equality or it equals zero if the constraint is non-binding.

Proof. See the Appendix. ■

As in the previous cases, storage allows to shift production across demand levels. Unlike the previous cases, however, it does not lead to a full flattening of production whenever the storage facilities are active. The reason is that the storage monopolist no longer equalizes prices, but rather marginal revenues when it sells (or marginal costs when it buys).[18] As it is standard in a monopoly problem (or symmetrically, in a monopsonist problem), marginal revenue is below the market price because an increase in supply (i.e., an increase in q_S) reduces the price at which the inframarginal units are sold. Symmetrically, an increase in demand (i.e., an increase in q_B) makes it more costly to buy the inframarginal units. Thus, the storage owner smooths storage in order to avoid a strong price reduction when it sells and a strong price increase when it buys. In turn, this prevents production and prices from being fully flattened, and production costs from being minimized.

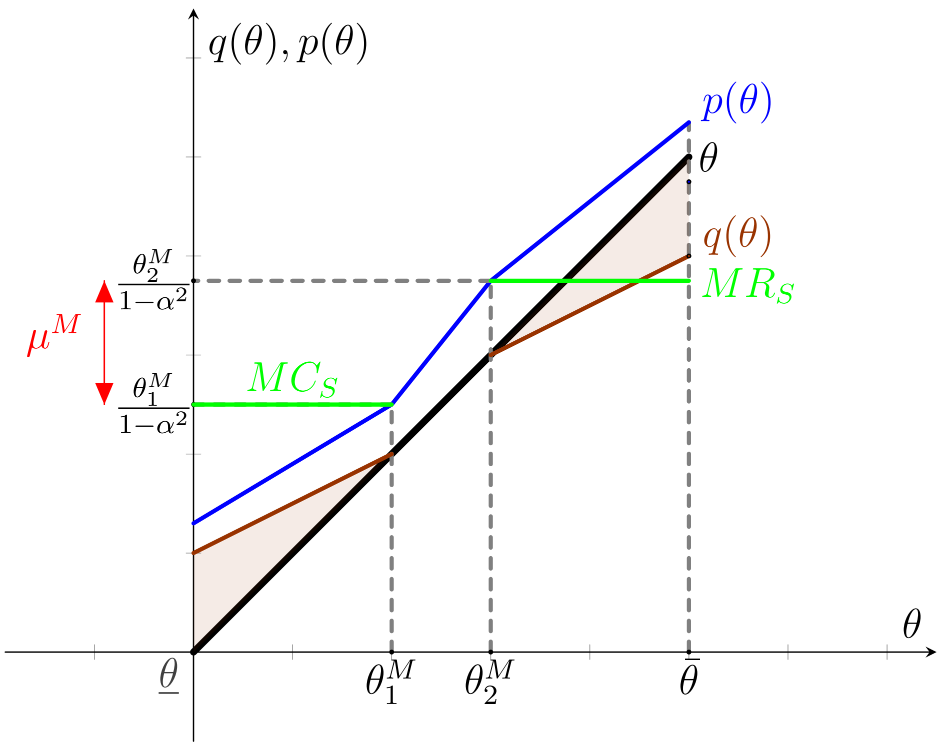
The comparison of Lemma 5 with Lemmas 1 and 3 shows that market power in storage creates an inefficient use of the storage capacity relative to both the first-best and the second-best. First, when the storage capacity is binding (> 0), the region over which the storage facilities are not active is inefficiently short. In other words, because of storage smoothing, the monopolist requires more demand levels to fill the same storage capacity.[19] Second, when the storage capacity constraint is not binding (= 0), the monopolist under-utilizes the existing storage capacity. In particular, a fraction of the storage capacity remains idle despite the scope for marginal arbitrage, which would help to reduce production costs. Again, another source of productive inefficiency.

Note that the degree of storage smoothing is positively related to the degree of market power in the product market, α. The higher α, the steeper is the marginal cost of the fringe, and hence the steeper is the residual demand function faced by the storage owner (see equation (7)). This makes the storage monopolist willing to smooth storage more in order to avoid sharp price changes.[20] In sum, market power in production amplifies the inefficient use of the storage capacity due to market power in storage.

[18] See Bushnell (2003) and Newbery (1990) provide similar results for hydro-power and commodities, respectively.

[19] For given K and the endogenous values of μ, we must have $\theta_1^M\left(\mu^M\right) \geq \theta_1^{FB}\left(\mu^{FB}\right)$ and $\theta_2^M\left(\mu^M\right) \leq \theta_1^{FB}\left(\mu^{FB}\right)$.

[20] If the storage monopolist was a Stackelberg leader, there would be less storage smoothing than under a simultaneous quantity choice model. In the releasing region, the storage monopolist would be able to commit to sell more knowing that the dominant producer would respond by increasing withholding,



Notes: This figure illustrates the solution provided by Lemma 5. The brown line represents market demand plus/minus storage decisions. The shaded area represents the amount of stored goods. The blue line gives prices at every demand level. As can be seen, the storage monopolist does not fully flatten production, but rather its marginal costs and revenues, as shown by the green lines.

Figure 3: Equilibrium storage decisions by the storage monopolist

For the storage monopolist, the marginal value of capacity is again made of two terms, a direct effect and a strategic effect:

$$\frac{d\pi_S}{dK} = \frac{\partial \pi_S}{\partial K} + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial \pi_S}{\partial q_D(\theta)} \frac{\partial q_D(\theta)}{\partial K} g\left(\theta\right) d\theta.$$

First, as in the case of competitive storage, an extra unit of capacity allows the firm to increase its arbitrage profit by buying an extra unit at $p\left(\theta_1^M\right)$ and selling it at $p\left(\theta_2^M\right)$, thus making extra profits $\left(\theta_2^M - \theta_1^M\right)/(1-\alpha^2)$. Due to storage smoothing, θ_1^M and θ_2^M are closer to each other than under competitive storage, thus implying that the marginal arbitrage profit is now lower.

However, there is now a second term that enhances the marginal value of capacity for the storage monopolist. In particular, when it adds new capacity and thus sells (buys) more output, the dominant producer restricts its own output (because of strategic

which would mitigate the price reduction. Under simultaneous quantity choices, this strategic effect is not present.

substitutability, see Lemma 2). This strategic effect partially mitigates the price reduction (increase), thus making storage capacity more valuable. Since the effects when the storage operator buys or sells are of the same magnitude, this is formally captured by

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial \pi_S}{\partial q_D(\theta)} \frac{\partial q_D(\theta)}{\partial K} g\left(\theta\right) d\theta = 2 \int_{\underline{\theta}}^{\theta_1} \left[\frac{\partial p\left(\theta\right)}{\partial q_D(\theta)} \frac{\partial q_D(\theta)}{\partial q_B(\theta)} \frac{\partial q_B(\theta)}{\partial K} q_B(\theta) \right] g(\theta) d\theta > 0.$$

This effect would not be present in the absence of market power in the product market (as the rivals' output decisions would not be affected by the storage decisions, $\partial q_D(\theta)/\partial q_B(\theta) = 0$). Similarly, it would not be present in the absence of market power in storage (as the storage operators would take prices as given, without internalizing the effects of their decisions on market prices, $\partial p\left(\theta\right)/\partial K = 0$). Hence, the combination of market power in both production and storage are necessary to uncover this effect.

Our next Proposition characterizes the equilibrium investment.

Proposition 4 *When storage is owned by an independent storage monopolist:*

(i) Equilibrium investment $K = K^M$ *is the unique solution to*

$$C'(K) = \frac{\theta_2^M\left(K\right) - \theta_1^M\left(K\right)}{1 - \alpha^2} + \frac{2\alpha K}{(1-\alpha^2) G[\theta_1^M(K)]}. \tag{14}$$

(ii) When $\alpha = 0$, $K^{SB} = K^{FB} > K^M$.

(iii) When $\alpha > 0$, *if* θ *is uniformly distributed and* $C'(K) = K$, *then* $K^{SB} > K^{FB} > K^M$.

Proof. See the Appendix. ■

The comparison of the storage monopolist's solution versus the second-best depends on countervailing forces. In the absence of market power in the wholesale market, storage smoothing reduces the marginal gain from arbitrage, thus leading to less investment than under the second-best. However, the presence of market power in generation pushes in the opposite direction. Whether one effect or the other dominates depends on the relative strength of the two sources of market power, which ultimately depends on the shape of $G\left(\theta\right)$ and $C(K)$, as well as on the value of α. We show that for uniformly distributed demand and a linear marginal cost function, the former effect dominates, thus leading to under-investment relative to both the first-best and the second-best.[21]

[21] This result holds for more general investment cost functions as long as this curve is not very steep for low levels of investment.

4.3 Vertically Integrated Storage Monopolist

We now consider the case in which the dominant producer owns all the storage facilities. Hence, the vertically integrated firm decides both on production as well as on storage. Its profit maximizing problem now becomes

$$\max_{q_D(\theta),q_S(\theta),q_B(\theta)} \pi^I = \int_{\underline{\theta}}^{\bar{\theta}} \left[p\left(\theta; q_S, q_B, q_D\right)\left[q_D(\theta) - q_B(\theta) + q_S(\theta)\right] - c_D\left(q_D(\theta)\right)\right] g\left(\theta\right) d\theta,$$

subject to the intertemporal constraints (1) and (2), with the market price given by (5). As compared to (6), the firm now internalizes how its output decisions affect the arbitrage profits made through its storage facilities. Also, as compared to (12), the firm now internalizes how its storage decisions affect the revenues made through its own production.

By replacing $q(\theta) = q_D(\theta) - q_B(\theta) + q_S(\theta)$, the problem would be equivalent to

$$\max_{q(\theta),q_S(\theta),q_B(\theta)} \pi^I = \int_{\underline{\theta}}^{\bar{\theta}} \left[p\left(\theta; q\right) q(\theta) - c_D\left(q(\theta) - q_S(\theta) + q_B(\theta)\right)\right] g\left(\theta\right) d\theta,$$

subject to the intertemporal constraints. As implicit in this formulation, the vertically integrated firm decides on how much output to offer to the market (regardless of whether it comes from its own production or from its storage facilities), and uses storage to minimize the costs of its in-house production. However, its own production is distorted by its incentives to push market prices up.[22]

Our next lemma characterizes the production and storage decisions of the vertically integrated firm, for given K. Figure 4 illustrates the solution.

Lemma 6 *When storage is owned by the dominant producer, for given K, the equilibrium storage and production decisions are given by:*

$$q_B^I(\theta) = \max\left\{\frac{\theta_1^I\left(\mu\right) - \theta}{2}, 0\right\} \text{ and } q_S^I(\theta) = \max\left\{\frac{\theta - \theta_2^I\left(\mu\right)}{2}, 0\right\}, \text{ and}$$

$$q_D^I(\theta) = \frac{\alpha}{1+\alpha} \max\left\{\theta_1^I\left(\mu\right), \theta\right\} \text{ for } \theta < \mathbb{E}(\theta) \quad (15)$$

$$q_D^I(\theta) = \frac{\alpha}{1+\alpha} \min\left\{\theta_2^I\left(\mu\right), \theta\right\} \text{ for } \theta > \mathbb{E}(\theta) \quad (16)$$

where

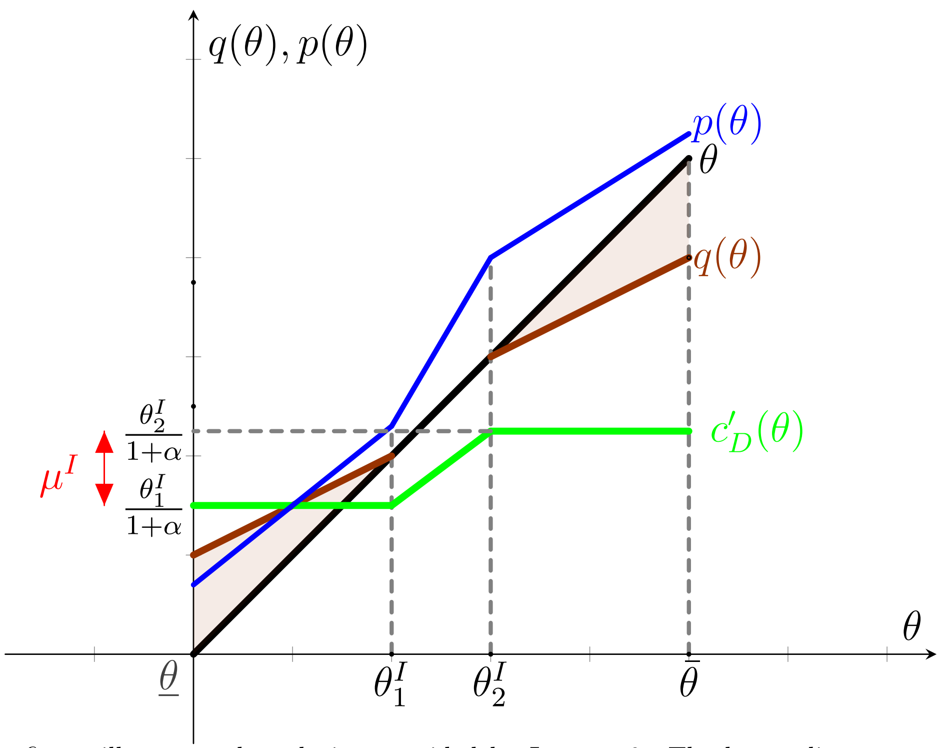
$$\theta_1^I\left(\mu\right) = \mathbb{E}(\theta) - \frac{\mu}{2}(1+\alpha) \leq \theta_2^I\left(\mu\right) = \mathbb{E}(\theta) + \frac{\mu}{2}(1+\alpha),$$

[22] This shows why this solution differs from the first-best, even when α approaches one. Indeed, the vertically integrated firm withholds output to push prices up, which leads to a distorted output allocation between the fringe and the dominant firm. This source of inefficiency is not present under the first-best.

and where μ solves the capacity constraint (1) with equality or it equals zero if the constraint is non-binding.

Proof. See the Appendix. ■

For given capacity K, the vertically integrated firm withholds output to push prices up and uses storage to smooth *its own* production across time. This minimizes its own costs, but gives rise to two sorts of productive inefficiencies. First, because it produces inefficiently little; and second, because it uses storage to flatten its own production, all the changes in demand are fully met by the fringe's production along its steeper marginal cost curve.



Notes: This figure illustrates the solution provided by Lemma 6. The brown line represents market demand plus/minus storage decisions. The shaded area represents the amount of stored goods. The blue line gives prices at every demand level. As can be seen, the vertically integrated firm operates the storage facilities to flatten its own production and thus its own marginal costs, as shown by the green line. Note that prices fall below marginal costs for low θs for which the firm is a net buyer.

Figure 4: Equilibrium storage decisions by the vertically integrated storage monopolist

Interestingly, vertical integration changes the pattern of market power over time given that the firm now internalizes the price effects on its net position $q_D(\theta) - q_B(\theta) + q_S(\theta)$. In particular, the vertically integrated firm no longer charges a constant markup at α (as it was the case for the stand-alone producer). Instead, its mark-up is increasing in demand. For $\theta < \theta_1^I$, the firm charges a markup below α because its net position $q_D(\theta) - q_B(\theta)$ is smaller than in the case of the stand-alone producer. This mark-up even becomes negative when $q_D(\theta) < q_B(\theta)$, which is when the firm is a net-buyer and hence exercises monopsony power by reducing prices below marginal costs. Instead, for $\theta > \theta_2^I$, the firm exercises more market power than in the stand-alone case because its net position $q_D(\theta) + q_S(\theta)$ is now larger. This is summarized below.

Corollary 1 *The demand-weighted mark-up charged by the vertically integrated firm is higher than in the stand-alone case. In particular, the firms charges a mark-up below (above) α for low demand levels $\theta < \theta_1^I$ (for high demand levels $\theta > \theta_2^I$).*

Proof. See the Appendix. ■

Using expressions (15) and (16), the dominant firm's marginal costs at θ_1^I and θ_2^I are $\theta_1^I/(1+\alpha)$ and $\theta_2^I/(1+\alpha)$, respectively. Hence, the marginal value of storage capacity is captured by the marginal cost savings from storing a unit of output that costs $\theta_1^I/(1+\alpha)$ in order to substitute production that would have cost $\theta_2^I/(1+\alpha)$. Accordingly, $\mu^I = (\theta_2^I - \theta_1^I)/(1+\alpha)$. Note that these are the marginal cost savings of the vertically integrated firm, which are below those at the industry level. As a result, there is inefficient under-investment in storage capacity as compared to the first-best. In turn, since the second-best capacity is above the first-best capacity, the equilibrium capacity is also inefficiently low with respect to the second-best.

Two key differences in investment incentives explain the departure from the first-best. The first difference comes from the ability of the vertically integrated firm to exercise market power. To see this, note that in both cases the marginal value of storage capacity is equal to the marginal costs savings for the dominant firm. However, in the first-best these coincide with the marginal cost savings at the industry level, as the dominant firm is producing the socially efficient share of output. In contrast, a dominant firm that behaves strategically withholds output and depresses its marginal costs, which reduces the need to smooth its total production costs by investing in storage capacity. Second, even if the dominant firm priced at marginal cost in the production stage, its incentives to invest in storage would still remain lower, as it does not internalize the cost savings that storage facilities would provide to the competitive fringe. Our next proposition characterizes the optimal investment decision of the vertically integrated firm.

Proposition 5 *When storage is owned by the dominant producer:*

(*i*) *Equilibrium investment $K = K^I$ is the unique solution to*

$$C'(K) = \frac{\theta_2^I(K) - \theta_1^I(K)}{1+\alpha}. \tag{17}$$

(*ii*) *There is inefficient under-investment in storage,* $K^{SB} > K^{FB} > K^{I}$. *The distortion in increasing in* α.

Proof. See the Appendix. ■

Interestingly, and in contrast with the previous cases, storage capacity K^I decreases in the degree of market power in the product market, α. A larger α implies that the dominant firm has lower and flatter marginal costs. Additionally, since a larger α implies that the dominant firm withholds more, the marginal cost savings are computed over a flatter region of the cost function. In sum, the marginal cost savings brought about by an extra unit of capacity are lower the more market power there is, thus making additional storage capacity less valuable the higher α.

5 Comparison across Market Structures

In this section, we compare equilibrium outcomes across market structures to assess the impacts on consumers surplus and overall efficiency. We start by performing the comparison for a given non-binding storage capacity, and then compare market outcomes under a binding capacity constraint. In all cases, we take K as given in order to understand how different players would use a given storage capacity chosen by the regulator.[23]

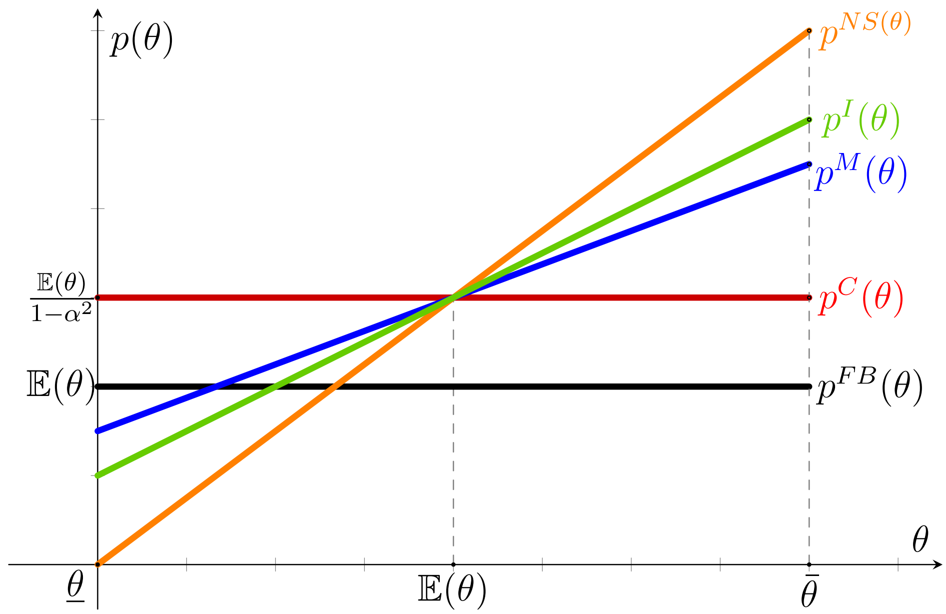
Consumer's surplus can be defined as

$$CS = \int_{\underline{\theta}}^{\bar{\theta}} \left[v - p^i(\theta)\right] \theta g(\theta) d\theta = v\mathbb{E}(\theta) - \mathbb{E}[p].$$

Hence, differences in consumer surplus across market structures are fully driven by differences in the market-weighted average price, denoted $\mathbb{E}[p]$. Market structures affect (*i*) the price levels for each demand realization, as well as (*ii*) the slope of the price pattern over time. Clearly, $\mathbb{E}[p]$ is higher under market structures that give rise to steeper price patterns, even if the unweighted average prices coincide.

To understand how the market structure affects the price level and the slope of the price patterns, it is useful to first consider the case in which the storage capacity constraint K is non-binding. Using our previous results, Figure 5 plots equilibrium prices as a function of demand θ under all market structures considered. We would like to highlight

[23] This approach facilitates the comparison across market structures, while providing a welfare ranking that extends to the case with endogenous storage capacity under some convexity conditions regarding the investment cost function.



Notes: For the case in which storage capacity K is non-binding, this figure depicts equilibrium prices for every demand level θ across all market structures: FB first-best (black), SB second-best and C competitive (red), M storage monopolist (blue), I vertically integrated firm (green) and NS no-storage (orange).

Figure 5: Equilibrium prices across market structures for non-binding storage capacity

three main results that come out of this figure. First, as compared to the case with no storage, storage smooths prices across time. However, only under competitive storage are prices fully equalized across demand levels (recall that we are assuming a non-binding capacity constraint). In contrast, market power in storage results in a steep price pattern, although not as steep as in the absence of storage. Second, regardless of who owns the storage facilities, market power in the product market increases the price level. This can be seen by comparing prices under competitive storage and the first best: both are flat, but the former are higher. Last, if storage facilities are monopoly owned, market power in the product market makes the price pattern both higher as well as steeper, more so under vertical integration than in the case of a stand-alone storage monopolist.

Averaging across all demand levels, the demand-weighted average prices under all market structures considered are given by:

$$\begin{aligned}
\mathbb{E}[p]^{FB} &= \mathbb{E}[\theta]^2 \\
\mathbb{E}[p]^{SB} &= \mathbb{E}[p]^{FB}\frac{1}{1-\alpha^2} \\
\mathbb{E}[p]^{C} &= \mathbb{E}[p]^{SB} \\
\mathbb{E}[p]^{M} &= \mathbb{E}[p]^{SB} + Var[\theta]\frac{1}{(1-\alpha)(2+\alpha)} \\
\mathbb{E}[p]^{I} &= \mathbb{E}[p]^{SB} + Var[\theta]\frac{1}{2(1-\alpha)} \\
\mathbb{E}[p]^{NS} &= \mathbb{E}[p]^{SB} + Var[\theta]\frac{1}{1-\alpha^2}
\end{aligned}$$

Average prices under the first-best simply reflect the average across marginal costs. In all other cases, prices are increasing in α, reflecting two types of mark-ups (i) a mark-up due to market power in the energy market (which is a function of α), and (ii) a markup due to market power in storage (which depends on α and $Var[\theta]$ as both affect the slope of the price pattern faced by storage owners). Since the first mark-up is common across all market structures, the price comparison solely depends on the distortions due to the use of storage. Comparing these expressions, it immediately follows that

$$\mathbb{E}[p]^{FB} < \mathbb{E}[p]^{SB} = \mathbb{E}[p]^{C} < \mathbb{E}[p]^{M} < \mathbb{E}[p]^{I} < \mathbb{E}[p]^{NS}.$$

Regarding total welfare, since demand is assumed to be price-inelastic, it can be expressed as simply the sum of gross consumer surplus minus total costs:

$$TW = v\mathbb{E}(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{q_D^2(\theta)}{2\alpha} + \frac{q_F^2(\theta)}{2(1-\alpha)}\right) g(\theta)d\theta$$

Similarly as before, total costs can be decomposed into two terms:[24] (i) one reflecting static productive inefficiencies, and (ii) another one reflecting dynamic production inefficiencies due to the distorted use of storage. On the one hand, total costs increase due to market power in the product market, which results in distorted market shares between the dominant firm and the fringe. Second, total costs increase due to the misuse of storage, which results in a lack of production equalization across time. As with prices, the second distortion is also amplified by market power in the product market.

The above results naturally carry over to the cases in which the storage capacity K is binding. In particular, for all K, the same outcome as under the second-best can be achieved by allocating K to competitive storage owners, which in turn deliver higher

[24] The expressions can be found in the Appendix.

consumer surplus and higher welfare than when storage is monopolized, either by a stand-alone firm or by a vertically integrated one. However, the comparison of consumer and total welfare in the stand-alone storage monopolist case versus the vertically integrated case depends on countervailing forces. On the one hand, the stand-alone monopolist spreads the use of storage more across time in order to avoid strong price effects. This results in higher production costs (recall that, for given K, $\theta_2^M(K) - \theta_1^M(K) < \theta_2^I(K) - \theta_1^M(I)$). On the other hand, when the dominant producer owns storage it has stronger incentives to withhold output in order to push market prices up. This creates larger static production inefficiencies as the dominant firm's output is replaced by the fringe's. Which of these two effects dominates depends on the degree of market power α and on the shape of the demand distribution $G(\theta)$. For reasonable assumptions on the demand distribution, e.g. uniformly distributed demand, the second effects dominates. This suggests that allocating storage capacity to vertically integrated firms may result in the lowest level of consumer surplus and overall efficiency.

The following Proposition summarizes the above results:

Proposition 6 *(i) For all K, the ranking of consumer surplus and total welfare across market structures is given by, for $j = I, M$:*

$$CS^{FB}(K) > CS^{SB}(K) = CS^C(K) > CS^j(K)$$
$$W^{FB}(K) > W^{SB}(K) = W^C(K) > W^j(K)$$

(ii) Let $\overline{K}$ be the storage capacity that the storage monopolist uses when K is non-binding. For any $K > \overline{K}$, or for any $K < \overline{K}$ and θ uniformly distributed,

$$CS^M(K) > CS^I(K)$$
$$W^M(K) > W^I(K)$$

Proof. See the Appendix. ■

In sum, it is not enough to promote investments in storage if market power in production remains. The reason is that storage facilities will be inefficiently operated if market prices are distorted due to market power. Also, regulators should avoid allocating storage capacity to dominant operators, particularly so if they are vertically integrated firms. This conclusion is further strengthened if regulators rely on pure market mechanisms to spur storage investments. As we have seen, the endogenous investment decisions of a vertically integrated firm depart from the second best solution, further compounding the inefficiencies arising from distortions in the use of storage.

One way for the regulator to implement the second-best is to allow small firms to invest up to K^{SB} and no more (recall that $K^{SB} < K^C$). An alternative would be to run storage auctions for a capacity equal to K^{SB}, but only allow small operators to participate. These options would allow the regulator to correct the distortions arising from competitive over-investment while relying on the market to efficiently operate storage facilities.

6 Simulation of the Spanish Electricity Market

In this section we illustrate some of our theoretical results using actual market data. In particular, we assess equilibrium market outcomes under different levels of storage capacity, and quantify the profitability of storage investment depending on whether electricity producers act competitively or strategically. For these purposes, we perform a series of simulations using the multi-unit auction model developed in De Frutos and Fabra (2012). The model characterizes equilibrium bidding by electricity generators who compete by submitting step-wise bid functions to the auctioneer. Production and prices are set according to a uniform-price auction.

The set of parameters used in the simulations closely replicate the Spanish wholesale electricity market. All simulations are conducted at the hourly level over a one year period (8,760 hours). The technology mix (in terms of capacities) has been set according to the 2030 energy targets, following the 2021-2030 Spanish National Energy and Climate Plan. This includes the deployment of new renewable capacity (mainly solar, but also wind) and the phase out of coal plants and half of the nuclear capacity. The objective is that by 2030, 74% of electricity generation will come from renewable sources.[25] For the plants' ownership structure, we have assumed that all new capacity additions are in the hands of competitive firms.[26] The hourly electricity demand patterns have been set as reported by the Spanish System Operator for 2017 (source: esios.ree.es). The hourly availability factors of the renewable resources have been set at the average of the previous five years, as also reported by the Spanish System Operator. By multiplying these factors times the installed capacity of each technology, we obtain the renewable production at an hourly basis. Hydro production has been allocated to shave the peaks of demand net of renewables. Last, the daily prices for CO2 (EUA) and gas (TTF Hub) have been set at the 2017 prices in international markets (source: Bloomberg). Last, with detailed

[25]See "Plan Nacional Integrado de Energía y Clima 2021-2030", MITECO (2020).

[26]Many capacity additions will certainly be in the hands of the large electricity producers. To the extent that this gives them more market power, our estimated mark-ups provide a lower lower bound of the degree of market power.

data about the gas plants' heat rates and CO2 emission factors, we have computed the marginal costs of these plants used in the simulations.[27]

6.1 Scenarios

For different levels of storage capacity, we compute the optimal storing and releasing decisions when the storage capacity is operated by either an unconstrained social planner (first-best), by a constrained social planner (second-best), or by a set of competitive storage operators (competitive storage). Since the arbitrage gains of competitive storage owners are above the ones internalized by storage owners with market power, our analysis provides an upper bound to the investment incentives provided by market prices. We assume that the time frame for storage operation (i.e. the full storage/release cycle) is the natural day,[28] and that the round-trip efficiency of storage is 0.85, i.e., there is a 15% efficiency loss. For each scenario, we compute the equilibria that would arise if electricity producers behaved competitively (i.e., by bidding at marginal cost) or strategically (i.e., by playing the Nash equilibria in bid functions as in De Frutos and Fabra (2012)).

6.2 Results

For illustrative purposes, Table 1 first provides the summary statistics of two sets of simulations, with and without storage. As it can be seen, more than three quarters of total demand will be covered by renewables, the rest being nuclear, hydro and gas. Demand-weighted average prices will be between 17-22 Euro/MWh, being lower in the competitive than in the strategic scenario, and slightly larger in the scenario with storage. Storage has a clear impact on the maximum and minimum prices, which go down and up respectively, leading to a flatter price curve across the day.[29]

[27] De Frutos and Fabra (2012) provide a detailed description of how these costs are computed.

[28] The time frame depends on several factors, such as the technical characteristics of the storage technology (batteries, pumped storage, etc...). Currently, the most common storage solutions use a 4-hour battery. Clearly, a longer time frame comes with bigger marginal arbitrage profits. However, when considering the lower usage over the life-cycle of the storage facility, daily cycles turn out to be more profitable.

[29] As we explain further down, the relationship between average consumers' prices and storage need not be monotonic. The reason is that storage decreases prices in some hours but increases them in others. The latter are not necessarily the hours with lower demand, but rather those in which prices are lower (which also depends on renewables output and market structure). This is compounded by the fact that the round-trip costs might imply that the lower price is raised more than the upper price is reduced.

	$K = 0$		$K = 10GWh/day$	
	Competitive	**Strategic**	**Competitive**	**Strategic**
Wind	46.9%	46.8%	47.1%	47.0%
Solar	27.0%	27.0%	27.3%	27.3%
Hydro	6.1%	6.1%	6.1%	6.1%
Nuclear	4.9%	4.5%	5.0%	4.7%
CCGT	3.6%	3.6%	3.0%	3.0%
Other renewables	11.5%	12.0%	11.7%	12.1%
Average price	17.25	21.65	17.67	21.88
Standard deviation	19.76	21.67	19.89	21.65
Price (max)	33.91	41.94	32.76	40.94
Price (min)	0.84	0.88	1.02	1.32

Table 1: Simulated market outcomes with and without storage

Notes: Over this period, hourly average demand in the Spanish market was 29,172 MWh. The table displays the average share of each technology in the generation mix (%), average demand-weighted hourly prices (Euro/MWh), average maximum and minimum prices (Euro/MWh) and their standard deviation (Euro/MWh). Two scenarios are considered, one without storage (first two columns) and the other one with storage (last two columns). In each case, we report the results when all generators behave competitively, or when they bid strategically. Storage is operated by competitive non-integrated firms.

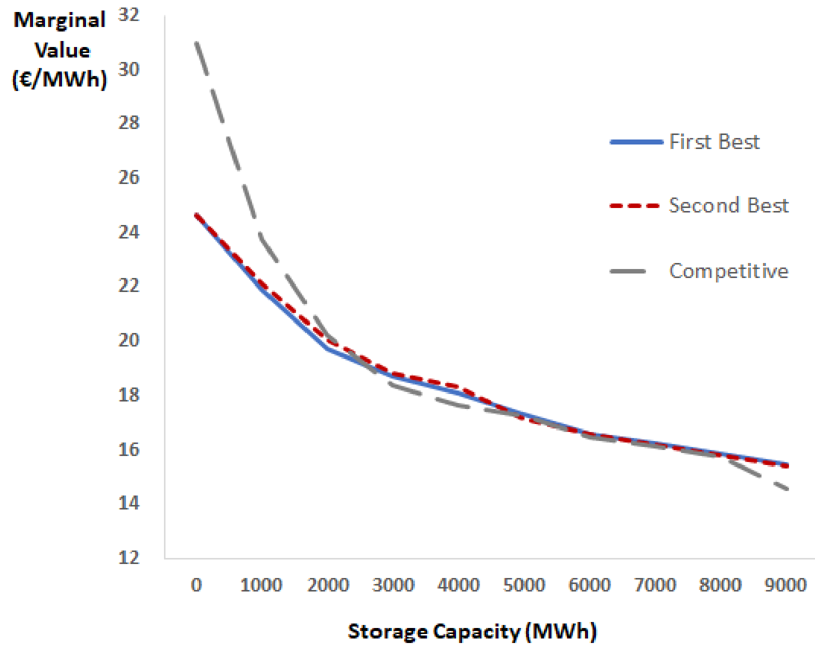
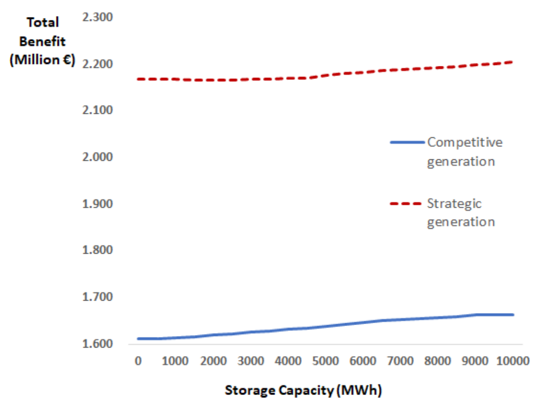


Figure 6: Marginal value of storage capacity (high renewables penetration)

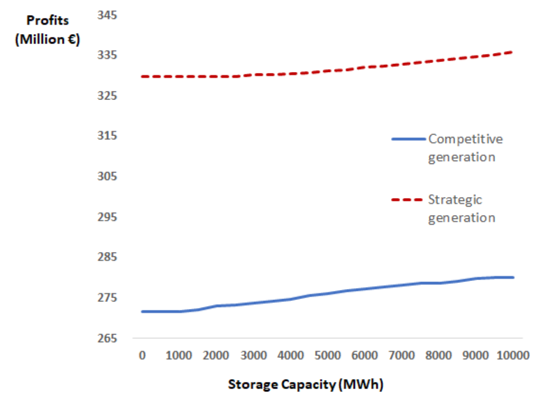
Notes: This figure shows the marginal value of storage capacity investments as a function of capacity, given the generation technology mix that is expected for 2030. It reports results for an unconstrained social planner (first-best), a constrained social planner (second-best), and competitive storage. The first best curve is computed as the system marginal cost saving when all generators supply at their marginal cost of production. The second best captures the marginal cost savings when generators pay the Nash equilibrium, i.e., possibly bidding above marginal production costs. The competitive curve is computed by calculating the marginal arbitrage profit of competitive storage owners at different capacity levels when generators behave strategically. The value displayed is the marginal value per cycle/day averaged across all hours of the year.

Figure 6 reports the marginal value of storage capacity, for different levels of investment, given the generation technology mix planned for 2030. It reports the results under the three storage ownership scenarios: first-best, second-best, and competitive storage. Recall that, for the social planner, the marginal value of storage capacity is given by the marginal cost savings from storing one extra unit of electricity. Likewise, for the competitive storage owners, the marginal value of storage capacity is given by the marginal arbitrage profits.

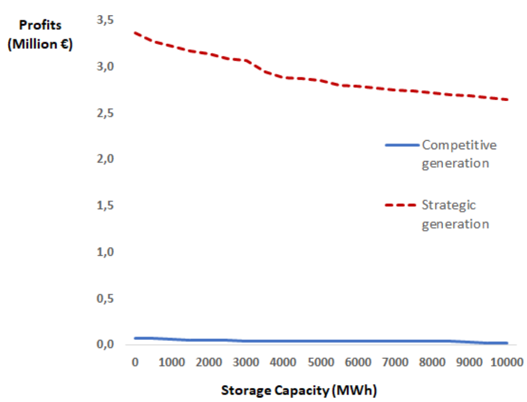
The first thing to notice is that, in line with our theoretical predictions, these curves are negatively sloped, i.e., adding storage capacity becomes less valuable and less profitable the greater the amount of existing storage capacity. The reason is that the most costly production units are first replaced by the least costly production units, and so on,



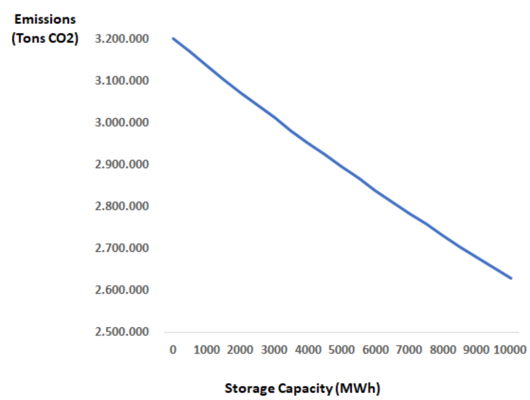
(a) Profits of wind



(b) Profits of solar PV



(c) Profits of CCGTs



(d) Emissions

Figure 7: Carbon emissions and profits by technology per year - Scenario 2030

Notes: Panels (a), (b) ans (c) display market profits by technology (not including investment costs) per year, for different levels of storage capacity, when generators behave competitively (solid line) or strategically (dashed line). Panel (d) reports carbon emissions per year. Profits of renewable technologies are increasing in storage capacity as these technologies produce more (because there is less curtailment) and they do so at higher prices. Not surprisingly, CCGTs make almost zero profits under the competitive scenario, and strictly positive profits otherwise.

in decreasing and increasing cost order, respectively. Hence, the cost savings (and thus the price differences) of storing become increasingly smaller as more storage capacity is introduced.

Also note that the differences across these three curves are small. A plausible explanation is that, with high renewables penetration, the degree of market power is low, leading to small production distortions and to prices that are close to marginal costs. This is particularly the case for high levels of storage capacity. Since this implies that marginal costs and prices are fairly similar in all scenarios, the marginal value of capacity is similar across all of them (see also Table 1). However, for lower levels of storage capacity, since the strategic producers can exercise more market power, the marginal arbitrage profits for the competitive storage owners exceed the marginal cost savings for the social planner. This might lead to inefficient over-investment under the market-based scenario, as we highlighted in our theoretical analysis.

If we consider an scenario with low renewables penetration, matters are strikingly different. To illustrate this, we have also run simulations with the 2017 market structure, when renewables' penetration was much lower: wind generation capacity was half and solar capacity was 8 times lower as compared to the 2030 targets. Results show that the marginal value of storage capacity was systematically negative. This implies that investing in storage capacity was neither profitable for competitive storage owners nor desirable from a social point of view, even if investment costs were negligible.[30] This is so for a two-fold reason. With few renewables, market prices are almost always set by the conventional technologies, whose marginal costs, and the resulting market prices, are fairly constant. This implies that the marginal cost savings and arbitrage profits become so small that they are more than offset by the round-trip efficiency losses of storing and releasing electricity.

The contrast between the positive and the negative marginal value of storage capacity under high and low renewables penetration, respectively, indicate that renewables boost the value of storage capacity. The complementarity goes both ways, as storage also makes investments in renewables more socially valuable and more profitable. This can be seen in Figures 7a and 7b, which depict the market revenues of wind and solar as storage capacity goes up. Both curves are increasing for two reasons. First, storage prevents renewable curtailment in periods of relatively high renewables production relative to

[30]We should keep in mind that this analysis is limited to the arbitrage value of energy storage, as we only focus on using energy storage to shift load from peak to off-peak periods. However, energy storage creates other positive externalities which should also be taken into account when computing the optimal investment.

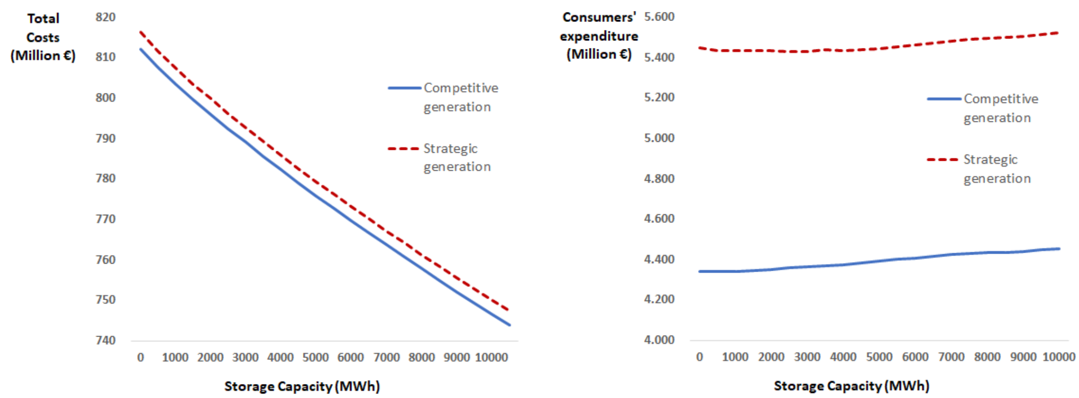
demand. This is particularly important for solar plants, as their production is strongly correlated during the sunny hours of the day, thus making curtailment more likely. And second, storage increases prices in low price hours (i.e., when renewables availability is high) and depresses prices in high price hours (i.e., when renewables availability is low).

Conventional technologies get the other side of the coin, as the reduced curtailment of renewables and the changes in price patterns imply that they sell less and they get paid lower prices on average. Hence, as can be seen in Figure 7c, their profits go down as storage capacity goes up. Because conventional production is increasingly replaced by renewables production, carbon emissions go down as well. This is shown in Figure 7d.

These various effects have important welfare implications. Figure 8a reports the effects of increasing storage capacity on generation costs, while Figure 8b reports the effects on consumers' expenditures. As expected, generation costs go down as storage capacity goes up. However, this does not necessarily translate into higher gains for end-consumers. The reason for this ambiguity is that storage decreases prices in some hours but increases them in others. The latter need not be the hours with lower demand, but rather those in which demand net of renewables is lower. Hence, the price increases might take place when consumers' consumption is high, thus implying that the average (demand-weighted) prices they face might well go up as storage capacity increases. This is compounded by the fact that the round-trip losses might imply that the lower price is raised more than the upper price is reduced. It follows that, in general, the relationship between consumers' prices and storage need not be monotonic.

Finally, to shed light on the profitability of storage investments, one would have to compare the arbitrage profits against the investment costs. Current figures report costs of battery storage at around 150 Euro/MWh (IRENA (2017)), well above the marginal values of storage reported in Figure 6, which are not greater than 24 Euro/MWh.[31] Hence, the costs of storage have to fall and the installed renewables capacity needs to ramp up for there to be a clear case for investments in storage. Over the last ten years, we have witnessed sharp cost reductions in renewables and battery storage (65% to 85% since 2010). Only if this trend continues in the future, will the costs of investing in storage fall below their marginal value. However, it is important to note that storage brings in additional social benefits beyond the pure arbitrage effects (notably, allowing to defer capacity investments, offering flexibility services, improving security of supply, promoting learning by doing externalities, etc.). If the value of these externalities makes those

[31]Note that IRENA (2017) provides the average costs of investment per unit of output that can be stored. Hence, its cost figure is fully comparable with our marginal value figure as they are both expressed in Euro/MWh.



(a) Generation costs (b) Consumers' expenditure

Figure 8: Productive efficiency and consumers' expenditure

Notes: For different levels of storage capacity, panel (a) reports generation costs (not including investment costs). Panel (b) shows firms' market revenues, thus reflecting the total amount that consumers pay to buy their electricity consumption at market prices. Both panels report the results when generators behave competitively (solid line) or strategically (dashed line).

investments socially desirable, regulators will have to provide investors with additional support in order to align private and social incentives.

7 Conclusions

There is consensus among the relevant institutions and industry analysts on the strong growth potential of energy storage over the next decade (see for instance, McCarthy and Eager (2020) and European Commission (2020)). However, whether these expectations fully materialise will depend on policy and regulatory decisions which will ultimately determine the incentives to efficiently operate and invest in storage facilities.

Our focus in this paper has been to analyze how such incentives depend on the market structure. Perfectly competitive markets replicate the first-best, absent other market imperfections. However, market power in storage and/or in generation reduce market efficiency through two channels: they induce an inefficient use of the storage facilities, and they distort investment incentives. Whereas market power in the wholesale electricity market tends to induce over-investment in storage, market power in storage tends to induce firms to under-invest. Under reasonable assumptions, the combination of the two through vertical integration gives rise to the most distorted outcome, both for consumers as well as for overall efficiency.

Our simulations of the Spanish electricity market show that the arbitrage profits made by storage owners are not enough to cover their investment costs. This implies that without public support, it is doubtful whether the socially optimal investments in energy storage (to the extent that they are positive) would actually take place. The mechanisms designed to grant public support should take into account that market structure matters, i.e., the same storage capacity in the hands of competitive storage owners is more socially valuable than if it is allocated to large storage firms or to generators.

Despite the scant attention given by academic research to these issues, there is an intense debate in the policy arena regarding the rules on who should own and operate storage facilities. In many jurisdictions, storage is considered a generation asset, which essentially bars system operators from owning and operating storage devices due to unbundling restrictions.[32] Yet, our analysis suggests that regulators should not put the spotlight on the integration between transmission and storage (which could potentially be positive),[33] but rather on the integration between generation and storage, as well as on the concentration in storage ownership. A vertically integrated firm or a large storage owner internalizes the price effects caused by storage on its own energy sale and purchase decisions. This causes them to distort the use of storage away from the cost-minimizing pattern, reducing its profitability, and thus weakening the firms' incentives to invest.

Throughout the analysis, we have assumed that storage owners are exposed to wholesale electricity prices. While this is generally true for large storage installations (e.g. pumped hydro), it need not be so for the distributed storage facilities (e.g. electric vehicles, or behind-the-meter batteries). In order to fully develop the potential of storage, it is paramount to foster dynamic electricity prices and time-of-use tariffs so that storage owners internalize the social benefits that they bring about.

[32]For instance, in May 2019, the European Commission ruled that only under exceptional circumstances are transmission and distribution operators allowed to own and operate storage facilities (European Commission (2019)). Similarly, in 2019, China decided not to allow network operators to include storage costs in their fees, which led to a sharp decline in storage investment. Yet, other jurisdictions (such as Australia or Chile) allow network operators to own and operate storage assets under certain conditions. And in the US, the debate is still on-going as regulators are currently reviewing the storage ownership rules, which differ widely across states (European Commission (2019, 2020)).

[33]For instance, that is the case if the transmission owner is required to operate storage so as to reduce system costs, just as a social planner would do. Also, storage in the hands of the System Operator could contribute to security of supply.

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Appendix: Proofs

Proof of Lemma 1

At the production stage, the problem of the social planner is to choose $q_S(\theta)$ and $q_S(\theta)$ to maximize total welfare W, taking capacity $K > 0$ and the demand distribution $G(\theta)$ as given. Therefore, we look for the solution to the following problem:

$$\max_{q_S(\theta),q_B(\theta),\forall\theta} W\big(q_S(\theta), q_B(\theta)\big) = \int_{\underline{\theta}}^{\bar{\theta}} \left[v\theta - \frac{\big(\theta - q_S(\theta) + q_B(\theta)\big)^2}{2} \right] g(\theta)d\theta$$

$$\text{s.t.} \quad h_1\big(q_S(\theta), q_B(\theta)\big) = \int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta)g(\theta)d\theta - \int_{\underline{\theta}}^{\bar{\theta}} q_S(\theta)g(\theta)d\theta \geq 0$$

$$h_2\big(q_B(\theta)\big) = K - \int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta)g(\theta)d\theta \geq 0$$

$$h_3\big(q_S(\theta)\big) = q_S(\theta) \geq 0, \forall\theta$$

$$h_4\big(q_B(\theta)\big) = q_B(\theta) \geq 0, \forall\theta$$

with $K > 0$. We can define the constraint set of the problem as:

$$C := \{q_S(\theta), q_B(\theta) \in X : h_j\big(q_S(\theta); q_B(\theta)\big) \geq 0, j = \{1, 2, 3, 4\}\}$$

The set $X = (0, +\infty)^2$ is open and convex because it is Cartesian product of open intervals which are open, convex sets. Note that the objective function $W(\cdot)$ and the constraints are continuously differentiable functions. Moreover, C is closed, bounded and compact, so the solution set to the problem is non-empty. Moreover, $W(\cdot)$ is strictly concave in $q_B(\theta)$ and $q_S(\theta)$. The constraints are (weakly) concave, so the solution to the problem is unique.

The Lagrangian of the problem is:

$$\mathbb{L}\big(q_B(\theta), q_S(\theta), \eta_S(\theta), \eta_B(\theta), \lambda, \mu\big) = \int_{\underline{\theta}}^{\bar{\theta}} \left[v\theta - \frac{\big(\theta - q_S(\theta) + q_B(\theta)\big)^2}{2} \right] g(\theta)d\theta + \int_{\underline{\theta}}^{\bar{\theta}} \eta_S(\theta)q_S(\theta)g(\theta)d\theta$$
$$+ \int_{\underline{\theta}}^{\bar{\theta}} \eta_B(\theta)q_B(\theta)g(\theta)d\theta + \lambda\Big[\int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta)g(\theta)d\theta - \int_{\underline{\theta}}^{\bar{\theta}} q_S(\theta)g(\theta)d\theta\Big] + \mu\Big[K - \int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta)g(\theta)d\theta\Big]$$

where $\lambda, \mu, \eta_S(\theta)$ and $\eta_B(\theta)$ are the multipliers associated with their respective constraints $h_1(\cdot), h_2(\cdot), h_3(\cdot), h_4(\cdot) \geq 0$. The Karush-Kuhn-Tucker (KKT) conditions are:

$$\theta - q_S(\theta) + q_B(\theta) - \lambda + \eta_S(\theta) = 0, \forall\theta \tag{18}$$

$$\theta - q_S(\theta) + q_B(\theta) - \lambda + \mu - \eta_B(\theta) = 0, \forall\theta \tag{19}$$

$$\eta_i(\theta) \geq 0, \forall\theta, i = \{S, B\}$$

$$q_i(\theta) \geq 0, \forall\theta, i = \{S, B\}$$

$$\eta_i(\theta)q_i(\theta) = 0, \forall\theta, i = \{S, B\}$$

$$\int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta)g(\theta)d\theta - \int_{\underline{\theta}}^{\bar{\theta}} q_S(\theta)g(\theta)d\theta \geq 0 \tag{20}$$

$$\lambda \geq 0 \tag{21}$$

$$\lambda\left[\int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta)g(\theta)d\theta - \int_{\underline{\theta}}^{\bar{\theta}} q_S(\theta)g(\theta)d\theta = 0\right] = 0 \tag{22}$$

$$\mu \geq 0 \tag{23}$$

$$K - \int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta)g(\theta)d\theta \geq 0 \tag{24}$$

$$\mu\left[K - \int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta)g(\theta)d\theta\right] = 0 \tag{25}$$

These conditions are necessary and sufficient, due the concavity of the objective function and the constraints. Without loss of generality, we can focus attention on cases in which for any $\theta \in [\underline{\theta}, \bar{\theta}]$, $q_B(\theta) > 0 \to q_S(\theta) = 0$ & $q_S(\theta) > 0 \to q_B(\theta) = 0$. We conjecture that there exists $\theta_1 \in [\underline{\theta}, \bar{\theta}]$ and $\theta_2 \in [\underline{\theta}, \bar{\theta}]$, with $\theta_1 \leq \theta_2$, such that:

$$\begin{cases} q_B(\theta) > 0 & \text{if} \quad \theta < \theta_1 \\ q_B(\theta) = 0 & \text{if} \quad \theta \geq \theta_1 \end{cases} \text{and} \begin{cases} q_S(\theta) = 0 & \text{if} \quad \theta \leq \theta_2 \\ q_S(\theta) > 0 & \text{if} \quad \theta > \theta_2 \end{cases}.$$

We proceed by finding the expressions for $q_B(\theta), q_S(\theta), \theta_1$ and θ_2 implied by this conjecture that satisfy all the KKT conditions. Note that $\lambda > 0$ must be satisfied in every possible solution of this problem, as if the associated constraint holds with inequality, one can always increase the value of the program by increasing $q_S(\theta)$ or reducing $q_B(\theta)$.

From condition (18):

$$q_S(\theta) = \theta - \lambda, \forall\theta > \theta_2$$

and from condition (19):

$$q_B(\theta) = \lambda - \mu - \theta, \forall\theta < \theta_1$$

By continuity:

$$q_S(\theta_2) = 0 \Rightarrow \theta_2 = \lambda \Rightarrow q_S^{FB}(\theta) = \theta - \theta_2, \forall \theta > \theta_2$$
$$q_B(\theta_1) = 0 \Rightarrow \theta_1 = \lambda - \mu \Rightarrow q_B^{FB}(\theta) = \theta_1 - \theta, \forall \theta < \theta_1$$

From condition 20:

$$\int_{\underline{\theta}}^{\theta_1} (\theta_1 - \theta) g(\theta) d\theta = \int_{\theta_2}^{\bar{\theta}} (\theta - \theta_2) g(\theta) d\theta. \tag{26}$$

We have two possible cases depending on the value of the exogenous parameter K. When K is binding, > 0 and $\theta_2 - \theta_1 = \mu > 0$. Define $x = \theta_2 - \theta_1$. By symmetry of $G(\theta)$, equation (26) implies that θ_2 and θ_1 must be symmetric around the mean, i.e.,

$$\theta_1 = \mathbb{E}(\theta) - \frac{x}{2} \Rightarrow \theta_1^{FB} = \mathbb{E}(\theta) - \frac{\mu^{FB}}{2} \tag{27}$$
$$\theta_2 = \mathbb{E}(\theta) + \frac{x}{2} \Rightarrow \theta_2^{FB} = \mathbb{E}(\theta) + \frac{\mu^{FB}}{2} \tag{28}$$

with μ^{FB} implicitly given by:

$$\int_{\underline{\theta}}^{\theta_1(\mu^{FB})} \big(\theta_1(\mu^{FB}) - \theta\big) g(\theta) d\theta = \int_{\theta_2(\mu^{FB})}^{\bar{\theta}} \big(\theta - \theta_2(\mu^{FB})\big) g(\theta) d\theta = K. \tag{29}$$

When K is not binding, so that $\mu = 0$, from equations (27) and (28) it is straightforward to establish that $\theta_1^{FB} = \theta_2^{FB} = \mathbb{E}(\theta)$. Therefore, the unique solution in this case is:

$$\begin{cases} q_B(\theta) = \mathbb{E}(\theta) - \theta & \text{if} \quad \theta < \mathbb{E}(\theta) \\ q_B(\theta) = 0 & \text{if} \quad \theta \geq \mathbb{E}(\theta) \end{cases} \text{and} \quad \begin{cases} q_S(\theta) = 0 & \text{if} \quad \theta \leq \mathbb{E}(\theta) \\ q_S(\theta) = \theta - \mathbb{E}(\theta) & \text{if} \quad \theta > \mathbb{E}(\theta) \end{cases}.$$

Proof of Proposition 1

Now we turn to the problem of choosing optimal K at the investment stage. The problem of the social planner at the investment stage it to maximize total welfare (which is a function of K alone) given the optimal operation of storage at the production stage.

Let $V(K)$ be the value function after substituting the optimal solutions $q_S^*(\theta, K)$ and $q_S^*(\theta, K)$ at the production stage. Thus, the problem of the social planner at the investment stage is

$$\max_K W(q_S^*(\theta, K), q_B^*(\theta, K), K) - C(K) = V(K) - C(K)$$

By the envelope theorem, we have that:

$$\frac{dV(K)}{dK} = \frac{\partial \mathbb{L}\big(q_B(\theta), q_S(\theta), \eta_S(\theta), \eta_B(\theta), \lambda, \mu\big)}{\partial K} = \mu^{FB}.$$

Therefore, the unique interior solution K^{FB} is given by:

$$\frac{\partial W}{\partial K} = 0 \Leftrightarrow \mu^{FB} - C'(K) = \theta_2(K^{FB}) - \theta_1(K^{FB}) - C'(K^{FB}) = 0 \tag{30}$$

with

$$\int_{\underline{\theta}}^{\theta_1(\mu(K))} \big(\theta_1(\mu(K)) - \theta\big) g(\theta) d\theta = \int_{\theta_2(K^{FB})}^{\bar{\theta}} \big(\theta - \theta_2(K^{FB})\big) g(\theta) d\theta = K^{FB}.$$

Proof of Lemma 2

The problem of the competitive fringe is:

$$\max_{q(\theta)} \left\{ p(\theta)q(\theta) - \frac{q^2(\theta)}{2(1-\alpha)} \right\}.$$

The first order condition, which is both necessary and sufficient, is:

$$p(\theta) - \frac{q^*(\theta)}{1-\alpha} = 0, \Leftrightarrow q_F^*(\theta) = (1-\alpha)p(\theta), \forall \theta$$

The dominant producer chooses its output in order to maximize its profits,

$$\max_{q_D(\theta)} \pi_D(\theta) = \frac{\theta - q_S(\theta) + q_B(\theta) - q_D(\theta)}{1-\alpha} q_D(\theta) - \frac{[q_D(\theta)]^2}{2\alpha}$$

Hence, the first order condition of problem is:

$$\frac{\partial \pi_D(\theta)}{\partial q_D(\theta)} = 0 \Leftrightarrow \frac{\theta - q_S(\theta) + q_B(\theta) - 2q_D(\theta)}{1-\alpha} - \frac{q_D(\theta)}{\alpha} = 0 \Leftrightarrow q_D(\theta) = \frac{\alpha}{1+\alpha}\left(\theta - q_S(\theta) + q_B(\theta)\right), \forall \theta \tag{31}$$

with second order condition satisfied. Note that the above implies

$$q_F(\theta) = \frac{1}{1+\alpha}\left(\theta - q_S(\theta) + q_B(\theta)\right).$$

The equilibrium price is

$$p(\theta) = \frac{1}{1-\alpha^2}\left(\theta - q_S(\theta) + q_B(\theta)\right).$$

Proof of Lemma 3

At the production stage, the problem of the social planner is to solve problem

$$\max_{q_B(\theta), q_S(\theta)} W = \int_{\underline{\theta}}^{\bar{\theta}} v\theta g(\theta) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \left[\frac{[q_D(\theta)]^2}{2\alpha} + \frac{[\theta - q_S(\theta) + q_B(\theta) - q_D(\theta)]^2}{2(1-\alpha)} \right] g(\theta) d\theta,$$

subject to constraints (2) and (1). The structure of the functional optimization problem is identical to the one in the Proof of Proposition 1 in Appendix 7, with concavity and compactness assumptions satisfied, so a unique solution to the problem exists. Without loss of generality, we can focus attention on cases in which for any $\theta \in [\underline{\theta}, \bar{\theta}]$, $q_B(\theta) > 0 \to q_S(\theta) = 0$ & $q_S(\theta) > 0 \to q_B(\theta) = 0$. The KKT conditions are:

$$\frac{\theta - q_S(\theta) + q_B(\theta) - q_D(\theta)}{1-\alpha} - \lambda = 0, \forall \theta \geq \theta_2 \tag{32}$$

$$\frac{\theta - q_S(\theta) + q_B(\theta) - q_D(\theta)}{1-\alpha} - \lambda < 0, \forall \theta < \theta_2$$

$$\frac{\theta - q_S(\theta) + q_B(\theta) - q_D(\theta)}{1-\alpha} - \lambda + \mu = 0, \forall \theta \leq \theta_1 \tag{33}$$

$$\frac{\theta - q_S(\theta) + q_B(\theta) - q_D(\theta)}{1-\alpha} - \lambda + \mu > 0, \forall \theta > \theta_1$$

$$\int_{\underline{\theta}}^{\theta_1} q_B(\theta) g(\theta) d\theta = \int_{\theta_2}^{\bar{\theta}} q_S(\theta) g(\theta) d\theta \tag{34}$$

with $\theta_1 \leq \theta_2$ and the complementary slackness conditions identical to equations (21)-(25) of the FB problem. Note that condition (34) already incorporates the fact that we must have $\lambda > 0$ in any optimal solution to the problem. Note that these conditions are necessary and sufficient, due the concavity of both the objective function and the constraints.

From conditions (32) and (33), and using the best response of the dominant firm, equation (31):

$$\big(\theta - q_S(\theta) + q_B(\theta)\big)\frac{1}{1-\alpha^2} = \lambda, \forall \theta > \theta_2$$

and from condition (33):

$$\big(\theta - q_S(\theta) + q_B(\theta)\big)\frac{1}{1-\alpha^2} = \lambda - \mu, \forall \theta < \theta_1$$

By continuity:

$$q_S(\theta_2) = 0 \Rightarrow \theta_2 = \lambda(1-\alpha^2) \Rightarrow q_S^{SB}(\theta) = \theta - \theta_2, \forall \theta > \theta_2$$

$$q_B(\theta_1) = 0 \Rightarrow \theta_1 = (\lambda - \mu)(1-\alpha^2) \Rightarrow q_B^{SB}(\theta) = \theta_1 - \theta, \forall \theta < \theta_1$$

From condition (34):

$$\int_{\underline{\theta}}^{\theta_1} (\theta_1 - \theta) g(\theta) d\theta = \int_{\theta_2}^{\bar{\theta}} (\theta - \theta_2) g(\theta) d\theta. \tag{35}$$

We have two possible cases depending on the value of the exogenous parameter K. When K is binding, $\mu > 0$ and $\theta_2 - \theta_1 = \mu(1-\alpha^2) > 0$. Define $x = \theta_2 - \theta_1$. By symmetry of $G(\theta)$, equation (44) implies that θ_2 and θ_1 must be symmetric around the mean, i.e.,

$$\theta_1 = \mathbb{E}(\theta) - \frac{x}{2} \Rightarrow \theta_1^{SB} = \mathbb{E}(\theta) - \frac{\mu^{SB}}{2}(1-\alpha^2) \tag{36}$$

$$\theta_2 = \mathbb{E}(\theta) + \frac{x}{2} \Rightarrow \theta_2^{SB} = \mathbb{E}(\theta) + \frac{\mu^{SB}}{2}(1-\alpha^2) \tag{37}$$

with μ^{SB} implicitly given by:

$$\int_{\underline{\theta}}^{\theta_1(\mu^{SB})} \big(\theta_1(\mu^{SB})-\theta\big)g(\theta)d\theta = \int_{\theta_2(\mu^{SB})}^{\bar{\theta}} \big(\theta-\theta_2(\mu)\big)g(\theta)d\theta = K. \tag{38}$$

Note that when K is not binding, so that $\mu = 0$, from equations (36) and (37) it is straightforward to establish that $\theta_1^{SB} = \theta_2^{SB} = \mathbb{E}(\theta)$.

Proof of Proposition 2

The problem of the constrained social planner is to choose K to maximize total welfare, conditional on optimal behavior of all agents at the production stage. Thus, the problem is:Second best:

$$\max_{q_B(\theta),q_S(\theta)} (K, q_S^*(\theta,K), q_B^*(\theta,K)) = vE[\theta] - \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{2\alpha}(q_D(\theta))^2 g(\theta)d\theta$$
$$+ \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{2(1-\alpha)}\big(\theta - q_S(\theta) + q_B(\theta) - q_D(\theta)\big)^2 g(\theta)d\theta.$$

By the envelope theorem,

$$\frac{dW}{dK} = \frac{\partial W}{\partial K} + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial W}{\partial q_D(\theta)}\frac{\partial q_D^*(\theta)}{\partial K} g(\theta)d\theta.$$

The first term is a direct effect and it equals μ^{SB}. The second term is a strategic effect which results from the impact of K on the dominant firm's output decision. Focusing on it,

$$\begin{aligned}
\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial W}{\partial q_D(\theta)}\frac{\partial q_D^*(\theta)}{\partial K} g(\theta)d\theta &= -\int_{\underline{\theta}}^{\bar{\theta}} \left[\frac{\partial q_D^*(\theta)}{\partial K}\frac{q_D^*(\theta) - \alpha\left(\theta - q_S^*(\theta) + q_B^*(\theta)\right)}{\alpha(1-\alpha)}\right] g(\theta)d\theta \\
&= \frac{\alpha}{(1-\alpha^2)}\int_{\underline{\theta}}^{\bar{\theta}} \left[\frac{\partial q_D^*(\theta)}{\partial K}\left(\theta - q_S^*(\theta) + q_B^*(\theta)\right)\right] g(\theta)d\theta,
\end{aligned}$$

where the second line follows from using the expression for $q_D^*(\theta)$.

Since

$$q_B(\theta) = \max\{\theta_1(\mu) - \theta, 0\} \text{ and } q_S(\theta) = \max\{\theta - \theta_2(\mu), 0\},$$

we can write,

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial W}{\partial q_D(\theta)}\frac{\partial q_D^*(\theta)}{\partial K} g(\theta)d\theta = \frac{\alpha}{(1-\alpha^2)}\left(\int_{\underline{\theta}}^{\theta_1} \frac{\partial q_D^*(\theta)}{\partial K}\theta_1 g(\theta)d\theta + \int_{\theta_2}^{\bar{\theta}} \frac{\partial q_D^*(\theta)}{\partial K}\theta_2 g(\theta)d\theta\right).$$

For $\theta \in (\underline{\theta}, \theta_1)$,

$$q_D^*(\theta) = \frac{\alpha}{1+\alpha}\theta_1 \Rightarrow \frac{\partial q_D^*(\theta)}{\partial K} = \frac{\alpha}{1+\alpha}\frac{\partial \theta_1}{\partial K} = \frac{\alpha}{1+\alpha}\frac{1}{G(\theta_1)}.$$

And for $\theta \in \left(\theta_2, \bar{\theta}\right)$,

$$q_D^*(\theta) = \frac{\alpha}{1+\alpha}\theta_2 \Rightarrow \frac{\partial q_D^*(\theta)}{\partial K} = \frac{\alpha}{1+\alpha}\frac{\partial \theta_2}{\partial K} = -\frac{\alpha}{1+\alpha}\frac{1}{1-G(\theta_2)}$$

Hence, the strategic effect is

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial W}{\partial q_D(\theta)} \frac{\partial q_D^*(\theta)}{\partial K} g(\theta) d\theta = -\frac{\alpha^2}{(1-\alpha^2)(1+\alpha)}(\theta_2 - \theta_1) < 0$$

Note that the strategic effect disappears if $\alpha = 0$. Furthermore, it is negative, and its absolute value is increasing in α.

Putting the direct and the strategic effects together

$$\begin{aligned}
\frac{dW}{dK} &= -\frac{\alpha^2}{(1-\alpha^2)(1+\alpha)}(\theta_2 - \theta_1) + \frac{1}{(1-\alpha^2)}(\theta_2 - \theta_1) - C'(K) \\
&= \frac{\alpha - \alpha^2 + 1}{(1-\alpha)(\alpha+1)^2}(\theta_2 - \theta_1) - C'(K).
\end{aligned}$$

Note that $\frac{\alpha-\alpha^2+1}{(1-\alpha)(\alpha+1)^2}$ is increasing in α, and it equals 1 for $\alpha = 0$. It follows that $K^{SB} > K^{FB}$. Note that the solution to the problem implies that the capacity constraint must be binding in the second stage. Otherwise, idle storage capacity would imply that the marginal benefit of storage capacity is lower than its marginal cost, violating the optimality condition at the first stage.

Proof of Lemma 4

At the production stage, the problem of the competitive storage operator is to solve problem (10) subject to constraints (2) and (1). The structure of the functional optimization problem is identical to the one in the Proof of Proposition 1, with concavity and compactness assumptions satisfied, so a unique solution to the problem exists. The Lagrangian of the problem, omitting the non-negativity constraints, is:

$$\begin{aligned}
\mathbb{L}\big(q_B(\theta), q_S(\theta), \lambda, \mu\big) = &\int_{\underline{\theta}}^{\bar{\theta}} p(\theta)\big[q_S(\theta) - q_B(\theta)\big] g(\theta) d\theta + \lambda\left[\int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta) g(\theta) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} q_S(\theta) g(\theta) d\theta\right] \\
&+ \mu\left[K - \int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta) g(\theta) d\theta\right]
\end{aligned}$$

where λ and μ are the Lagrange multipliers. Without loss of generality, we can focus attention on cases in which for any $\theta \in [\underline{\theta}, \bar{\theta}]$, $q_B(\theta) > 0 \rightarrow q_S(\theta) = 0$ & $q_S(\theta) > 0 \rightarrow q_B(\theta) = 0$. The KKT conditions are:

$$p(\theta) - \lambda = 0, \forall \theta \geq \theta_2 \tag{39}$$

$$p(\theta) - \lambda < 0, \forall \theta < \theta_2 \tag{40}$$

$$p(\theta) - \lambda + \mu = 0, \forall \theta \leq \theta_1 \tag{41}$$

$$p(\theta) - \lambda + \mu > 0, \forall \theta > \theta_1 \tag{42}$$

$$\int_{\underline{\theta}}^{\theta_1} q_B(\theta) g(\theta) d\theta = \int_{\theta_2}^{\bar{\theta}} q_S(\theta) g(\theta) d\theta \tag{43}$$

with $\theta_1 \leq \theta_2$ and the complementary slackness conditions identical to equations (21)-(25) of the FB problem. Note that condition (43) already incorporates the fact that we must have $\lambda > 0$ in any optimal solution to the problem. Note that these conditions are necessary and sufficient, due the concavity of both the objective function and the constraints.

From condition (39):

$$p(\theta) = \lambda, \forall \theta > \theta_2$$

and from condition (41):

$$p(\theta) = \lambda - \mu, \forall \theta < \theta_1$$

Since $p(\theta)$ is the best response of the dominant firm,

$$\lambda = p(\theta) = \frac{\theta - q_S(\theta)}{1 - \alpha^2} \Leftrightarrow q_S(\theta) = \theta - (1 - \alpha^2)\lambda, \forall \theta > \theta_2$$
$$\lambda - \mu = p(\theta) = \frac{\theta + q_B(\theta)}{1 - \alpha^2} \Leftrightarrow q_S(\theta) = (1 - \alpha^2)(\lambda - \mu) - \theta, \forall \theta < \theta_1$$

By continuity:

$$q_S(\theta_2) = 0 \Rightarrow \theta_2 = (1 - \alpha^2)\lambda \Rightarrow q_S^C(\theta) = \theta - \theta_2, \forall \theta > \theta_2$$
$$q_B(\theta_1) = 0 \Rightarrow \theta_1 = (1 - \alpha^2)(\lambda - \mu) \Rightarrow q_B^C(\theta) = \theta_1 - \theta, \forall \theta < \theta_1$$

From condition (43):

$$\int_{\underline{\theta}}^{\theta_1} (\theta_1 - \theta) g(\theta) d\theta = \int_{\theta_2}^{\bar{\theta}} (\theta - \theta_2) g(\theta) d\theta \tag{44}$$

We have two possible cases depending on the value of the exogenous parameter K. When K is binding, $\mu > 0$ and $\theta_2 - \theta_1 = \mu(1 - \alpha^2) > 0$. Define $x = \theta_2 - \theta_1$ and assume that $G(\theta)$ has a well-defined mean given by $\mathbb{E}(\theta)$. By symmetry of $G(\theta)$, equation (44) implies that θ_2 and θ_1 must be symmetric around the mean i.e.

$$\theta_1 = \mathbb{E}(\theta) - \frac{x}{2} \Rightarrow \theta_1^C = \mathbb{E}(\theta) - \frac{\mu^C(1-\alpha^2)}{2} \tag{45}$$

$$\theta_2 = \mathbb{E}(\theta) + \frac{x}{2} \Rightarrow \theta_2^C = \mathbb{E}(\theta) + \frac{\mu^C(1-\alpha^2)}{2} \tag{46}$$

with μ^C implicitly given by:

$$\int_{\underline{\theta}}^{\theta_1(\mu^C)} \big(\theta_1(\mu^C) - \theta\big) g(\theta) d\theta = \int_{\theta_2(\mu}^{\bar{\theta}} \big(\theta - \theta_2(\mu\big) g(\theta) d\theta = K. \tag{47}$$

Note that when K is not binding, so that $\mu = 0$, from equations (45) and (46) it is straightforward to establish that $\theta_1^C = \theta_2^C = \mathbb{E}(\theta)$.

Proof of Proposition 3

The free-entry and perfect competition assumptions imply that entry/investments take place until expected profits are zero, conditional on operating the storage facilities optimally, i.e., $\pi_S\big(q_S^C(\theta), q_B^C(\theta)\big) = 0$. Profits of the storage operator at the investment stage are:

$$\begin{aligned} \pi_S\big(K, \mu(K)\big) &= \int_{\underline{\theta}}^{\bar{\theta}} p^C(\theta)\big[q_S^C(\theta) - q_B^C(\theta)\big] g(\theta) d\theta - C(K) \\ &= \mu^C(K)K - C(K) \end{aligned} \tag{48}$$

with $\mu^{SB}(K)$ implicitly given by equation (47).

Thus, under the zero-profit condition, the equilibrium investment $K = K^C$ is the unique solution to

$$\pi_S\big(q_S^C(\theta), q_B^C(\theta)\big) = 0 \Leftrightarrow \frac{C(K)}{K} = \mu^{SB}(K) = \frac{\theta_2^C - \theta_1^C}{1-\alpha^2}. \tag{49}$$

Note that the solution to the problem implies that the capacity constraint must be binding in the second stage, for the same reasons as in the first-best problem.

Now, we show that $K^C > K^{FB}$. Assume on the contrary that $K^C \leq K^{FB}$. From equations (29) and (47), this implies that:

$$\int_{\underline{\theta}}^{\theta_1^C} (\theta_1^C - \theta) g(\theta) d\theta \leq \int_{\underline{\theta}}^{\theta_1^{FB}} (\theta_1^{FB} - \theta) g(\theta) d\theta \to \theta_1^C \leq \theta_1^{FB}$$

$$\int_{\theta_2^C}^{\bar{\theta}} (\theta - \theta_2^C) g(\theta) d\theta \leq \int_{\theta_2^{FB}}^{\bar{\theta}} (\theta - \theta_2^{FB}) g(\theta) d\theta \to \theta_1^C \geq \theta_1^{FB}$$

Thus, from the optimal solutions (4) and (14),

$$\theta_2^C - \theta_1^C \geq \theta_2^{FB} - \theta_1^{FB} \Rightarrow \mu^C(1-\alpha^2) \geq \mu^{FB} \Rightarrow \frac{C(K)}{K}(1-\alpha^2) \geq C'(K).$$

A contradiction, by strict convexity of the cost function $C(K)$ and $\alpha > 0$.

On the other hand, $K^C > K^{SB}$ follows directly from the strict convexity of $C(K)$.

Proof of Lemma 5

At the production stage, the problem of the storage monopolist is:

$$\max_{q^S(\theta), q^B(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \frac{\theta - q^S(\theta) + q^B(\theta) - q^D(\theta)}{1-\alpha}(q^S(\theta) - q^B(\theta)) g(\theta)\, d\theta,$$

subject to constraints (1) and (2). The structure of the functional optimization problem is identical to the one in the Proof of Proposition 1, with concavity and compactness assumptions satisfied, so a unique solution to the problem exists. The Lagrangian of the problem, omitting the non-negativity constraints, is given by:

$$\begin{aligned}\mathcal{L} = & \frac{1}{1-\alpha} \int_{\underline{\theta}}^{\bar{\theta}} \left[\theta - q^S(\theta) + q^B(\theta) - q^D(\theta)\right]\left[q^S(\theta) - q^B(\theta)\right] g(\theta)\, d\theta \\ & + \lambda \left[\int_{\underline{\theta}}^{\bar{\theta}} q^B(\theta) g(\theta)\, d\theta - \int_{\underline{\theta}}^{\bar{\theta}} q^S(\theta) g(\theta)\, d\theta\right] + \mu \left[K - \int_{\underline{\theta}}^{\bar{\theta}} q^B(\theta) g(\theta)\, d\theta\right]\end{aligned}$$

where λ and μ are the Lagrange multipliers. Without loss of generality, we can focus attention on cases in which for any $\theta \in [\underline{\theta}, \bar{\theta}]$, $q_B(\theta) > 0 \to q_S(\theta) = 0$ & $q_S(\theta) > 0 \to q_B(\theta) = 0$. The KKT conditions are:

$$\begin{aligned}\frac{\theta - 2q^S(\theta) - q^D(\theta)}{1-\alpha} - \lambda = 0, \forall \theta \in \left(\theta_2, \bar{\theta}\right) \\ \frac{\theta}{1-\alpha} - \lambda < 0, \forall \theta < \theta_2 \\ \frac{\theta + 2q^B(\theta) - q^D(\theta)}{1-\alpha} - \lambda + \mu = 0, \forall \theta \in (0, \theta_1) \\ -\frac{\theta + 2q^B(\theta) - q^D(\theta)}{1-\alpha} + \lambda - \mu < 0, \forall \theta > \theta_1 \\ \int_{\underline{\theta}}^{\theta_1} q^B(\theta) g(\theta)\, d\theta = \int_{\theta_2}^{\bar{\theta}} q^S(\theta) g(\theta)\, d\theta = K\end{aligned}$$

with $\theta_1 \leq \theta_2$ and the complementary slackness conditions identical to equations (21)-(25). These conditions are necessary and sufficient, due the concavity of both the objective function and the constraints.

Pointwise optimality implies that the system of reaction functions is:

$$\begin{aligned}
q^S(\theta) &= \frac{\left(\theta - q^D(\theta)\right) - \lambda\left(1-\alpha\right)}{2} \\
q^B(\theta) &= \frac{\left(\lambda-\mu\right)\left(1-\alpha\right) - \left(\theta - q^D(\theta)\right)}{2} \\
q^D(\theta) &= \left(\theta - q^S(\theta) + q^B(\theta)\right)\frac{\alpha}{1+\alpha}
\end{aligned}$$

Thus, we have that:

$$\begin{aligned}
q_S(\theta) &= \frac{\theta - \left(1-\alpha^2\right)\lambda}{\alpha+2}, \forall\theta > \theta_2 \\
q_B(\theta) &= \frac{\left(1-\alpha^2\right)\left(\lambda-\mu\right) - \theta}{\alpha+2}, \forall\theta < \theta_1
\end{aligned}$$

By continuity:

$$\begin{aligned}
q_S(\theta_2) = 0 \Rightarrow \theta_2 = \left(1-\alpha^2\right)\lambda \Rightarrow q^S(\theta) = \frac{\theta-\theta_2}{2+\alpha}, \forall\theta > \theta_2 \\
q_B(\theta_1) = 0 \Rightarrow \theta_1 = \left(1-\alpha^2\right)\left(\lambda-\mu\right) \Rightarrow q^B(\theta) = \frac{\theta_1-\theta}{2+\alpha}, \forall\theta < \theta_1
\end{aligned}$$

As for the market price,

$$\begin{aligned}
p(\theta) &= \frac{\theta - q^S(\theta) + q^B(\theta) - q^D(\theta)}{1-\alpha} \\
&= \frac{1}{\left(1-\alpha^2\right)}\left(\theta - q^S(\theta) + q^B(\theta)\right)
\end{aligned}$$

Using the expressions for $q^S(\theta)$ and $q^B(\theta)$,

$$\begin{aligned}
p(\theta) &= \frac{\theta\left(1+\alpha\right)+\theta_2}{\left(\alpha+2\right)\left(1-\alpha^2\right)}, \ \forall\theta > \theta_2 \\
p(\theta) &= \frac{\theta\left(1+\alpha\right)+\theta_1}{\left(\alpha+2\right)\left(1-\alpha^2\right)}, \ \forall\theta < \theta_1
\end{aligned}$$

Note that

$$\begin{aligned}
p(\theta) - p(\theta_2) &= \frac{\theta-\theta_2}{\left(\alpha+2\right)\left(1-\alpha^2\right)}, \ \forall\theta > \theta_2 \\
p(\theta_1) - p(\theta) &= \frac{\theta_1-\theta}{\left(\alpha+2\right)\left(1-\alpha^2\right)}, \ \forall\theta < \theta_1
\end{aligned}$$

From condition (2):

$$\int_{\underline{\theta}}^{\theta_1} \frac{\theta_1-\theta}{2+\alpha} g(\theta)d\theta = \int_{\theta_2}^{\bar{\theta}} \frac{\theta_1-\theta}{2+\alpha} g(\theta)d\theta = K \tag{50}$$

We have two possible cases depending on the value of the exogenous parameter K. When K is binding, $\mu > 0$ and $\theta_2 - \theta_1 = \mu(1-\alpha^2) > 0$. Define $x = \theta_2 - \theta_1$. By symmetry of $G(\theta)$, equation (50) implies that θ_2 and θ_1 must be symmetric around the mean i.e.,

$$\theta_1 = \mathbb{E}(\theta) - \frac{x}{2} \Rightarrow \theta_1^M = \mathbb{E}(\theta) - \frac{\mu(1-\alpha^2)}{2} \tag{51}$$

$$\theta_2 = \mathbb{E}(\theta) + \frac{x}{2} \Rightarrow \theta_2^M = \mathbb{E}(\theta) + \frac{\mu(1-\alpha^2)}{2} \tag{52}$$

with $\mu = \mu^M(K)$ implicitly given by:

$$\int_{\underline{\theta}}^{\theta_1(\mu^M(K))} \frac{\theta_1(\mu^M(K)) - \theta}{2+\alpha} g(\theta)d\theta = \int_{\theta_2(\mu^M(K))}^{\bar{\theta}} \frac{\theta - \theta_2(\mu^M(K))}{2+\alpha} g(\theta)d\theta = K \tag{53}$$

Note that when K is not binding, so that $\mu = 0$, from equations (51) and (52) it is straightforward to establish that $\theta_1^M = \theta_2^M = \mathbb{E}(\theta)$. However, as shown below, $\mu = 0$ cannot be a solution to the first stage problem.

Proof of Proposition 4

(i) The problem of the storage firm is to maximize profits, conditional on operating the storage facilities optimally. Thus, the problem is:

$$\begin{aligned}
\max_K \pi_S\big(K, \mu(K)\big) &= \int_{\underline{\theta}}^{\bar{\theta}} p^M(\theta)\big[q_S^M(\theta) - q_B^M(\theta)\big]g(\theta)d\theta - C(K) \\
&= \frac{1}{(1-\alpha^2)(2+\alpha)^2} \int_{\theta_2(\mu(K)}^{\bar{\theta}} \big[\theta(1+\alpha) + \theta_2^M\big(\mu(K)\big)\big]\big[\theta - \theta_2^M\big(\mu(K)\big)\big]g(\theta)d\theta \\
&- \frac{1}{(1-\alpha^2)(2+\alpha)^2} \int_{\underline{\theta}}^{\theta_1(\mu(K)} \big[\theta(1+\alpha) + \theta_1^M\big(\mu(K)\big)\big]\big[\theta_1^M\big(\mu(K)\big) - \theta\big]g(\theta)d\theta \\
&- C(K)
\end{aligned} \tag{54}$$

with $\mu^M(K)$ implicitly given by equation (53).

Taking the derivative with respect to K we have:

$$\frac{\partial \pi_S}{\partial K} = \frac{-1}{(1-\alpha^2)(2+\alpha)^2}\left[\int_{\theta_2}^{\bar{\theta}} \big[\alpha\theta + 2\theta_2\big]\frac{\partial \theta_2}{\partial K} g(\theta)d\theta + \int_{\underline{\theta}}^{\theta_1} \big[\alpha\theta + 2\theta_1\big]\frac{\partial \theta_1}{\partial K} g(\theta)d\theta\right] - C'(K). \tag{55}$$

Applying the implicit function theorem to equation (53), we can obtain:

$$\frac{\partial \theta_1}{\partial K} = \frac{\partial \theta_1}{\partial \mu}\frac{\partial \mu}{\partial K} = \frac{2+\alpha}{G(\theta_1)}$$

$$\frac{\partial \theta_2}{\partial K} = \frac{\partial \theta_2}{\partial \mu}\frac{\partial \mu}{\partial K} = -\frac{2+\alpha}{1-G(\theta_2)}$$

Thus:

$$\frac{\partial \pi_S}{\partial K} = \frac{1}{(1-\alpha^2)(2+\alpha)G(\theta_1)}\left[2G(\theta_1)(\theta_2-\theta_1)+\alpha\left(\int_{\theta_2}^{\bar{\theta}}\theta g(\theta)d\theta - \int_{\underline{\theta}}^{\theta_1}\theta g(\theta)d\theta\right)\right]-C'(K)$$
$$= \frac{2\alpha K}{(1-\alpha^2)G(\theta_1)}+\frac{\theta_2-\theta_1}{1-\alpha^2}-C'(K).$$

Therefore,
$$\frac{\partial \pi_S}{\partial K}=0 \Leftrightarrow \frac{2\alpha K}{(1-\alpha^2)G(\theta_1)}+\frac{\theta_2-\theta_1}{1-\alpha^2}=C'(K). \tag{56}$$

(ii) Optimality conditions when $\alpha = 0$ are given by:

$$\begin{aligned}
C'(K) &= \theta_2^{FB}(K)-\theta_1^{FB}(K)\\
C'(K) &= \theta_2^{SB}(K)-\theta_1^{SB}(K)\\
C'(K) &= \theta_2^{M}(K,\alpha=0)-\theta_1^{M}(K,\alpha=0)
\end{aligned}$$

Equations (29), (38) and (53) imply that

$$\theta_2^{FB}(K)-\theta_1^{FB}(K)=\theta_2^{SB}(K)-\theta_1^{SB}(K)>\theta_2^{M}(K,\alpha=0)-\theta_1^{M}(K,\alpha=0)$$

Thus, $K^M < K^{SB} = K^{FB}$ for $\alpha = 0$.

(iii) We just need to show that $K^M < K^{FB}$, as $< K^{FB} < K^{SB}$ is always true for $\alpha > 0$. With demand uniformly distributed on $[\underline{\theta}, \bar{\theta}]$, optimal investment in storage capacity is given by:

$$\begin{aligned}
\theta_2^{FB}(K^{FB})-\theta_1^{FB}(K^{FB}) &= C'(K^{FB})\\
\bar{\theta}-\underline{\theta}-2\sqrt{2(\bar{\theta}-\underline{\theta})K^{FB}} &= K^{FB}\\
K^{FB} &= \left[5-2\sqrt{6}\right](\bar{\theta}-\underline{\theta})
\end{aligned}$$

Note that, for θ uniformly distributed on $[\underline{\theta}, \bar{\theta}]$, the marginal investment revenue for the monopolist $MR^M(K)$ is:

$$\begin{aligned}
MR^M(K) &= \frac{\theta_2^M-\theta_1^M(K)}{1-\alpha^2}+\frac{2\alpha K}{(1-\alpha^2)G(\theta_1^M(K))}\\
&= \frac{(\bar{\theta}-\underline{\theta})-2\sqrt{2(2+\alpha)(\bar{\theta}-\underline{\theta})K}}{1-\alpha^2}+\frac{\alpha 2K(\bar{\theta}-\underline{\theta})}{(1-\alpha^2)\sqrt{2(2+\alpha)(\bar{\theta}-\underline{\theta})K}}\\
&= \frac{1}{(1-\alpha^2)\sqrt{2+\alpha}}\left(\sqrt{2+\alpha}(\bar{\theta}-\underline{\theta})-(4+\alpha)\sqrt{2(\bar{\theta}-\underline{\theta})K}\right)
\end{aligned}$$

Evaluated at K^{FB}:

$$MR^M(K = K^{FB}) = \frac{\bar{\theta} - \underline{\theta}}{(1-\alpha^2)\sqrt{2+\alpha}} \left(\sqrt{2+\alpha} - (4+\alpha)\sqrt{2(5-2\sqrt{6})}\right) < 0$$

Moreover,

$$\begin{aligned}\frac{\partial MR^M(K)}{\partial K} =& \frac{2}{(2+\alpha)(1-\alpha^2)}\left(\frac{\partial \theta_2^M(K)}{\partial K} - \frac{\partial \theta_1^M(K)}{\partial K}\right) \\ &+ \frac{\alpha}{(2+\alpha)(1-\alpha^2)}\left(-\theta_2^M g(\theta_2^M) - \theta_1^M g(\theta_1^M)\right) < 0\end{aligned}$$

for all K, as $\frac{\partial \theta_2^M(K)}{\partial K} < 0$ and $\frac{\partial \theta_1^M(K)}{\partial K>0}$. Thus, $K^M(K) < K^{FB}(K)$.

Proof of Lemma 6

The structure of the functional optimization problem is identical to the one in the Proof of Proposition 1, with concavity and compactness assumptions satisfied, so a unique solution to the problem exists. For a more formal treatment of the problem, we refer the reader to the characterization of the first-best. The Lagrangian of the problem, omitting the non-negativity constraints, is given by:

$$\begin{aligned}\mathbb{L}\big(p(\theta), q_B(\theta), q_S(\theta), \lambda, \mu\big) =& \int_{\underline{\theta}}^{\bar{\theta}} \left[p(\theta) D\,(p;\theta) - \frac{[D\,(p;\theta) - q_S(\theta) + q_B(\theta)]^2}{2\alpha}\right] g\,(\theta)\, d\theta \\ &+ \lambda\left[\int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta) g(\theta) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} q_S(\theta) g(\theta) d\theta\right] + \mu\left[K - \int_{\underline{\theta}}^{\bar{\theta}} q_B(\theta) g(\theta) d\theta\right]\end{aligned}$$

where λ and μ are the Lagrangian multipliers and $D\,(p;\theta) = \theta - (1-\alpha)p(\theta)$. Without loss of generality, we can focus attention on cases in which for any $\theta \in [\underline{\theta}, \bar{\theta}]$, $q_B(\theta) > 0 \rightarrow q_S(\theta) = 0$ and $q_S(\theta) > 0 \rightarrow q_B(\theta) = 0$. The KKT conditions are:

$$\theta - (1-\alpha)\big[q_S(\theta) - q_B(\theta)\big] - (1-\alpha^2)p(\theta) = 0, \forall \theta \tag{57}$$

$$\frac{1}{\alpha}\big[\theta - q_S(\theta) - (1-\alpha)p(\theta)\big] - \lambda = 0, \forall \theta \geq \theta_2 \tag{58}$$

$$\frac{1}{\alpha}\big[\theta - (1-\alpha)p(\theta)\big] - \lambda < 0, \forall \theta < \theta_2$$

$$\frac{1}{\alpha}\big[\theta + q_B(\theta) - (1-\alpha)p(\theta)\big] - \lambda + \mu = 0, \forall \theta \leq \theta_1 \tag{59}$$

$$\frac{1}{\alpha}\big[\theta - (1-\alpha)p(\theta)\big] - \lambda + \mu > 0, \forall \theta > \theta_1$$

$$\int_{\underline{\theta}}^{\theta_1} q_B(\theta) g(\theta) d\theta = \int_{\theta_2}^{\bar{\theta}} q_S(\theta) g(\theta) d\theta \tag{60}$$

with $\theta_1 \leq \theta_2$ and the complementary slackness conditions identical to those in equations (21)-(25). These conditions are necessary and sufficient, due the concavity of both the objective function and the constraints.

Combining conditions (57) and (58):

$$q_S(\theta) = \frac{\theta - \lambda(1+\alpha)}{2}, \forall \theta \geq \theta_2$$
$$p(\theta) = \frac{\theta + \lambda(1-\alpha)}{2(1-\alpha)}, \forall \theta \geq \theta_2$$

From conditions (57) and (59):

$$q_B(\theta) = \frac{(\lambda - \mu)(1+\alpha) - \theta}{2}, \forall \theta \leq \theta_1$$
$$p(\theta) = \frac{\theta + (\lambda - \mu)(1-\alpha)}{2(1-\alpha)}, \forall \theta \leq \theta_1$$

And from condition (57):

$$p(\theta) = \frac{\theta}{1-\alpha^2} \text{ for } \theta_1 < \theta < \theta_2.$$

By continuity:

$$q_S(\theta_2) = 0 \Rightarrow \theta_2 = (1+\alpha)\lambda \Rightarrow \begin{cases} q_S^I(\theta) = \frac{\theta - \theta_2}{2}, \forall \theta > \theta_2 \\ p(\theta) = \frac{\theta}{2(1-\alpha)} + \frac{\theta_2}{2(1+\alpha)}, \forall \theta > \theta_2 \end{cases}$$
$$q_B(\theta_1) = 0 \Rightarrow \theta_1 = (1+\alpha)(\lambda - \mu) \Rightarrow \begin{cases} q_B^I(\theta) = \frac{\theta_1 - \theta}{2}, \forall \theta < \theta_1 \\ p(\theta) = \frac{\theta}{2(1-\alpha)} + \frac{\theta_1}{2(1+\alpha)}, \forall \theta < \theta_1 \end{cases}$$

From condition 60:

$$\int_{\underline{\theta}}^{\theta_1} \frac{\theta_1 - \theta}{2} g(\theta) d\theta = \int_{\theta_2}^{\bar{\theta}} \frac{\theta - \theta_2}{2} g(\theta) d\theta. \tag{61}$$

We have two possible cases depending on the value of the exogenous parameter K. When K is binding, $\mu > 0$ and $\theta_2 - \theta_1 = \mu(1+\alpha) > 0$. Define $x = \theta_2 - \theta_1$ and assume that $G(\theta)$ has a well-defined mean given by $\mathbb{E}(\theta)$. By symmetry of $G(\theta)$, equation (61) implies that θ_2 and θ_1 must be symmetric around the mean i.e.,

$$\theta_1 = \mathbb{E}(\theta) - \frac{x}{2} \Rightarrow \theta_1^C = \mathbb{E}(\theta) - \frac{\mu(1+\alpha)}{2} \tag{62}$$
$$\theta_2 = \mathbb{E}(\theta) + \frac{x}{2} \Rightarrow \theta_2^C = \mathbb{E}(\theta) + \frac{\mu(1+\alpha)}{2} \tag{63}$$

with $\mu = \mu^I(K)$ implicitly given by:

$$\int_{\underline{\theta}}^{\theta_1(\mu^I(K))} \frac{\theta_1(\mu^I(K)) - \theta}{2} g(\theta) d\theta = \int_{\theta_2(\mu^I(K))}^{\bar{\theta}} \frac{\theta - \theta_2(\mu^I(K))}{2} g(\theta) d\theta = K \tag{64}$$

Note that when K is not binding, so that $\mu = 0$, from equations (62) and (63) it is straightforward to establish that $\theta_1^I = \theta_2^I = \mathbb{E}(\theta)$. However, as shown below, $\mu = 0$ cannot be a solution to the first stage problem.

Proof of Corollary 1

Let us use $L(\theta)$ to denote the Lerner Index, i.e., the ratio between price minus marginal cost over price. Using the equilibrium storage decisions,

$$L(\theta) = \begin{cases} \frac{\theta(1+\alpha) - \theta_1^I(1-\alpha)}{\theta(1+\alpha)+\theta_1^I(1-\alpha)} & \text{if} \quad \theta < \theta_1^I \\ \alpha & \text{if} \quad \theta_1^I \leq \theta \leq \theta_2^I \\ \frac{\theta(1+\alpha) - \theta_2^I(1-\alpha)}{\theta(1+\alpha)+\theta_2^I(1-\alpha)} & \text{if} \quad \theta > \theta_2^I \end{cases}$$

These mark-ups are continuous in θ. They are constant for $\theta_1^I \leq \theta \leq \theta_2^I$ and increasing in θ otherwise. Hence, for $\theta < \theta_1^I$, $L(\theta) < \alpha$. And for $\theta > \theta_2^I$, $L(\theta) > \alpha$. Since the two expressions are a mirror image of each other, while the markups for high demand levels are weighted more, the demand-weighted average mark-up is greater than α.

Proof of Proposition 5

By identical arguments to those in the proof for optimal First Best capacity investment (Section 7), the unique interior solution K^I is given by the solution to:

$$\mu - C'(K) = \frac{\theta_2^I(K) - \theta_1^I(K)}{1+\alpha} - C'(K) = 0 \tag{65}$$

with $\mu = \mu^I(K)$ implicitly given by the solution to

$$\int_{\underline{\theta}}^{\theta_1(\mu(K))} \frac{\theta_1(\mu(K)) - \theta}{2} g(\theta) d\theta = \frac{1}{2} \int_{\theta_2(\mu(K))}^{\bar{\theta}} \frac{\theta - \theta_2(\mu(K))}{2} g(\theta) d\theta = K.$$

Now we show that $K^I < K^{FB}$. Assume on the contrary that $K^I \geq K^{FB}$. Then, by strict convexity of the cost function:

$$K^I \geq K^{FB} \Rightarrow C'(K^I) \geq C'(K^{FB}) \tag{66}$$

Moreover:

$$K^I \geq K^{FB} \Rightarrow \theta_2^I - \theta_1^I < \theta_2^{FB} - \theta_1^{FB} \Rightarrow (1+\alpha)C'(K^I) < C'(K^{FB}) \tag{67}$$

where the first implication comes from the capacity constraints in the optimal solution (equations (29) and (64)), and the second from first stage optimality conditions (4) and (17). Putting (66) and (67) together:

$$(1+\alpha)C'(K^I) < C'(K^{FB}) \leq C'(K^I)$$

Proof of Proposition 6

From Lemma 4, we know that, for given K, equilibrium storage decisions under the second-best and under competitive storage coincide. Thus, $W^{SB}(K) = W^C(K)$ and $CS^{SB}(K) = CS^C(K)$, $\forall K$. Moreover,

$$\{q_B^{SB}(\theta,K), q_S^{SB}(\theta,K)\} = \underset{q_B(\theta,K),q_S(\theta,K)}{\text{argmax}} W(K)$$

Thus, as $q_B^M(\theta,K) \neq q_B^{SB}(\theta,K)$ and $q_B^I(\theta,K) \neq q_B^{SB}(\theta,K)$ for some θ and all K, we have that $W^{SB}(K) > W^M(K)$ and $W^{SB}(K) > W^I(K)$, $\forall K$.

For the second part of the proposition, we want to show that $W^M(K) > W^I(K)$, which is equivalent to $TC^M(K) > TC^I(K)$. Note that:

$$\begin{aligned}
\lim_{K\to\infty}[TC^M(K) - TC^I(K)] &= V[\theta]\frac{5\alpha}{8(1-\alpha)(2+\alpha)^2} > 0 \\
\lim_{K\to\infty}[TC^M(K) - TC^I(K)] &= 0
\end{aligned}$$

Thus, $TC^M(K) < TC^I(K)$ if:

$$\frac{\partial TC^M(K)}{\partial K} < 0 \tag{68}$$

$$\frac{\partial TC^I(K)}{\partial K} < 0 \tag{69}$$

$$\left|\frac{\partial TC^M(K)}{\partial K}\right| \geq \left|\frac{\partial TC^I(K)}{\partial K}\right| \tag{70}$$

for all $K < K^M(max)$, where:

$$K^M(max) = \int_{\underline{\theta}}^{\mathbb{E}[\theta]} \frac{\mathbb{E}[\theta]-\theta}{2+\alpha} g(\theta)d\theta.$$

Recall that

$$TC(K) = \int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{\alpha q_F^2(\theta,K)}{2} + \frac{q_F^2(\theta,K)}{2(1-\alpha)}\right) g(\theta)d\theta$$

For the monopolist:

$$TC^M(K) = \frac{1+\alpha-\alpha^2}{2(1-\alpha)}\Bigg[\int_{\underline{\theta}}^{\theta_1^M(K)} \left(\frac{\theta(1+\alpha)+\theta_1^M(K)}{(2+\alpha)(1+\alpha)}\right)^2 g(\theta)d\theta + \int_{\theta_1^M(K)}^{\theta_2^M(K)} \left(\frac{\theta}{(1+\alpha)}\right)^2 g(\theta)d\theta$$
$$+ \int_{\theta_2^M(K)}^{\bar{\theta}} \left(\frac{\theta(1+\alpha)+\theta_2^M(K)}{(2+\alpha)(1+\alpha)}\right)^2 g(\theta)d\theta\Bigg]$$

For the vertically integrated firm

$$TC^I(K) = \int_{\underline{\theta}}^{\theta_1^M(K)} \left(\frac{1}{2\alpha}\left(\frac{\alpha\theta_1^I(K)}{1+\alpha}\right)^2 + \frac{1}{2(1-\alpha)}\left(\frac{(1+\alpha)\theta+(1-\alpha)\theta_1^I(K)}{2(1+\alpha)}\right)^2\right) g(\theta)d\theta$$
$$+ \frac{1+\alpha-\alpha^2}{2(1-\alpha)}\int_{\theta_1^M(K)}^{\theta_2^M(K)} \left(\frac{\theta}{(1+\alpha)}\right)^2 g(\theta)d\theta$$
$$+ \int_{\theta_2^M(K)}^{\bar{\theta}} \left(\frac{1}{2\alpha}\left(\frac{\alpha\theta_2^I(K)}{1+\alpha}\right)^2 + \frac{1}{2(1-\alpha)}\left(\frac{(1+\alpha)\theta+(1-\alpha)\theta_2^I(K)}{2(1+\alpha)}\right)^2\right) g(\theta)d\theta$$

For the independent monopolist, after some computations we have that:

$$\frac{\partial TC^M(K)}{\partial K} = \frac{1+\alpha-\alpha^2}{(1-\alpha)(2+\alpha)(1+\alpha)^2 G\left(\theta_1^M(K)\right)}\Bigg[\int_{\underline{\theta}}^{\theta_1^M(K)} \left[\theta(1+\alpha)+\theta_1^M(K)\right] g(\theta)d\theta$$
$$- \int_{\theta_2^M(K)}^{\bar{\theta}} \left[\theta(1+\alpha)+\theta_2^M(K)\right] g(\theta)d\theta\Bigg] < 0$$
$$\left|\frac{\partial TC^M(K)}{\partial K}\right| = \frac{1+\alpha-\alpha^2}{(1-\alpha)(2+\alpha)(1+\alpha)^2}\left[\theta_2^M(K)-\theta_1^M(K)\right]$$
$$+ \frac{1+\alpha-\alpha^2}{(1-\alpha)(2+\alpha)(1+\alpha)G\left(\theta_1^M(K)\right)}\left[\int_{\theta_2^M(K)}^{\bar{\theta}} \theta g(\theta)d\theta - \int_{\underline{\theta}}^{\theta_1^M(K)} \theta g(\theta)d\theta\right]$$

and for the vertically integrated firm:

$$\frac{\partial TC^I(K)}{\partial K} = \frac{1}{2(1+\alpha)^2}\frac{1}{G\left(\theta_1^I(K)\right)}\Bigg[\int_{\underline{\theta}}^{\theta_1^M(K)} \left[(1+\alpha)\theta+(1+3\alpha)\theta_1^I(K)\right] g(\theta)d\theta$$
$$- \int_{\theta_2^I(K)}^{\bar{\theta}} \left[(1+\alpha)\theta+(1+3\alpha)\theta_2^I(K)\right] g(\theta)d\theta\Bigg] < 0$$
$$\left|\frac{\partial TC^I(K)}{\partial K}\right| = \frac{(1+3\alpha)}{2(1+\alpha)^2}\left[\theta_2^I(K)-\theta_1^I(K)\right] + \frac{1}{2(1+\alpha)G\left(\theta_1^I(K)\right)}\left[\int_{\theta_2^I(K)}^{\bar{\theta}} \theta g(\theta)d\theta - \int_{\underline{\theta}}^{\theta^I(K)} \theta g(\theta)d\theta\right]$$

Assuming that θ is uniformly distributed on $[\underline{\theta}, \bar{\theta}]$, we have that:

$$\left|\frac{\partial TC^M(K)}{\partial K}\right| = \frac{1+\alpha-\alpha^2}{(1-\alpha)(2+\alpha)(1+\alpha)^2}\left[(2+\alpha)(\bar{\theta}-\underline{\theta})-(3+\alpha)\sqrt{2(2+\alpha)(\bar{\theta}-\underline{\theta})K}\right]$$
$$\left|\frac{\partial TC^I(K)}{\partial K}\right| = \frac{1}{(1+\alpha)^2}\left[(1+2\alpha)(\bar{\theta}-\underline{\theta})+(7\alpha+3)\sqrt{(\bar{\theta}-\underline{\theta})K}\right]$$

Thus:

$$\left|\frac{\partial TC^M(K)}{\partial K}\right| - \left|\frac{\partial TC^I(K)}{\partial K}\right| > 0 \Leftrightarrow K < \left(\frac{\alpha^2(2+\alpha)\sqrt{\bar{\theta}-\underline{\theta}}}{(1+\alpha-\alpha^2)(3+\alpha)\sqrt{2(2+\alpha)}-(7\alpha+3)(1-\alpha)(2+\alpha)}\right)^2$$

Note that:

$$\left(\frac{\alpha^2(2+\alpha)\sqrt{\bar{\theta}-\underline{\theta}}}{(1+\alpha-\alpha^2)(3+\alpha)\sqrt{2(2+\alpha)}-(7\alpha+3)(1-\alpha)(2+\alpha)}\right)^2 < \frac{\left(\mathbb{E}[\theta]-\underline{\theta}\right)^2}{2(\bar{\theta}-\underline{\theta})(2+\alpha)} = K^M(max)$$

Thus, $\left|\frac{\partial TC^M(K)}{\partial K}\right| > \left|\frac{\partial TC^I(K)}{\partial K}\right|$, so that $TC^M(K) < TC^I(K)$ and $W^M(K) > W^I(K)$.

For comparing consumer surplus, we follow the same approach. First note that:

$$\lim_{K\to\infty}[CS^M(K) - CS^I(K)] = \frac{\alpha}{2+\alpha}V[\theta] > 0$$
$$\lim_{K\to\infty}[CS^M(K) - CS^I(K)] = 0$$

Second, note that:

$$\frac{\partial CS^M(K)}{\partial K} = \frac{1}{(1-\alpha^2)G(\theta_1^M(K))}\left[\int_{\theta_2^M(K)}^{\bar{\theta}}\theta g(\theta)d\theta - \int_{\underline{\theta}}^{\theta_1^M(K)}\theta g(\theta)d\theta\right] > 0$$

for all K and

$$\frac{\partial CS^I(K)}{\partial K} = \frac{1}{(1+\alpha)G(\theta_1^I(K))}\left[\int_{\theta_2^I(K)}^{\bar{\theta}}\theta g(\theta)d\theta - \int_{\underline{\theta}}^{\theta_1^I(K)}\theta g(\theta)d\theta\right] > 0$$

for all K.

Finally, when θ is uniformly distributed on $[\underline{\theta}, \bar{\theta}]$, we have that:

$$\frac{\partial CS^M(K)}{\partial K} - \frac{\partial CS^I(K)}{\partial K} > 0 \Leftrightarrow \frac{\bar{\theta}-\underline{\theta}-\sqrt{2(2+\alpha)(\bar{\theta}-\underline{\theta})K}}{1-\alpha^2} - \frac{\bar{\theta}-\underline{\theta}-2\sqrt{(\bar{\theta}-\underline{\theta})K}}{1+\alpha} > 0$$
$$\left(\frac{\alpha\sqrt{\bar{\theta}-\underline{\theta}}}{\sqrt{2(2+\alpha)}+2\alpha-2}\right)^2 > K$$

Note that:

$$\left(\frac{\alpha\sqrt{\bar{\theta}-\underline{\theta}}}{\sqrt{2(2+\alpha)}+2\alpha-2}\right)^2 < \frac{\left(\mathbb{E}[\theta]-\underline{\theta}\right)^2}{2(\bar{\theta}-\underline{\theta})(2+\alpha)} = K^M(max)$$

Thus, $\frac{\partial CS^M(K)}{\partial K} > \frac{\partial CS^I(K)}{\partial K}$, so that $CS^M(K) > CS^I(K)$, for all K.