

Global carbon price asymmetry

EPRG Working Paper 2116

Cambridge Working Paper in Economics 2145

Robert A. Ritz

Abstract This paper studies a social planner who chooses countries' carbon prices so as to maximize global welfare. Product markets are characterized by firm heterogeneity, market power, and international trade. Because of the market-power distortion, the planner's optimal policy is second-best. The main insight is that optimal carbon prices may be highly asymmetric: zero in some countries and above the social cost of carbon in countries with relatively dirty production. This result obtains even though a uniform global carbon price is always successful at reducing countries' emissions. Competition policy that mitigates market power may enable stronger and more balanced climate action.

Keywords Carbon leakage, carbon pricing, imperfect competition, international trade, second best

JEL Classification H23 (externalities), L11 (market structure), Q54 (climate)

Contact
Publication
Funding

Robert Ritz – r.ritz@jbs.cam.ac.uk
May 2021
none

Global carbon price asymmetry

Robert A. Ritz*

Energy Policy Research Group

Judge Business School

University of Cambridge

This version: December 2020

Abstract

This paper studies a social planner who chooses countries' carbon prices so as to maximize global welfare. Product markets are characterized by firm heterogeneity, market power, and international trade. Because of the market-power distortion, the planner's optimal policy is second-best. The main insight is that optimal carbon prices may be highly asymmetric: zero in some countries and above the social cost of carbon in countries with relatively dirty production. This result obtains even though a uniform global carbon price is always successful at reducing countries' emissions. Competition policy that mitigates market power may enable stronger and more balanced climate action.

Keywords: Carbon leakage, carbon pricing, imperfect competition, international trade, second best

JEL codes: H23 (externalities), L11 (market structure), Q54 (climate)

*I am grateful to Lassi Ahlvik, Geoffroy Dolphin, Sam Fankhauser, Felix Grey, Chris Harris, Cameron Hepburn, Frank Jotzo and David Newbery for helpful comments on earlier versions of this paper. All views expressed and any errors are mine.

1 Introduction

Carbon pricing is increasingly being used as a key policy instrument to combat climate change. Yet carbon prices around the world remain low and uneven: around €30 per ton of CO₂ in Europe’s flagship cap-and-trade system—and even higher for some national carbon taxes—but much lower in most other jurisdictions (World Bank 2020). This picture stands in marked contrast to the Pigouvian ideal of a uniform global carbon price set at the social cost of carbon (SCC).

So far, carbon pricing has focused on power generation and emissions-intensive industrial sectors like aluminium, cement and steel. Three characteristics of these regulated industries are striking. First, firms within each industry often have widely varying carbon intensities of production. This enhances the potential for market-based regulation to achieve significant gains in abatement-cost efficiency. Second, emissions-intensive industries are often highly concentrated with long-standing concerns about the exercise of market power. This makes relevant the theory of the second best. Third, international trade is important as the scope of the product market in which regulated firms compete is often wider than that of the carbon price they face. This has led to concerns about the potential for leakage of emissions to less regulated jurisdictions.

This paper studies the optimal design of carbon prices in a model in which these three characteristics are crucial. The model considers a social planner who chooses countries’ carbon prices so as to maximize global welfare. Because of a market-power distortion in the product market, the planner’s optimal policy is second-best. The central trade-off is that a higher carbon price reduces a country’s domestic emissions but also increases deadweight losses in the product market (due to pass-through of carbon costs to consumers) and leads to a degree of carbon leakage to the other country.¹ Thereby, the country with relatively clean firms is more vulnerable to carbon leakage as a policy-induced loss in production to the dirtier country translates into a larger increase in emissions. In the special case without market power and without carbon leakage, the planner sets a uniform global carbon price at the SCC, restoring the first-best outcome.

The main insight is that second-best carbon prices can be highly asymmetric across countries. Market power, on its own, pushes countries’ optimal carbon prices downwards as the planner seeks to cushion the increase in consumer prices. The presence of international trade introduces a further effect: if carbon leakage for the country with relatively clean firms is sufficiently pronounced, its optimal carbon price is zero. This, in turn, limits deadweight losses in the product market and enables the planner to choose a higher carbon price for the dirtier country—which creates additional climate benefits as it reshuffles

¹The leakage channel in the model arises from the market-share losses of more tightly regulated firms.

production to cleaner firms. As long as market power is not too pronounced, the dirtier country’s optimal carbon price may lie above the SCC. This finding obtains even though a uniform global carbon price is always successful in reducing countries’ emissions.

The result should not be overplayed given the model’s very simple welfare function.² The more general point is that, while carbon prices around the world today are almost certainly far too low, failing to implement a uniform global carbon price does not necessarily imply the wrong response to climate change. Moreover, competition policy to mitigate market power may also enable stronger and more balanced climate action.

Related literature. This paper relates to several strands of literature on the theory of environmental economics. First, when climate action is exogenously restricted to a subset of countries, it is second-best for those countries to set lower carbon prices for sectors with internationally-traded products—unless corrective trade tariffs are available (Hoel 1996). This paper, by contrast, endogenizes the extent of climate action across all countries and is concerned with cross-country (rather than cross-sectoral) differences in carbon prices.

Second, with imperfect competition in product markets, the optimal emissions price typically falls short of the Pigouvian rule (Buchanan 1969). This theory applies to local environmental problems in which a government sets a single domestic emissions price. This paper studies second-best carbon prices that maximize global welfare in an international context in which each country may set a different carbon price.^{3,4}

Third, cross-country differences in marginal abatement costs can be optimal due to equity concerns: a less rich country may have a higher marginal utility of income (Chichilnisky & Heal 1994). This result can be interpreted as a second-best policy due to restrictions on international financial transfers. This paper directly characterizes countries’ carbon prices and obtains an extreme version of non-uniform pricing in a model without equity concerns.

2 Model

Consider a global industry in which $n_k \geq 1$ firm(s) are based in country $k = i, j$. Firm m from country k produces x_k^m units of output with an emissions intensity $z_k \equiv e_k^m/x_k^m$, where e_k^m is firm m ’s emissions and $E_k \equiv \sum_m e_k^m$ ($k = i, j$). The global demand curve

²The model considers a partial-equilibrium setting without further distortions in factor markets or wider tax interactions. The social planner does not have access to additional policy instruments (e.g., output subsidies) to directly address the market-power distortion.

³A second-best domestic emissions tax can exceed social cost if the demand curve is highly convex or if this alleviates the problem of “excess entry” by firms. Neither consideration arises in the present model. See Requate (2006) for a valuable survey of environmental regulation under imperfect competition.

⁴Babiker (2005) finds carbon leakage rates up to 130% in a general-equilibrium model with similar ingredients to the present paper but does not study countries’ optimal carbon prices.

is linear $p(X) = \alpha - X$ where $X \equiv X_i + X_j$ is total industry output ($X_k \equiv \sum_m x_k^m$ for $k = i, j$). Firms have constant unit costs, which are set to zero to facilitate the exposition. Faced with a carbon price τ_k in its country, firm m of k 's profits are $\Pi_k^m = px_k^m - \tau_k e_k^m$.⁵

The product market features a generalized version of Cournot competition. The conduct parameter $\theta \in (0, 1]$ indexes the intensity of competition. Formally, firms' equilibrium outputs $(\hat{x}_k^m)_{k=i,j}$ satisfy:

$$\hat{x}_k^m = \arg \max_{x_k^m \geq 0} \left\{ \left[p \left(\theta(x_k^m - \hat{x}_k^m) + \sum_{m=1}^{n_i} \hat{x}_i^m + \sum_{m=1}^{n_j} \hat{x}_j^m \right) - \tau_k z_k \right] x_k^m \right\}. \quad (1)$$

Firm m in country k , in deviating its output by $(x_k^m - \hat{x}_k^m)$, conjectures that industry output will change by $\theta(x_k^m - \hat{x}_k^m)$ as a result. In this ‘‘conduct equilibrium’’ (Weyl & Fabinger 2013), a lower θ corresponds to more intense competition while $\theta > 0$ means that competition is imperfect. The Cournot-Nash equilibrium occurs where $\theta = 1$.

Profit-maximization implies that, for each firm, a generalized version of marginal revenue, $p - \theta x_k^m$, is equal to its marginal carbon cost, $z_k \tau_k$. Equivalently, its marginal abatement cost $(p - \theta x_k^m)/z_k$ equals the carbon price. Let $X_k(\tau_i, \tau_j)$ and $E_k(\tau_i, \tau_j)$ denote outputs and emissions in this product-market equilibrium.

Global welfare $W = U - sE$ reflects consumer utility $U \equiv \int_0^X p(v)dv$ and the global SCC, s .⁶ The social planner's problem is to $\max_{\tau_i, \tau_j} W(\tau_i, \tau_j)$ subject to the constraint that firms make non-negative profits, $\Pi_k^m \geq 0$ ($k = i, j$). Assume that $W(0, 0) > 0$ so the market is socially viable without carbon pricing—and the planner therefore never shuts it down. A necessary condition for this is that some consumers' willingness-to-pay exceeds environmental damages, $\alpha > s \max\{z_i, z_j\}$.⁷

3 Carbon prices and global emissions

The first results characterize basic properties of carbon pricing. The rate of carbon leakage associated with carbon pricing by country i is:

$$L_i^C \equiv \frac{dE_j(\tau_i, \tau_j)/d\tau_i}{-dE_i(\tau_i, \tau_j)/d\tau_i}. \quad (2)$$

⁵The assumption of constant emissions intensities is a common and useful benchmark in the literature. Similar insights would obtain in a richer model with end-of-pipe abatement. Production costs being equal across firms and countries switches off the welfare channel that carbon pricing can affect industry production-cost efficiency (Hepburn, Quah & Ritz 2013) so as to focus sharply on consumer welfare and climate damages.

⁶Product-market revenues are a transfer from consumers to firms, carbon-pricing revenues are a transfer from firms to governments, and production costs are zero.

⁷It is easy to construct examples in which carbon pricing makes socially viable a market that—given the SCC—was otherwise not, with $W(\tau_i, \tau_j) > 0 > W(0, 0)$ for some $\tau_i, \tau_j > 0$.

This measures the fraction of i 's emissions reduction that leaks to j . Also define output leakage $L_i^O \equiv (dX_j/d\tau_i)/(-dX_i/d\tau_i)$.

Lemma 1 *An increase in country i 's carbon price τ_i reduces its domestic production, $dX_i/d\tau_i < 0$ and its domestic emissions, $dE_i/d\tau_i < 0$, where:*

- (a) *the rate of output leakage $L_i^O = n_j/(n_j + \theta) > 0$;*
- (b) *the rate of carbon leakage $L_i^C = (z_j/z_i)[n_j/(n_j + \theta)] > 0$;*
- (c) *carbon cost pass-through $dp(\tau_i, \tau_j)/d\tau_i = [n_i/(n_i + n_j + \theta)] z_i > 0$.*

Output leakage is more pronounced with (i) more rivals in j engaging in “business stealing” from those in i as a result of the unilateral cost increase (higher n_j); and (ii) more competitive conduct (lower θ).

Carbon leakage equals output leakage scaled by the relative emissions intensity z_j/z_i . A higher carbon price by i increases in global emissions if its carbon leakage exceeds 100%. This is ruled out by symmetry but occurs if j 's production is sufficiently more polluting.⁸

Carbon pricing reduces i 's profit margin as less than 100% of its carbon cost is passed on to consumers; pass-through decreases with market power and with more rivals in j .

Global action “works” in the following sense:

Lemma 2 *An increase in a uniform global carbon price ($\tau_k = \tau$ for $k = i, j$):*

- (a) *reduces global emissions, $dE(\tau, \tau)/d\tau < 0$;*
- (b) *reduces country k 's emissions, $dE_k(\tau, \tau)/d\tau \leq 0$, if and only if $L_k^C \leq 1$.*

A uniform tightening in carbon prices is always successful at reducing aggregate emissions—even if it may induce higher emissions by an individual country. Intuitively, if unilateral action by i has carbon leakage above 100%, then i 's firms are significantly cleaner than j 's so a higher global carbon price improves their competitiveness and they expand production and emissions.⁹

4 Carbon prices and global welfare

Now consider the second-best carbon prices chosen by a social planner. At a global level, carbon pricing involves a trade-off between lower consumer utility and the potential for

⁸Large intra-industry heterogeneity is borne out in practice (Lyubich, Shapiro & Walker 2018).

⁹A direct corollary is that global carbon pricing may not achieve the lowest global emissions. To illustrate, suppose that i and j each have a carbon price of zero or the SCC. Due to the linearity of emissions in carbon prices, the model exhibits the property that countries' emissions reductions are additive. Letting $\Delta E(\tau_i, \tau_j) \equiv [E(\tau_i, \tau_j) - E(0, 0)]$, unilateral action and coordinated action are related according to:

$$\underbrace{\Delta E(s, s)}_{\text{both } i \text{ and } j \text{ tighten}} = \underbrace{\Delta E(s, 0)}_{i \text{ tightens unilaterally}} + \underbrace{\Delta E(0, s)}_{j \text{ tightens unilaterally}} < 0,$$

So the inability of i 's carbon price to reduce global emissions ($\Delta E(s, 0) > 0$) is just the flipside of unilateral carbon pricing achieving lower global emissions than a global carbon price ($\Delta E(0, s) < \Delta E(s, s) < 0$).

lower environmental damages. Under some conditions, the former dominates:

Lemma 3 *If country i 's rate of carbon leakage is sufficiently high,*

$$L_i^C \geq 1 - \frac{\theta}{(n_j + \theta)} \frac{\theta}{(n_i + n_j + \theta)} \frac{(\alpha/s)}{z_i} \equiv \underline{L}_i^C,$$

then a zero carbon price is welfare-dominant, $W(0, \tau_j) \geq W(\tau_i, \tau_j)$ for all $\tau_i, \tau_j \geq 0$.

The result is immediate if i 's carbon leakage exceeds 100%. Then a “reverse leakage” argument applies: a reduction in i 's carbon price raises its own emissions but this is outweighed by the induced reduction in j 's emissions—so global emissions decline. As consumers also gain, global welfare rises. Because its leakage rate is constant, this logic holds at any level of countries' carbon prices. Put simply, the extent of i 's carbon leakage precludes any effective climate action by way of carbon pricing for its firms.

This conclusion applies as long as i 's leakage rate is sufficiently high, $L_i^C \geq \underline{L}_i^C$, where $\underline{L}_i^C < 1$ because of market power, $\theta > 0$. The critical value \underline{L}_i^C declines with the ratio α/s , which is a measure of the size of market-power distortion (via α) relative to the climate problem (via s).¹⁰ If the former is sufficiently important, \underline{L}_i^C turns negative.

The main interest of the paper lies in global carbon price asymmetry, so suppose that i 's firms are cleaner with $z_i/z_j < 1$. The next result shows how the problem is then resolved by the three industry characteristics described in the introduction:

Lemma 4 *Suppose that country i 's carbon price $\tau_i = 0$. Then an interior solution $\tau_j^* > 0$ for country j that maximizes $W(0, \tau_j)$ satisfies:*

$$\frac{\tau_j^*}{s} = 1 - \underbrace{\frac{\theta}{n_j} \left(\frac{\alpha/s - z_j}{z_j} \right)}_{\text{market power}} + \underbrace{\frac{n_i}{n_j} \left[1 + \left(\frac{n_i + n_j + \theta}{\theta} \right) \left(1 - \frac{z_i}{z_j} \right) \right]}_{\text{international competition \& firm heterogeneity}}.$$

The first deviation of τ_j^* from the SCC is driven by market power. The standard result that a second-best domestic emissions tax is below social cost is nested where $n_i = 0$. Lemma 4 then reduces to $\tau_j^*|_{n_i=0} = [s - (\theta/n_j)(\alpha/z_j - s)] < s$ (recalling that $\alpha > s \max\{z_i, z_j\}$). With perfect competition, $\tau_j^*|_{n_i=0, \theta=0} = s$ follows the Pigouvian rule.

The second deviation from the SCC instead pushes τ_j^* upwards—driven by firm heterogeneity and cross-border competition. An increase in j 's carbon price shifts production to i 's cleaner firms. This has two implications. First, output leakage to i limits the contraction in industry output due to j 's carbon price, mitigating the incremental product-market distortion. Second, the contraction in industry output leads to a greater reduction

¹⁰The initial $p(0, 0) = \theta\alpha/(n_i + n_j + \theta) > 0$ so $DWL(0, 0) = \frac{1}{2}[p(0, 0)]^2$ increases with α .

in global emissions precisely because i 's firms are cleaner. These factors limit deadweight losses and amplify environmental benefits, pushing upwards j 's optimal carbon price.

A related observation is that the social planner regards countries' carbon prices as strategic substitutes.¹¹ A higher carbon price by j raises the product price and so exacerbates the market-power distortion. This sharpens the planner's trade-off against emissions cuts by i , and reduces the welfare gain from i 's own carbon price.

The main result shows how this international-competition effect can dominate the planner's calculus and yield extreme asymmetry in global carbon prices:

Proposition 1 *Suppose that country i 's firms are sufficiently cleaner than j 's, with*

$$\frac{z_i}{z_j} \leq 1 - \frac{\theta n_j}{[(n_i + \theta)(n_i + \theta) + n_j(n_i + n_j + \theta)]} \equiv \delta < 1.$$

Then, for the range of parameter values given by

$$\frac{\alpha}{s} \in \left[\Phi, \Phi + \frac{n_i n_j}{\theta} (z_j - z_i) \right] \text{ where } \Phi \equiv \left(1 + \frac{n_i}{\theta} \right) \left[z_j + \frac{n_i}{\theta} (z_j - z_i) \right],$$

i 's second-best carbon price $\tau_i^ = 0$ while j 's satisfies $\tau_j^* \in [s, s[1 + (n_i/\theta)(1 - z_i/z_j)]]$.*

Proposition 1 establishes in equilibrium the logic underlying Lemmas 3 and 4. The range on α/s ensures that, on one hand, the market-power distortion is small enough for τ_j^* to exceed the SCC by Lemma 4 but, on the other hand, it is also large enough for j 's firms to remain profitable at a very high carbon price.¹² The condition $z_i/z_j \leq \delta$ ensures that indeed $\tau_i^* = 0$ because i 's leakage is sufficiently pronounced as per Lemma 3.

To illustrate, let $n_i = n_j = 3$, $\theta = \frac{1}{2}$, $z_j = 1$, and $s = 50$. Competitive conduct then lies “half-way” between Cournot-Nash and perfect competition, broadly in line with empirical estimates for many industrial markets. Suppose that i 's firms are modestly cleaner with $z_i = 0.9$; Proposition 1's condition becomes $z_i \leq \delta = \frac{121}{127}$ and is therefore met. With $\alpha = 600$, j 's optimal carbon price is $\tau_j^* = 73\frac{1}{3}$ by Lemma 4—almost 50% above the SCC. If instead $\alpha = 560$, $\tau_j^* = 80$ makes j 's firms just indifferent about being active while τ_j^* remains above the SCC as long as $\alpha \leq 740$. For these parameter values, $L_i^C = .952$ and $L_j^C = .771$ (independent of the value of α , by Lemma 1), confirming that the result does not hinge on carbon leakage exceeding 100% so global action works as per Lemma 2.¹³

¹¹Global welfare, $W(\tau_i, \tau_j) = U(\tau_i, \tau_j) - sE(\tau_i, \tau_j)$ is submodular in countries' carbon prices:

$$\frac{d}{d\tau_j} \left[\frac{dW(\tau_i, \tau_j)}{d\tau_i} \right] = \frac{d}{d\tau_j} \left[p(\tau_i, \tau_j) \frac{dX}{d\tau_i} - s \frac{dE_i}{d\tau_i} (1 - L_i^C) \right] = \frac{dp}{d\tau_j} \frac{dX}{d\tau_i} < 0,$$

since $dX/d\tau_i$, $dE_i/d\tau_i$ and L_i^C are all constants, $dp/d\tau_j > 0$, and $dX/d\tau_i < 0$ (Lemma 1).

¹²Note that i 's firms are always profitable given that $\tau_i^* = 0$.

¹³For $\alpha < 560$, Lemma 4's τ_j^* violates the constraint $\Pi_j^m \geq 0$ so the planner instead chooses the highest τ_j such that $\Pi_j^m = 0$. For $\alpha > 740$, τ_j^* remains interior as per Lemma 4 until it becomes zero for $\alpha \geq 1040$.

References

- Babiker, Mustafa H. (2005). Climate Change Policy, Market Structure, and Carbon Leakage. *Journal of International Economics* 65, 421–445.
- Buchanan, James (1969). External Diseconomies, Corrective Taxes, and Market Structure. *American Economic Review* 59, 174–177.
- Chichilnisky, Graciela and Geoffrey Heal (1994). Who Should Abate Carbon Emissions? An International Viewpoint. *Economics Letters* 44, 443–449.
- Hepburn, Cameron J., John K.-H. Quah and Robert A. Ritz (2013). Emissions Trading with Profit-Neutral Permit Allocations. *Journal of Public Economics* 98, 85–99.
- Hoel, Michael (1996). Should a Carbon Tax be Differentiated Across Sectors? *Journal of Public Economics* 59, 17–32.
- Lyubich, Eva, Joseph S. Shapiro and Reed Walker (2018). Regulating Mismeasured Pollution: Implications of Firm Heterogeneity for Environmental Policy. *American Economic Association: Papers & Proceedings* 108, 136–142.
- Requate, Till (2006). Environmental Policy Under Imperfect Competition: A Survey. In: Tom Tietenberg and Henk Folmer (Eds.), *International Yearbook of Environmental and Resource Economics 2006/2007*, Edward Elgar, Cheltenham: UK.
- Weyl, E. Glen and Michal Fabinger (2013). Pass-Through as an Economic Tool: Principles of Incidence under Imperfect Competition. *Journal of Political Economy* 121, 528–583.
- World Bank (2020). *State and Trends of Carbon Pricing 2020*. World Bank Group, Washington DC: USA.

Online Appendix

Proof of Lemma 1. Faced with a carbon price τ_k , the first-order condition for firm m in country k , which satisfies (1), is:

$$(p - z_k \tau_k) - \theta x_k^m = 0. \quad (3)$$

Summing over all $n_i + n_j$ firms shows that the industry output and product price are equal to:

$$X(\tau_i, \tau_j) = \frac{n_i(\alpha - z_i \tau_i) + n_j(\alpha - z_j \tau_j)}{(n_i + n_j + \theta)} \text{ and } p(\tau_i, \tau_j) = \frac{\theta \alpha + n_i z_i \tau_i + n_j z_j \tau_j}{(n_i + n_j + \theta)}. \quad (4)$$

The optimality conditions (3) imply $X_i = n_i(\alpha - X - z_i \tau_i)/\theta$ for i and so:

$$X_i(\tau_i, \tau_j) = \frac{n_i}{(n_i + n_j + \theta)} \left[\alpha - \frac{(n_j + \theta)}{\theta} z_i \tau_i + \frac{n_j}{\theta} z_j \tau_j \right]. \quad (5)$$

For part (a), this pins down the output responses to i 's own carbon price as well as to j 's:

$$\frac{dX_i}{d\tau_i} = -\frac{n_i(n_j + \theta)}{\theta(n_i + n_j + \theta)} z_i < 0 \text{ and } \frac{dX_i}{d\tau_j} = \frac{n_i n_j}{\theta(n_i + n_j + \theta)} z_j > 0 \quad (6)$$

So output leakage equals $L_i^O \equiv (dX_j/d\tau_i)/(-dX_i/d\tau_i) = n_j/(n_j + \theta)$. For part (b), in terms of emissions, using the definition $E_k \equiv z_k X_k$ ($k = i, j$),

$$\frac{dE_i}{d\tau_i} = -\frac{n_i(n_j + \theta)}{\theta(n_i + n_j + \theta)} z_i^2 < 0 \text{ and } \frac{dE_i}{d\tau_j} = \frac{n_i n_j}{\theta(n_i + n_j + \theta)} z_i z_j > 0. \quad (7)$$

So carbon leakage rate $L_i^C \equiv (dE_j/d\tau_i)/(-dE_i/d\tau_i) = (z_j/z_i)[n_j/(n_j + \theta)]$. Finally, for part (c), carbon cost pass-through follows directly from (4).

Proof of Lemma 2. For part (a), using (5) and $E_k \equiv z_k X_k$ ($k = i, j$), global emissions $E \equiv E_i + E_j$ are given by:

$$E(\tau_i, \tau_j) = \frac{\alpha(n_i z_i + n_j z_j)}{(n_i + n_j + \theta)} - \frac{n_i z_i [(n_j + \theta) z_i - n_j z_j]}{\theta(n_i + n_j + \theta)} \tau_i - \frac{n_j z_j [(n_i + \theta) z_j - n_i z_i]}{\theta(n_i + n_j + \theta)} \tau_j \quad (8)$$

For the special case with equal carbon prices $\tau_i = \tau_j = \tau$, this becomes:

$$\begin{aligned} E(\tau, \tau) &= \frac{\alpha(n_i z_i + n_j z_j)}{(n_i + n_j + \theta)} - \frac{[n_i z_i [(n_j + \theta) z_i - n_j z_j] + n_j z_j [(n_i + \theta) z_j - n_i z_i]]}{\theta(n_i + n_j + \theta)} \tau \\ &= \frac{\alpha(n_i z_i + n_j z_j)}{(n_i + n_j + \theta)} - \frac{[n_i n_j (z_i - z_j)^2 + \theta(n_i z_i^2 + n_j z_j^2)]}{\theta(n_i + n_j + \theta)} \tau \end{aligned} \quad (9)$$

so that $dE(\tau, \tau)/d\tau < 0$ holds for any values of firms' emissions intensities z_i, z_j . For part (b), using (5) and $E_i \equiv z_i X_i$ for country i , say, and again imposing $\tau_i = \tau_j = \tau$ shows that $dE_i(\tau, \tau)/d\tau \leq 0$ holds if and only if $n_j/(n_j + \theta) \leq z_i/z_j$. Using Lemma 1(b), this is the same condition as that for $L_i^C \leq 1$.

Proof of Lemma 3. Using the expression for global welfare $W(\tau_i, \tau_j) = \int_{v=0}^{X(\tau_i, \tau_j)} p(v)dv - sE(\tau_i, \tau_j)$ and the definition of the carbon leakage rate L_i^C , the impact of a change in country i 's carbon price satisfies:

$$\frac{dW}{d\tau_i}(\tau_i, \tau_j) = p(\tau_i, \tau_j) \frac{dX}{d\tau_i} - s \frac{dE}{d\tau_i} \quad (10)$$

$$= \frac{dX}{d\tau_i} \left[p(\tau_i, \tau_j) - s \left(\frac{dE_i}{d\tau_i} / \frac{dX}{d\tau_i} \right) (1 - L_i^C) \right]. \quad (11)$$

$$= \frac{dX}{d\tau_i} \left[p(\tau_i, \tau_j) - \frac{(n_j + \theta)}{\theta} s z_i (1 - L_i^C) \right], \quad (12)$$

where the last line uses (4) and (7) from the proof of Lemma 1. Apart from $p(\tau_i, \tau_j)$, all terms on the right-hand side are constants with respect to carbon prices, and, by Lemma 1(c), carbon cost pass-through is positive, $dp/d\tau_k > 0$ for $k = i, j$, so for any $\tau_i, \tau_j \geq 0$ this expression is bounded above according to:

$$\frac{dW}{d\tau_i}(\tau_i, \tau_j) \leq \frac{dX}{d\tau_i} \left[p(0, 0) - \frac{(n_j + \theta)}{\theta} s z_i (1 - L_i^C) \right]. \quad (13)$$

Using $p(0, 0) = \theta\alpha/(n_i + n_j + \theta)$ from (4) and rearranging shows that if $L_i^C \geq \underline{L}_i^C$ then $dW(\tau_i, \tau_j)/d\tau_i \leq 0$ and so $W(0, \tau_j) \geq W(\tau_i, \tau_j)$ for all $\tau_i, \tau_j \geq 0$.

Proof of Lemma 4. By assumption, $\tau_i = 0$ for country i and the optimal $\tau_j^* > 0$ for country j is interior so it solves the analogous expression to (12):

$$\frac{dW}{d\tau_j}(0, \tau_j^*) = \frac{dX}{d\tau_j} \left[p(0, \tau_j^*) - s z_j \left[1 + \frac{n_i}{\theta} \left(1 - \frac{z_i}{z_j} \right) \right] \right] = 0, \quad (14)$$

where $dX/d\tau_j < 0$ by Lemma 1 and using $L_j^C = (z_i/z_j)[n_i/(n_i + \theta)]$. By (4), the equilibrium product price satisfies:

$$p(0, \tau_j^*) = \frac{(\theta\alpha + n_j z_j \tau_j^*)}{(n_i + n_j + \theta)}. \quad (15)$$

Using these two expressions to solve for τ_j^* initially yields:

$$\frac{n_j z_j \tau_j^*}{(n_i + n_j + \theta)} = s z_j \left[1 + \frac{n_i}{\theta} \left(1 - \frac{z_i}{z_j} \right) \right] - \frac{\theta\alpha}{(n_i + n_j + \theta)} \quad (16)$$

and so:

$$\frac{\tau_j^*}{s} = \left[1 + \frac{(n_i + \theta)}{n_j} \right] \left[1 + \frac{n_i}{\theta} \left(1 - \frac{z_i}{z_j} \right) \right] - \frac{\theta(\alpha/s)}{n_j z_j}. \quad (17)$$

Now isolating the terms that hinge on $\theta > 0$ and $n_i > 0$, respectively, gives:

$$\frac{\tau_j^*}{s} = 1 + \frac{\theta}{n_j} \left(1 - \frac{\alpha/s}{z_j} \right) + \left[\frac{n_i}{\theta} \left(1 - \frac{z_i}{z_j} \right) + \frac{n_i}{n_j} \left[1 + \frac{n_i}{\theta} \left(1 - \frac{z_i}{z_j} \right) \right] + \frac{n_i}{n_j} \left(1 - \frac{z_i}{z_j} \right) \right] \quad (18)$$

and some further rearranging gives the expression as claimed.

Proof of Proposition 1. The proof has three steps. First, to identify conditions under which $\tau_j^* \geq s$. Second, to identify conditions under which j 's firms remain profitable under this τ_j^* . Third, to obtain a condition under which $\tau_i^* = 0$ is indeed optimal.

Step 1. Suppose that $\tau_i^* = 0$ and that optimal $\tau_j^* > 0$ for country j is interior. If so, then τ_j^* satisfies the expression in Lemma 4, and therefore $\tau_j^* \geq s$ holds if and only if:

$$\frac{n_i}{n_j} \left[1 + \left(\frac{n_i + n_j + \theta}{\theta} \right) \left(1 - \frac{z_i}{z_j} \right) \right] \geq \frac{\theta}{n_j} \left(\frac{\alpha/s - z_j}{z_j} \right). \quad (19)$$

Rearranging this expression in terms of α/s gives:

$$\alpha/s \leq \left(1 + \frac{n_i}{\theta} \right) z_j + \frac{n_i}{\theta} \left(\frac{n_i + n_j + \theta}{\theta} \right) (z_j - z_i) \equiv A. \quad (20)$$

Step 2. By the first-order condition in (3), j 's firms remain profitable (with $\Pi_j^m \geq 0$) under this τ_j^* as long as $p(0, \tau_j^*) \geq z_j \tau_j^*$. Using (4) and rearranging shows that this is equivalent to:

$$\frac{\theta(\alpha/s)}{(n_i + \theta)z_j} \geq \frac{\tau_j^*}{s}. \quad (21)$$

Now inserting the expression for τ_j^*/s from Lemma 4 gives:

$$\frac{\theta(\alpha/s)}{(n_i + \theta)z_j} \geq 1 - \frac{\theta}{n_j} \left(\frac{\alpha/s - z_j}{z_j} \right) + \frac{n_i}{n_j} \left[1 + \left(\frac{n_i + n_j + \theta}{\theta} \right) \left(1 - \frac{z_i}{z_j} \right) \right]. \quad (22)$$

Simplifying this expression in terms of α/s yields:

$$(\alpha/s) \geq \left(1 + \frac{n_i}{\theta} \right) \left[z_j + \frac{n_i}{\theta} (z_j - z_i) \right] \equiv B. \quad (23)$$

Step 3. By Lemma 3, $L_i^C \geq \underline{L}_i^C$ is a sufficient condition for $\tau_i^* = 0$ to be optimal for the social planner. This condition can instead be written as:

$$\frac{z_j}{z_i} \frac{n_j}{(n_j + \theta)} \geq \left[1 - \frac{\theta}{(n_j + \theta)} \frac{\theta(\alpha/s)}{(n_i + n_j + \theta)z_i} \right] \quad (24)$$

and so

$$(\alpha/s) \geq \frac{(n_i + n_j + \theta)}{\theta} \left[z_i - \frac{n_j}{\theta} (z_j - z_i) \right] \equiv C. \quad (25)$$

Now obtain a further condition on the relative emissions intensity under which $B \geq C$:

$$\left(1 + \frac{n_i}{\theta} \right) \left[z_j + \frac{n_i}{\theta} (z_j - z_i) \right] \geq \frac{(n_i + n_j + \theta)}{\theta} \left[z_i - \frac{n_j}{\theta} (z_j - z_i) \right] \quad (26)$$

which rearranges to:

$$\frac{z_i}{z_j} \leq 1 - \frac{\theta n_j}{[(n_i + \theta)(n_i + \theta) + n_j(n_i + n_j + \theta)]} \equiv \delta < 1. \quad (27)$$

In summary, whenever α/s exceeds B , and also exceeds C given that $z_i/z_j \leq \delta$, $\tau_i^* = 0$ is indeed optimal and j 's firms remain profitable under the τ_j^* of Lemma 4 which, as long as α/s is less than A , exceeds the SCC. It is easy to check that $A > B$ then also always holds:

$$A - B = \frac{n_i}{\theta} \frac{n_j}{\theta} (z_j - z_i) > 0. \quad (28)$$

The maximum value given by $\tau_j^* = s[1 + (n_i/\theta)(1 - z_i/z_j)]$ occurs where j 's are just indifferent about being active at $\alpha/s = B$.