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The target-consistent carbon price for an electricity sector decarbonizing through massive variable renewable electricity (VRE) depends sensitively on the VRE penetration level, as the marginal curtailment of VRE rises rapidly beyond a certain level. This paper develops a simple linear model to illustrate the relation between the shadow carbon price (SCP) and VRE penetration and calibrates it for the island of Ireland's 2026 target VRE penetration of 55%. The SCP rises rapidly with increased VRE investment beyond a certain point, and can be used to direct mitigating investment in storage, interconnectors, and other flexibility options. The SCP for the final efficient portfolio will be the target-consistent carbon price for electricity that can help judge the appropriateness of the original target level of VRE penetration.

Keywords: social cost of carbon, variable renewable electricity, marginal curtailment

JEL Classification H23; L94; Q28; Q42; Q48

1 Introduction

Economists have a marked preference for taxes and subsidies as instruments to correct externalities like greenhouse gas emissions or learning-by-doing spill-overs. Policy makers are clearly more attracted to quotas and targets despite their risk of inefficient allocations (Weitzman, 1974). Sometimes these inefficiencies can be mitigated by the compromise solution of a tradable permit, as in the EU Emissions Trading System (ETS), where the apparently superior alternative of a regionally agreed uniform carbon tax runs into the problem that taxes are devolved decisions

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jealously defended by Member States. Without trading, carbon taxes are more likely to be introduced once targets have been agreed and accepted (Dolphin et al., 2019), particularly in sectors like electricity where they are demonstrably effective in delivering low-cost carbon savings through fuel-switching (Chyong et al., 2020; Newbery et al., 2018). In some sectors that are otherwise not taxed like electricity carbon taxes can be readily monitored, but most energy-using sectors are likely to be subject to a variety of taxes (road transport fuel is a good example, see Newbery, 2005).[1] As such an additional carbon tax will be hard to externally verify, whereas targets can be easily monitored.

Targets have clear political attractions in encouraging agreement between nation states. The 2015 COP 21 Paris Agreement[2] on climate change set ambitious targets for 2050 that have been translated into "nationally determined contributions" by each of the 196 Parties.[3] Thus the EU requires Member States to publish *National Energy and Climate Plans* (Newbery, 2021). In the UK the *Climate Change Act 2008* requires the Government to set periodic carbon budgets for 5-year periods.

Perhaps the most valuable advantage of target setting is that it addresses the club goods problem of financing public goods (Buchanan, 1965). If the club, such as the EU, agrees a set of targets for the share of renewable energy to be met by each country by a target date (e.g. as in the EU's *20-20-20 Directive*, EC, 2009) then each county will jointly finance the club good, in this case ensuring a level of investment that is justified by the learning-by-doing spillovers that are (partly) captured by the club. Reducing carbon emissions is a global public good for which national targets make sense.

The other underappreciated advantage of target setting is its greater credibility and durability, particularly if associated with monitoring. Thus the UK Committee on Climate Change annually assesses progress to meeting carbon targets to which it holds the Government to account (see e.g. CCC, 2023). While taxes and charges are often changed annually at Budget time, for a target to have any meaning it needs to have some durability, only changed in response to a clearer estimate of the underlying purpose of the target (hence the value of independent monitoring). Measures to reduce emissions normally require investment decisions, whose viability depends on future carbon prices, which if subject to political pressure will likely lack credibility. Uncertainty about future carbon prices increases investment risk (and its cost) or encourages delays. The

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[1] The total fuel tax can be justified as a road user charge, the external costs of air and water pollution (including CO_2), the exercise of international market power against OPEC, and as an administratively simple revenue-raising tax that is largely unseen at the point of purchase. Disentangling the size of the carbon tax within this mix would be challenging.

[2] https://unfccc.int/process-and-meetings/the-paris-agreement

[3] https://unfccc.int/most-requested/key-aspects-of-the-paris-agreement

history of the EU ETS demonstrates high volatility and periodic collapse of the EU Allowance (EUA) price for CO_2. In the 2011 Budget, the UK Government introduced a Carbon Price Floor (CPF) for the GB electricity sector, starting in 2013 and to escalate in real terms until it reached £$_{2011}$70/tonne by 2030. This was an attempt to create greater forward certainty of the carbon price in reaction to the EU ETS volatility and to underwrite the commercial viability of planned nuclear power stations. The CPF was implemented as a Carbon Price Support (CPS), an annually adjustable carbon tax addition to the EUA to bring the total carbon price paid by fossil fuel for electricity generation up to the CPF trajectory. It had a dramatic effect in driving coal off the system and improving the commercial viability of renewable electricity (Chyong et al., 2020).

Its credibility was almost immediately undermined in the 2014 Budget: "the Government announced that the CPS component of the floor price would be capped at a maximum of £18/tCO$_2$ from 2016 to 2020 to limit the competitive disadvantage faced by business and reduce energy bills for consumers. This price freeze was extended to 2021 in Budget 2016."[4] Later the EUA price reached high levels, amplified by the Ukrainian gas crisis that drove up the demand for coal and hence the net demand for EUAs by the power sector. The CPS remained frozen, further increasing the GB price of electricity. Although the Government subsidized domestic consumers, industry suffered this additional penalty. The CPS became a useful stealth tax for the Treasury in a period of high fiscal deficits. Carbon taxes thus lack credibility, clarity, and are more subject to transitory political whim – unhelpful for investors.

Finally, targets stimulate the necessary coordination of the supply chains needed to deliver the investments required to meet the targets. This has been dramatically demonstrated in the GB Offshore Wind Industry. The first allocation round announced in 2015 bureaucratically set the strike price for offshore wind at £$_{2012}$119.89/MWh for delivery in 2017/18. After that prices were set in periodic auctions. The 2017 auction cleared at £$_{2012}$74.75/MWh for delivery starting in 2021/2 and £$_{2012}$57.5/MWh starting in 2022/23, less than half the set price of the first round. The 2019 auction cleared at £$_{2012}$39.65/MWh for delivery in 2023/4.[5] The 2022 round cleared at £$_{2012}$37.35/MWh for delivery starting in 2026/7. The dramatic fall in the support price was partly due to a fall in the cost of finance (with growing experience of the new contracts for difference) and partly due to the development of a supply chain able to deliver the required platforms and reliable turbines. The increasingly credible government commitment to developing offshore wind as a major part of the UK's decarbonization strategy, combined with a planned sequence of offshore sites identified and auctioned off by the Crown Estate combined with a

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[4] https://commonslibrary.parliament.uk/research-briefings/sn05927/

[5] See https://www.gov.uk/government/publications/contracts-for-difference-cfd-allocation-round-3-results

well-designed route for financing the offshore link to the onshore network, made it attractive to develop the capability (in blade, turbine and rig production) to ramp up delivery.

Policy-makers nevertheless need a shadow price of carbon (SPC) for cost-benefit analysis if they are to make efficient choices within and across different sectors of the economy (Santos, 2022; Stiglitz, 2019; Hepburn et al., 2020). Ideally this would be the social cost of carbon (SCC). According to Stern et al. (2023) the US first legally required an SCC in policy making in 2008[6] while the latest estimated SCC (of \$50/ton CO_2) was published by IAWG (2016). However, as Stern et al., (2023) carefully document, the methodology used to calculate the SCC as the marginal social damage caused by the release of 1 tonne of CO_2 is highly sensitive to a large number of imperfectly known parameters and assumptions.

Instead they argue, and this paper agrees, that it is more useful to estimate a target-consistent carbon price, where the target, at least at a high level, derives from the Paris Agreement devolved down to nation states, and within each, to sectoral levels. The UK Government now calculates a target-consistent shadow carbon price for sectors not subject to the Emissions Trading System carbon price (BEIS, 2019). This paper shows that with the target-driven high levels of renewable electricity planned, the standard approach to calculating the target-consistent carbon price no longer works and needs to be replaced by the method described below.

2 Carbon pricing in the electricity sector

The route to rapid decarbonization invariably starts with decarbonizing electricity, first, because the technology exists and is cost-competitive while requiring no change in the final product (and hence no need for behavioural change), and second, because electrification is the key to decarbonizing sectors such as transport, heating and industrial processes. Clearly that only makes sense if the electricity itself has been decarbonized. While there is a powerful case for target setting, it remains important to estimate the shadow price of carbon (SPC) for each target to aid external target monitoring. Significantly different SPCs imply that the choice of targets will not be least-cost and will need adjustment.

Given the key role of electricity, this paper sets out a method for calculating its target-consistent carbon price, illustrated for the island of Ireland (which has a challenging renewables target). Most countries decarbonizing electricity require a massive increase in Variable Renewable Electricity (VRE, e.g. wind and solar PV). The problem is that the ratio of peak to average on-shore wind can be 3-4:1, and even in the most favourable sites and offshore, at least 2:1. In northern European counties solar PV has ratios of 8+:1. The share of VRE in total annual

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[6]Ctr. for Biological Diversity v. Nat'l Highway Traffic Safety Admin., 508 F.3d 508 (9th Cir. 2007).

generation will be limited by its average capacity factor (i.e. the fraction of full operating hours per year, 25%-33% for most on-shore wind, 10%-13% for northern PV). At high VRE penetration (e.g. above 50%) peak output will exceed total demand (including exports and storage), and the resulting surplus must be curtailed (i.e. spilled or wasted). When it comes to calculating the target-consistent carbon price for such systems, this paper argues that a quite different approach is needed from the normal approach based on the carbon intensity of different generating technologies.

The key insight is that marginal curtailment is typically more than three times average curtailment, so the contribution of each additional MW of VRE will displace less and less carbon from the fossil generation needed to maintain system reliability and capacity adequacy. The standard approach to choosing the least-cost technology mix assumes that each technology is controllable and so the main question is balancing capital and operating (mainly fuel and carbon) costs to meet variable demand. High-capital low-operating cost plant should run on base load (i.e. almost all the time) while the peak demands are best met with low-capital high-operating cost plant.[7] Given any set of fuel and carbon prices, a short-run merit order can be calculated and the SPC of meeting any level of demand can be calculated from the emissions intensity of the marginal plant, while in long-run equilibrium screening curve analysis (Stoft, 2002) determines the optimal plant mix. For any level of demand (or average over the year) the SPC can be determined from the merit order (e.g. as set out in Newbery, 2018). This standard approach breaks down once VRE reaches the point at which it is necessary or economic to curtail excess generation, and a different approach is needed to compute both the efficient plant mix and, the subject of this paper, the target price of carbon.

2.1 Marginal Abatement Cost and target-consistent carbon prices

Marginal abatement cost (MAC) schedules rank the marginal abatement cost of reducing CO_2 (or greenhouse gas emissions) for each technology, with the x-axis measuring the cumulative tonnes abated. As such, once the target amount of abatement is set, the target-consistent marginal cost can be read off the graph, and will, if all the earlier choices are implemented, be the least cost way of reaching the target. The MAC will then be the target-consistent carbon price (RFF, 2021). The MAC schedule is typically constructed from the bottom up, drawing on detailed cost estimates for each abatement option. The problem with this approach is that it requires detailed technical and cost information for all the options available, including those yet to be discovered and deployed in the market. High carbon prices or taxes may stimulate the discovery of new

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[7]This can be extended to take account of future fuel and carbon prices uncertainty by using mean-variance portfolio analysis (Roques et al., 2006, 2008).

solutions, while high transactions costs, inertia and/or ignorance may cause some apparently attractive solutions not to be adopted – reality is likely to diverge from the MAC schedule. The UK Government uses MAC methodology to set the SPC for sectors not subject to the carbon prices set by the Emissions Trading System (BEIS, 2019).

The alternative approach, illustrated by the net-zero CO_2 emissions target approach of Kaufman et al. (2020), starts from energy-emissions models and a choice of pathway to the target (in their case, net zero) to compute the near-term path of carbon prices. Their defence is that the best models are used to plan the pathways so using the same models should give a policy-consistent carbon price. By concentrating on the near term their method avoids the extreme sensitivity of the SCC to distant and highly uncertain damages and the discount rate.

That is not to deny that the MAC and associated target-consistent carbon price are also sensitive to the discount rate and technology costs (which should include the rather uncertain external learning benefits), but the sensitivity is typically an order of magnitude less. Dietz and Fankhauser (2010) observe that at interest rate of 4% "the range of the present MAC (Marginal Abatement Cost) is ~\$0 to ~\$68/tCO$_2$, which is a factor of 10 narrower than the uncertainty around the SCC." Newbery (2018) provides further criticisms of the MAC approach, pointing out in particular its sensitivity to fuel costs, and proposes the closely related concept of a break-even carbon price. That is "the carbon price at which a zero or low-carbon technology (e.g. an on-shore wind turbine) has the same cost (including this carbon cost) as the most competitive carbon-intensive alternative (e.g. a combined cycle gas turbine, CCGT). ... It also lends itself to exploring the sensitivity of this carbon price to key uncertain parameters, such as the fossil fuel price, the discount rate and capital costs." The target-consistent carbon price is essentially the same concept but applied to an electricity system as a whole but, crucially, now allowing for curtailment.

2.2 The relation between VRE penetration and the shadow price of carbon

The argument of this paper is that the shadow price of carbon, SPC, in electricity can be derived from the marginal curtailment schedule. For practical applications this can be estimated or simulated from detailed dispatch models (e.g. as illustrated in Newbery, 2021), but such models are opaque and the determinants of the SPC not easy to understand. The starting point is residual demand, that is, domestic demand *less* VRE output.[8] When this becomes negative, the surplus VRE can either be stored (if there is spare storage capacity), exported (if neighbouring countries are able to accept increased imports and only up to the export capacity

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[8]and nuclear output, where adjusting output is problematic or very costly. Demand side response can considerably mitigate curtailment and needs to be included in residual demand.

of the interconnectors), or will have to be curtailed (wasted). Curtailment in each hour will vary depending on the state of these variables (storage, exports), which, in the case of storage, will also require a forward look to determine the optimal time and rate at which storage should be increased or run down. In addition to these hourly varying state variables, the System Operator will need to maintain adequate fast-acting flexible capacity to maintain system stability and reliability, imposing additional constraints on the volume of VRE that can be accepted, discussed below.

The higher is VRE penetration, the higher will be the average curtailment factor, while the marginal curtailment of adding more VRE capacity can be 3+ times this average factor. The carbon displaced per MW of extra VRE will be the carbon-intensity of the displaced fossil generation, e tonnes CO_2/MWh, *times* the capacity factor of this last MW of VRE, allowing for its *marginal* curtailment. Given the costs of adding VRE and the cost saving from displacing fossil generation, the cost per tonne of CO_2 abated then gives the SPC for each level of VRE.

This paper develops a simple model to gain greater insights into the relationship between VRE penetration and the shadow price of carbon, which in turn gives the target-consistent carbon price for electricity. It addresses the question whether 100% decarbonization of electricity is a sensible mid-term goal (in the UK for 2035, BEIS, 2022), or one that should be delayed closer to 2050, as other sectors increase their degree of decarbonization. The next section sets out this simple model while the following sections illustrate the magnitude of the SPC and suggest ways to extend the results to more complex systems with other mitigation options.

3 The model

The simplest model has flexible dispatchable fossil generation (assumed to be efficient gas turbines) with output $G(t)$ at time t and VRE generation (in the example, wind) with capacity W MW and potential output $\phi(t)W$. The annual average value of $\phi(t) = \phi$, the capacity factor. Demand is $D(t)$, and fossil generation is required to meet residual demand, $R(t) = D(t) - w(t)$, where $w(t)$ is the amount of wind that can be accepted. The carbon intensity of fossil generation depends on the carbon intensity of fuel (gas), γ tonnes CO_2/MWh$_{th}$, and the efficiency with which the fuel is converted into electricity, e, MWh/MWh$_{th}$, where subscript $_{th}$ indicates the thermal content of the fuel (unsubscripted MWh is used only for electricity). The fuel cost is f €/MWh$_{th}$ so the variable fuel cost is $v_F = f/e$ €/MWh. For the moment assume there is adequate dispatchable capacity to provide system reliability (in other words, as VRE increases, some fossil plant is retired but enough retained for security of supply), although it will require an annual payment, r_F, to cover the fixed cost of remaining available (readily measured in GB by the annual capacity payment determined in the capacity auction, €/MW/yr). The annual

additional capital cost of wind is r_W €/MW/yr – the full cost less its capacity credit, δr_F. Similarly its relevant variable cost is taken as zero for convenience as it is the excess of fossil over VRE variable cost that matters.

The system operator has to maintain balance between supply and demand at every moment. If supply were to suddenly fall (or demand increase) demand will extract more energy from the inertia in the spinning mass of the turbines but at the expense of a fall in frequency. The system must be designed and managed so that the rate of change of frequency (RoCoF) is no greater than a specified amount (in the Single Electricity Market, SEM, of the island of Ireland, 1 herz/second; Eirgrid/Soni, 2020) and the volume of synchronized spinning mass is sufficient to maintain frequency within specified limits (in the SEM the normal range is between 49.9 and 50.1 herz, or cycles per second, with load shedding at 48.85 herz; Eirgrid/Soni, 2011). VRE such as wind is connected to the system through inverters (that convert DC to AC) and as a result VRE has no effective inertia and is non-synchronous. The share of synchronous generation must be kept above a specified fraction of demand to ensure frequency stability.

This limit is normally described as the maximum level of System Non-Synchronous Penetration (of VRE), SNSP, and will depend on the plant mix and operating requirements of RoCoF and the acceptable frequency range. Thus for the SEM, the target SNSP for 2020 was 75% (achieved by 2022), with more ambitious targets later in the decade. Britain, on the other hand, has substantial base-load zero carbon nuclear power with enough inertia to handle current levels of VRE and still meet acceptable RoCoF standards. In addition, the SO has to keep enough flexible capacity ready to dispatch at a moment's notice against the risk of plant or line failure, normally equal in amount to the largest single infeed (from a generator or over a transmission link) – the N-1 constraint. Where the SNSP limit binds (i.e. in the SEM with no zero-carbon synchronized generation), and provided generation units are small enough relative to demand, this constraint is less important than the SNSP limit, unless that is very high. In countries with a significant share of nuclear power like GB, SNSP may not bind but the N-1 constraint can require significant balancing reserves, as the largest nuclear plant that might trip is large (1,600 MW when Hinckley Point C is commissioned later this decade). In the worst case, balancing reserves require fossil generation synchronized and running at minimum load, m. In the best case balancing reserves can be met by storage, and hence zero-carbon.[9]

The level of SNSP is therefore normally the critical determinant of the amount of curtailment. The required share of synchronous generation is at least $\beta = 1-$ SNSP (so $\beta = 25\%$ in the

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[9]In GB, regulating and fast reserve must be available within 2 minutes and be sustainable for at least 15 minutes, while increasing output in excess of 25MW/min. Batteries and pumped storage, of which there are 4,000 MW in 2022, satisfy these requirements. See https://www.nationalgrideso.com/industry-information/balancing-services/reserve-services/balancing-reserve

SEM), and the constraint is $G(t) \geq Max(\beta D(t), m)$. If the N-1 requirement is non-binding, the curtailment function is

$$k(\phi(t)W, t) = Max(0, \phi(t)W - (1 - \beta)D(t)). \tag{1}$$

Thus while potential wind output is $\phi(t)W$, actual or useful wind output is $w(t) = \phi(t)W - k(\phi(t)W, t)$. Residual demand is then $R(t) = D(t) - w(t)$ and the curtailment function $k(\phi(y)W, y) \equiv k(y, W)$ is defined for the y hours of curtailment ranked in descending order with $k' < 0$ over the range $[0, y^*(W))$, where y^* is the solution to

$$k(y^*, W) = 0. \tag{2}$$

As an example, Figure 1 graphs the curtailment function for the SEM using demand and actual wind data for 2018, but scaling wind up every hour by a factor of nearly three to achieve a 55% penetration in 2026, for three cases. The base case for 2026 has SNSP of 75%, but there are ambitions to raise this limit, possibly to 85% by then. The Celtic interconnector of 700MW (adding to additional export capacity of 900 MW) may be commissioned as early as 2026, and clearly has a considerable impact by allowing additional exports before the need to curtail excess wind. Finally, trebling planned battery electrical storage (BES) has a further impact in reducing the volume of curtailment. In each case the curtailment functions are ranked and graphed as $k(y, W)$ on $[0, y^*(W))$ with $k' < 0$.

The normal way to measure curtailment is the volume of wind curtailed, $\int_0^{y^*} k(W, y)dy$, which in general will be higher than $y^* W \phi$ as curtailment hours are likely to be hours of above average capacity factor, ϕ. Marginal curtailment caused by the entry of 1 MW of extra wind capacity is

$$\frac{d}{dW} \int_0^{y^*} k(W, y)dy = k(W, y^*)\frac{dy^*}{dW} + \int_0^{y^*} \frac{dk(W, y)}{dW}dy,$$
$$= \int_0^{y^*} \frac{dk}{dW}dy = \int_0^{y^*} \left(\frac{\partial k}{\partial W} + \frac{\partial k}{\partial y^*}\frac{\partial y^*}{\partial W}\right)dy. \tag{3}$$

The ratio of the marginal to average curtailment is $W \int_0^{y^*} \frac{dk}{dW}dy / \int_0^{y^*} kdy$. In figure 1, the curtailment function is roughly linear in y over much of its range and can be approximated by

$$k = \alpha(W - W_0)(1 - y/y^*), \tag{4}$$

with W_0 the level of wind at which curtailment first appears. In this SEM calibrated case for 75% SNSP and no Celtic interconnector, $W_0 = 4,534$ MW, $W = 10,234$ MW),[10] $\alpha = 0.47$,

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[10] The peak capacity factor for the whole of GB is 93.4% (as wind farms in different locations are not perfectly correlated) so the peak wind output is divided by 0.934 to derive the implied capacity. The increase in 1,000 MW gives an increase in peak wind output of 934 MW.

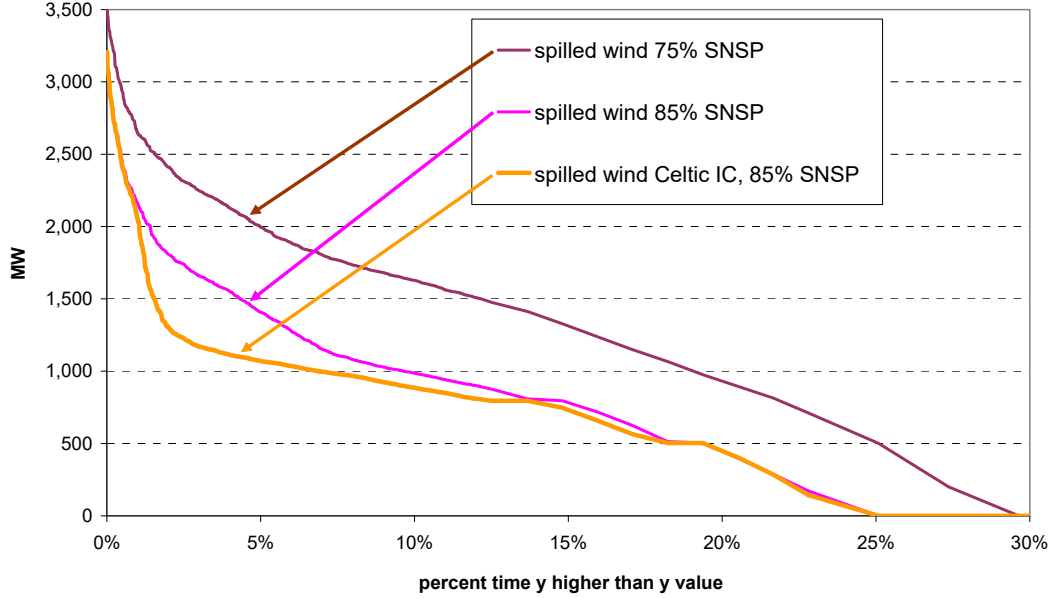**Illustrative curtailment curves for SEM 2026**

Figure 1: Curtailment curves for the island of Ireland projected for 2026

and $y^* = 29.5\%$ (see Table 1). The Appendix shows that the ratio of the marginal to average curtailment from (A3) is just $2W/(W - W_0)$ or 3.6, considerably greater than 2. In the linear case Figure 2 show how this can be demonstrated geometrically, as the curtailment function defines a triangle whose area is total curtailment, $\frac{1}{2}\alpha(W - W_0).y^*$, divided by W to give average curtailment.

Marginal curtailment is found by displacing the curtailment function upwards by $\alpha\Delta W$, and the marginal curtailment is then the area of the trapezium, the parallelogram $\alpha\Delta W.y^*$ plus the triangle $\frac{1}{2}\alpha\Delta W.\Delta y^*$. The ratio of the marginal to the average curtailment is then directly $2W/(W - W_0)$, replacing ΔW by 1, and ignoring the (in the limit) vanishingly small term in $\Delta y^*/y^*$. As W/W_0 increases, the ratio will tend to 2. In more realistic cases in which the curtailment function is concave but the marginal curtailment is an almost linear displacement, the ratio will be higher as the area under the curtailment function will be smaller than the triangle defined by its extreme points. This is illustrated in in Figure 1 where curtailment takes account of storage and exports and the marginal curtailment is found by incrementing wind capacity by a modest amount (100 MW). As the area is below the relevant triangle the ratio is higher at 3.66.
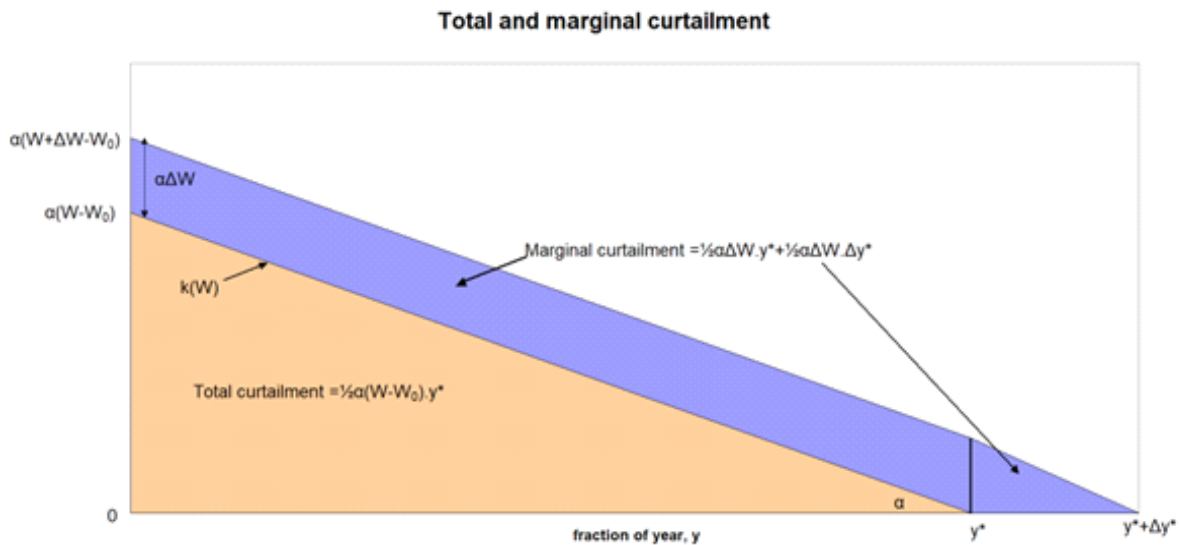
Total and marginal curtailment

$\alpha(W+\Delta W - W_0)$

$\alpha\Delta W$

$\alpha(W-W_0)$

$k(W)$

Marginal curtailment $=\frac{1}{2}\alpha\Delta W.y^* + \frac{1}{2}\alpha\Delta W.\Delta y^*$

Total curtailment $=\frac{1}{2}\alpha(W-W_0).y^*$

α

0

fraction of year, y

y^*

$y^* + \Delta y^*$

Figure 2: Geometrical illustration of total and marginal curtailments

3.1 Curtailment with wind and solar PV

Paul Simshauser has kindly provided data from Queensland, Australia that demonstrates the potentially high ratio of the marginal to average curtailment factors, not just for wind but also for solar PV. Simshauser (2023) simulates the optimal combination of wind and solar PV for a Renewable Energy Zone (REZ) which is connected to the main grid by a line of limited capacity. As the thermal limit on the line increases with wind speed, when wind is generating the line can carry more than its nominal rating. Using this dynamic rating greatly improves the economics of the REZ, as developers have to pay to connect to the main grid. Table 1 gives the capacity and output details for the optimal configuration, showing that 4,284 MW of VRE can connect within the REZ. With suitable Frequency Containment Ancillary Services to maintain stability and address N-1 contingencies, it is possible to connect to the grid by a double circuit with nominal capacity of just 1,536 MW. As PV peaks in the daytime when wind falls, the two are complementary, allowing the REZ to meet its desired level of curtailment of no more than 1% averaged over five years. However, increasing capacity of either technology by 10 MW gives rise to a ratio of marginal to average curtailment of 4.47 for wind and 2.61 for solar PV. These ratios also increase rapidly with additional increments of capacity, holding the line capacity constant.

Table 1 Queensland REZ: Five-year average simulated results

Technology	Capacity	Potential	Curtailed	Actual CF	curtailment		Ratio
	MW	GWh	GWh	%	Avg %	Marg %	
Wind	2,853	8,673	250	33.6%	1.00%		
Wind	2,863	8,673	254	33.6%	1.01%	4.5%	4.47
PV	1,431	3,315	122	25.5%	0.98%		
PV	1,441	3,338	124	25.5%	0.99%	2.6%	2.61

Simshauser's example shows that curtailment of VRE is inevitable in an optimized secure system beyond a certain penetration level. As wind and solar have different output profiles, their marginal curtailments are relevant for determining their annual output and hence profitability, and it is these factors that should guide the fraction of VRE capacity added by wind and PV.

Consider an REZ, which has been optimized to just achieve zero curtailment with W_0 MW of wind and P_0 MW of solar PV, and suppose that the curtailment function is $k(W, P, y)$ with $k(W_0, P_0, y) = 0$. Let the profit functions be Π_j, assume zero avoidable operating costs and annual fixed costs (capitla charge and fixed O&M) r_j, $j = W, P$. Price in hour t is $p(t)$ and capacity factors are $\phi_j(t)$. There is no curtailment for hours $[0, 1 - y^*)$. Annual (unchanging) profits are

$$\Pi_j = K_j \left(\int_0^{1-y^*} \phi_j(t)p(t)dt + \int_0^{y^*} (\phi_j(y) - k(W, P, y))p(y)dy - r_j \right), \ K_W = W, \ K_P = P.$$

At subject to the constraint that there is zero curtailment (so $y^* = 0$), the optimal plant mix is when $d\Pi_W/dW = d\Pi_P/dP$, or when

$$\int_0^1 (\phi_W(t) - \phi_P(t))p(t)dt = r_W - r_P. \tag{5}$$

Clearly much depends on the time pattern of the two VRE outputs and their correlation with prices. For an isolated REZ with little impact on prices the shares of wind and PV are independent of the total size of the REZ, but this ceases to be true once curtailment starts. Again the marginal net profits should be equilibrated for any given y^*, so

$$\begin{aligned}\frac{d\Pi_W}{dW} - \frac{d\Pi_P}{dP} &= 0 = \int_0^{1-y^*} (\phi_W(t) - \phi_P(t))p(t)dt + r_P - r_W \\ &+ \int_0^{y^*} (\frac{dk}{dP} - \frac{dk}{dW})p(y)dy.\end{aligned}$$

For small amounts of curtailment the first term with be close to zero, so the expansion rule is to equate the price-weighted marginal curtailment factors of each technology. Thus in a PV-dominated system dk/dP will be more strongly negatively correlated with price than dk/dW, suggesting that the balance of PV to wind should deliver lower marginal curtailment factors for PV than wind.

4 The shadow price of carbon

In this simple model residual demand is assumed to be met by a sufficient number of small combined cycle gas turbines (to handle the N-1 constraint), whose annual output G is $\int_0^1 R(h)dh = \int_0^1 (D(h) - w(h))dh$, where the upper limit is one year, and so hours are 1/8,760 of a year. If $\int_0^1 D(h)dh = Y$ MWh/yr, then

$$G = Y - \phi W + \int_0^{y*} k dy. \tag{6}$$

Annual CO_2 emissions are $E = \gamma G/e$ tonnes CO_2 , where γ is the emissions intensity of the fuel (gas) in tonnes CO_2/MWh_{th} and e is the efficiency of the gas turbine (%). The average emission intensity of electricity is

$$E/Y = \frac{\gamma}{e}\left(1 - \frac{\phi W - \int_0^{y*} k dy}{Y}\right),$$

where the term in bracket is the share of fossil generation in total production, or 1 - VRE penetration.

The impact of extra wind capacity on gas consumption from (6) is

$$\frac{dG}{dW} = -\left(\phi - \int_0^{y*} \frac{dk}{dW}dy\right) < 0,$$

negative as more wind reduces gas consumption. The marginal impact of an extra 1 MW of wind capacity on emissions, E, is then $dE/dW = (\gamma/e).dG/dW$ (tonnes $CO_2/MW/yr$), negative as a reduction.[11]

$$\frac{dE}{dW} = -\frac{\gamma}{e}\left(\phi - \int_0^{y*} \frac{dk}{dW}dy\right).$$

The annual (net) cost of this extra wind is r_W €/MW/yr and the extra cost of the gas generation is $(f/e).dG/dW < 0$, so the net cost of displacing emissions is $r_W + (f/e).dG/dW$. The shadow price of carbon, SPC, is therefore

$$\text{SPC} = \frac{r_W + (f/e).dG/dW}{-(\gamma/e).dG/dW} = \frac{r_W}{\frac{\gamma}{e}\left(\phi - \int_0^{y*} \frac{dk}{dW}dy\right)} - \frac{f}{\gamma}. \tag{7}$$

For the linear case Appendix equation (13) gives $\int_0^{y*} \frac{dk}{dW}dy = \alpha y^*$ and the SPC is

$$\text{SPC} = \frac{e}{\gamma}\left(\frac{r_W}{\phi - \alpha y^*} - \frac{f}{e}\right), \tag{8}$$

$$= \frac{e}{\gamma}\left(\frac{r_W}{\phi - \alpha y_r^*(\frac{(W/W_r - W_0/W_r)}{1 - W_0/W_r})} - \frac{f}{e}\right), \tag{9}$$

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[11] Care must be taken with units. γ/e is $t.CO_2/MWh$, G is MWh/yr and W is MW, so the product is $t.CO_2.MW^{-1}year^{-1}$.

where αy^* has been rewritten in terms of the reference level of wind capacity, W_r, the zero-curtailment wind capacity, W_0, and the scaling factor, W/W_r using Appendix equation (15) . The first term in (8) rises rapidly as αy^* increases towards ϕ as figure 3 demonstrates. As in Newbery (2018), $\partial\tau/\partial f = -1/\gamma$, which for gas is 5, so a €1/MWh$_{th}$ increase in the price of gas lowers the SPC by €5/tonne. Equation (8) shows that if the cost of VRE, r_W, falls to $f(\phi - \alpha y^*)/e$ then there is no carbon cost in replacing fossil fuel by VRE, but as (9) shows, as W rises, so the breakeven VRE cost, r_W, falls perhaps to the point where it is no longer costless to replace fossil fuel. It is simple and intuitive (if tedious) to show that $d\text{SCP}/dW < 0$, as is $d\text{SCP}/df$ and $d\text{SCP}/d\phi$.

4.1 Numerical estimates

Newbery (2020) provides two carefully calibrated cases[12] of the Single Electricity Market (SEM) of the island of Ireland projecting forward to 2026, using the hourly demand profile from 2018 but scaling wind (which then had a capacity factor $\phi = 28.4\%$) to a full wind capacity $W_r = 10,234$ MW[13] to achieve a target level of 55% penetration (ignoring curtailment).[14] In the base case SNSP is set at 75%, the Celtic Interconnector (700 MW) is not commissioned, and battery electrical storage (BES) is just the expected 334 MWhrs. In the ambitious case, SNSP is 85%, the Celtic Interconnector is commissioned and BES has been trebled to 1,000 MWhr. The relevant parameter estimates are given in Table 2.

Table 2 Parameters for base and ambitious scenarios

Parameters	Base case	Ambitious case
y^* curtailed hours	29.5%	21.7%
$\int k\,dh$, spilled wind	3,388 GWh	1,674 GWh
$A = \alpha(W_r - W_0)$, intercept	2,678 MW	1,762 MW
$\alpha = \partial k/\partial W$, slope	0.47	0.37
W_0 Zero curtailment level	4,534 MW	5,472 MW
W_0/W_r	0.443	0.535
$\phi - \alpha y^*$	14.53%	20.4%

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[12] These do not assume a linear curtailment function and compute $\partial k^*/\partial W$ by simulation.

[13] The highest wind output in the scaled year is 8,673 MW, or only 85% of actual capacity, because of the imperfect correlation of wind output across the island.

[14] Newbery (2020) also provides a data annex with the hourly wind and demand data, togethe with a spread sheet that simulates storage and export decisions.
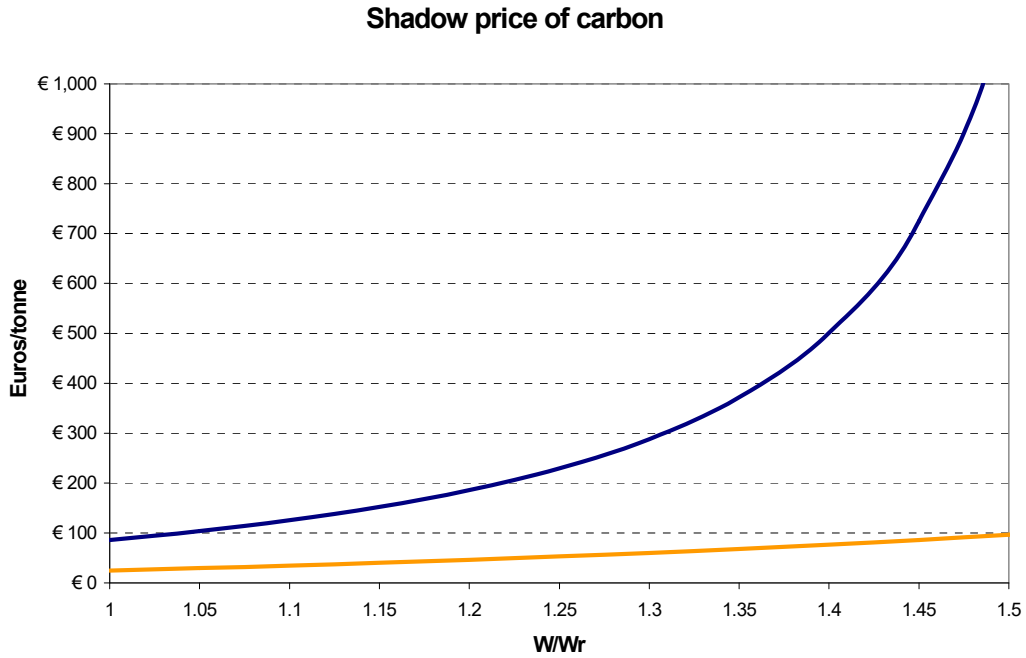
Shadow price of carbon

Figure 3: Shadow price of carbon for base and ambitious SEM 2026 cases

From the same source, $r_W = €120,132/\text{MW/yr} = €13.7/\text{MWh}$.[15] The gas price f was taken as €21.4/MWh (FES, 2019, before the pandemic and the Russian gas crisis) while $\gamma = 0.2$ and the implied average efficiency in gas generation, e, is 41% (implied by the emissions intensity of gas generation, γ/e, of 0.49 tonnes CO_{2e}/MWh).[16] In order to derive an SPC in €/tonne, prices including r_W need to be in €/MWh. A graph of the SPC against the ratio of wind capacity, W, to the reference wind level at which the model is calibrated, W_r, is shown in figure 3 for both the base case and the ambitious case. In the base case the initial value of the SPC is €86/tonne CO_2, rising rapidly (14-fold by $W/W_r = 1.5$, while in the ambitious case the initial value is only €25/tonne CO_2, increasing less than four-fold over the same range. The lesson to draw from these numerical illustrations is that the shadow price of carbon is highly sensitive to marginal curtailment, which in turn is highly sensitive to the options for using curtailed VRE (storage or exports). The discussion above also showed that the SPC is highly sensitive to the price of the displaced fossil fuel and the cost of VRE.

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[15] The capacity credit could reduce this by €28/kWyr (the first clearling price in the GB capacity auction, which subsequently fell considerable, of €3.5/MWh, but δ could be below 10% at high VRE penetration, reducing this to €0.35/MWh, small compared to the learning benefit discussed below.

[16] from IPCC Annex 3 at https://www.ipcc.ch/site/assets/uploads/2018/02/ipcc_wg3_ar5_annex-iii.pdf#page=7

4.2 Interpreting the Shadow Price of Carbon

The SPC will be lower the smaller is y^*, which figure 1 shows will be lowered by more storage and increased interconnection. It will also be lower the lower is r_W, and thus depends on the extent to which learning externalities are included or not. As these can be material $(10-20\%)$,[17] this will depend on whether the SPC is to be interpreted as a global measure (internalizing such spill-overs) or purely national (in which case the spill-over will be far less, confined to the stimulus given to developing a local supply chain). If additional flexible fossil capacity is needed, then its cost will add to r_W but the (rather small) capacity credit of VRE will reduce the cost of the fossil plant displaced. Adding more VRE in a curtailed system leads to a marginal curtailment that is a multiple of average curtailment. Thus in the SEM base case of Table 1 although the initial curtailment is 13%, the marginal curtailment is 48% or 3.5 times as large. Thus an extra MW of VRE will effectively contribute only 52% of its nominal capacity factor, which in effect doubles the cost of delivering electricity.

The SPC and the underlying marginal curtailment function can also be used as a screening device for other uses of surplus wind such as hydrogen production, in two complementary ways. Thus spilled wind has zero opportunity cost while if converted into hydrogen and stored it can displace future CO_2 as well addressing periods of low VRE and high scarcity prices. On the upside, an electrolyser built to make use of its otherwise spilled output will have a capacity factor of $28.4\% \times 48\% = 13.6\%$ instead of $28.4\% \times 13\% = 3.8\%$, the apparent capacity factor based on average curtailment. The energy efficiency of electrolysis is about $75-80\%$[18] and if converted back into electricity in a hydrogen-fired turbine at 50% efficiency and allowing a round trip storage efficiency of 85% (Elberry et al., 2021) I MWh of spilled wind could deliver 0.27 MWh electricity at some future date. If this displaced 0.49 tonnes CO_{2e}/MWh the electrolysis route would avoid 0.133 tonnes CO_{2e}/MWh of spilled wind, or 159 tonnes CO_{2e}/MWyr of installed extra wind capacity.

Equation (8) also applies to export-constrained zones within a country (e.g. Scotland within the GB electricity market), where a high SPC gives a signal that further VRE investment should be directed to zones with a lower SPC. One solution to discouraging excessive VRE to locate behind transmission constraints is to offer non-firm connections for any VRE that results in increased congestion, equivalent to last-on first-off curtailment rule, with no compensation for a failure to export (the solution proposed by the Irish regulator) (CER, 2001).

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[17]Newbery (2018) shows how to calculate the globally desirable level of subsidy and Newbery (2020) derives values for the SEM case here, with a central estimate of 10% of the capital value.

[18]https://newatlas.com/energy/hysata-efficient-hydrogen-electrolysis/

5 Conclusions

This paper has developed a simple model of VRE curtailment to give a shadow price of carbon, SPC, which can be interpreted as the target-consistent cost of carbon once the electricity system has made the necessary complementary investments (in transmission, storage and interconnectors) to equalize the SPC of various investments. The model has been calibrated for the Single Electricity Market (SEM) of the island of Ireland for its 2026 target wind penetration of 55%, with a base case contrasted with an ambitious case that aims to increase storage, interconnection, and raise SNSP (the penetration above which the system becomes insecure). The key driver of the SPC is the observation that the ratio of the marginal abatement to the average abatement of VRE is typically 3-4, gradually tending towards 2 at very high penetration levels (but at which point the marginal abatement will have reached very high levels). To put this in perspective, unless the SEM can improve it capacity to absorb surplus VRE, by about 2026 with an average curtailment of 13%, the marginal curtailment of 48% would make the delivered cost of the marginal MWh of VRE twice the nominal cost.

The SPC appears to be highly sensitive to actions taken to reduce curtailment, as a comparison of the ambitious and base case for the SEM demonstrates, as is the rate at which the SPC increases with further VRE investment. It is also highly sensitive to the cost of fossil generation and the cost of the VRE. The rather cautionary conclusion is that, at least in small island systems like the SEM, and by extension, larger island system like Britain, the SPC of full electricity decarbonization (the UK 2035 target) could be prohibitively high, and far higher than the cost of decarbonizing other sectors more rapidly. In contrast, storage and the wider-area VRE averaging offered by interconnection will become increasingly lower cost ways of reducing CO_2 emissions, as are other complemetary policies (R&D, demand side flexibility and other socio-technical innovations and policies designed "target emissions reductions with very high abatement costs" (World Bank, 2021, p. 9). Cheap hydro storage (multi-month dams) become hugely attractive but are in limited geographical availability (Newbery, 2023).

Appendix Linear curtailment functions

The linear curtailment function is

$$k = \alpha(W - W_0)(1 - y/y^*), \tag{10}$$

with W_0 the level of wind at which curtailment first appears. Differentiating (10) totally gives

$$\frac{dk}{dW} = \frac{\partial k}{\partial W} + \frac{\partial k}{\partial y^*}\frac{dy^*}{dW} = \alpha(1 - y/y^*) + \alpha(W - W_0)\frac{y}{y^{*2}}\frac{dy^*}{dW}. \tag{11}$$

Totally differentiating (2) and from (4) gives:

$$\frac{dy^*}{dW} = -\frac{\partial k/\partial W}{\partial k/\partial y} = \frac{y^*}{W - W_0}. \tag{12}$$

Substitute (12) in (11) to give

$$\int_0^{y^*} \frac{dk}{dW}dy = \int_0^{y^*} \left(\alpha(1 - y/y^*) + \alpha y/y^*\right)dy = \alpha y^*, \tag{13}$$

while total curtailment is just the area of the triangle under the linear curtailment schedule (half base times height)

$$\int_0^{y^*} k\,dy = \alpha(W - W_0)y^*/2.$$

The ratio of the marginal to the average curtailment is

$$\frac{W \int_0^{y*} \frac{\partial k}{\partial W}dy}{\int_0^{y*} k\,dy} = \frac{2W}{W - W_0} > 2, \tag{14}$$

For a reference level of wind, W_r, and associated fraction of time curtailed, y_r^*, around which to consider deviations, simple geometry (see Figure 2) on the function (10) gives

$$y^*(W) = y_r^* \frac{W - W_0}{W_r - W_0}. \tag{15}$$

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